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Fuzzy Relational Equation Based on Triangular Norms to Detect the Problems of Bangladeshi Weavers

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ABSTRACT

In this work, Fuzzy relational equation $P \circ Q = R$ is considered as model, where P is an $m \times n$ fuzzy matrix formed by taking the t-norms "minimum", "algebraic product", "bounded difference" and "drastic product" (respectively) between the attributes related to the labourers and the attributes related to the owners of the labourers's, Q is an $n \times 1$ fuzzy matrix formed by the owner's expectations and R is the resultant matrix giving the status of the bonded labourers. Here "$\circ$" means max-T composition. In this work we consider T as t-norms "minimum", "algebraic product", "bounded difference" and "drastic product" one after another. Finally, we have given a comparative study among the results of application of all t-norms used here.

Keywords: Fuzzy relational equation, minimum, algebraic product, bounded difference, drastic product.

1. Introduction

Since its introduction by Sanchez [9] in 1976, fuzzy relational equations have been studied by many researchers [3-4], [6] in the theoretical aspects. Pappis and Adamopoulos [2] have contributed in the computational aspects. Lettieri and Liguori [4-5] have investigated fuzzy relational equations provided with one solution. Pedrycz [8] and Vasantha Kandasamy and Smarandache [10] have worked with a goal to applicational aspects and Eli Sanchez [19] has investigated solution of fuzzy relational equations with extended operations. Alamet el. [1] has done the same work on cottage industry labourers using 'minimum'. But here we have used the t-norms 'minimum($\wedge$)', 'algebraic product(*)", "bounded difference" and drastic product respectively to measure the effect of globalization on Bangladeshi weavers.

2. Inspiration of this work

For last few years, we worked on fuzzy relational equation (FRE), To do so, we were looking for an empirical study on FRE with main focus on Triangular norms. In our

observation, we see that some researchers [7,10] worked on the application of FRE but they were completely dependent on expert's or reviewer's opinions to form fuzzy relational matrix. As a result of which, the fuzzy matrices, for same information, vary due to unlike comments by the reviewers. Situation like this, leads us to drive a method to form fuzzy relational matrices based on Triangular norms.

Here, we studied weaver's problems and the effect of globalization on their lives using Fuzzy Relational Equations (FRE). We have arrived at interesting conclusions to understand and assay this grave problem.

3. Technique of study

We have made a sample survey of around 30 families out of 200 families. They are interviewed using a linguistic questionnaire. Some of the notable facts about their lives are as follows:

1. They do not know any other trade or work but most of them like to learn some other work.
2. They are living now below the poverty line because of the advent of electrical or power looms which has drastically affected their income.
3. The whole family works for over 10 hours.
4. Only one had completed his school final. All others have no education for they have to learn the trade while very young.
5. They don't have even a square meal a day.
6. Owners of the bonded labourers are not able to give enough work to these labourers.
7. The maximum salary a family of 3 to 4 members is around Tk.5000. 20% only get this, 70% of the families get below Tk.4000 per month.
8. Paying as rent, electricity, water etc makes them live below poverty line.

The following attributes are taken as the main point for study:

B_1-Knowledge of other works.

B_2-The effect of advent of power looms and globalization (modern textile machinery) on their economy.

B_3-Salary they earn in a month.

B_4-Level of savings or debts.

B_5-Government interferes.

B_6-Hours of work.

We have taken these six heads $B_1, B_2, B_3, B_4, B_5, B_6$ related to the bonded labourers as the rows of the fuzzy relational matrix.

The main attributes O_1, O_2, O_3, O_4 related to the owners of the bonded labourers are:

O_1- Globalization/ introduction of modern textile machines.

O_2- Profit or loss.

O_3- Availability of raw materials.

O_4- Demand for finished goods.

We have taken these four heads O_1, O_2, O_3, O_4 as the columns of the fuzzy relational matrix. Then by operating pairwise these two sets of attributes $\{B_1, B_2, B_3, B_4, B_5, B_6\}$ and$\{O_1, O_2, O_3, O_4\}$ with the help of t-norms 'minimum ($\wedge$)', 'algebraic product(*)", "bounded difference" and drastic product respectively we find the fuzzy relational matrix P.

The following are the grading policy and limit sets using the questionnaire:

$B_1 = \frac{1}{n+1}$;where n=numbers of work one knows.

$B_1 \geq 0.5$ Means no knowledge of other work, hence live in poverty.

B_2: Good(0), Bad(0.5) and Worse(1)

$B_2 \geq 0.5$ implies power loom/other textile machinery had made their condition from bad to worse.

$B_3 = \frac{n}{8000}$; where n= monthly income in Bangladeshi taka.

$B_3 \geq 0.5$implies earning is mediocre. ($B_3 < 0.5$implies the earning does not help them to meet both ends)

$B_4 = \frac{1}{1+\frac{E}{I}}$;where E =amount of expenditure per month, I = amount of income per month.

$B_4 = 0.5$ implies no saving no debt.($B_4 < 0.5$ implies they are in debt).

B_5: Not helped(1), Medium level helped(0.5) and Highly helped(0)

$B_5 \geq 0.5$ implies government interference has not helped. ($B_5 < 0.5$ implies Government interference have helped).

$B_6 = \frac{n}{10}$; where$n$ = working hours.

$B_6 = 0.5$ implies 5 hours work. ($B_6 > 0.5$implies more than 5 hours of work).

O_1: Highly affected (1), Medium affected (0.5) and Not affected (0)

$O_1 \geq 0.5$ implies the globalizations/ government has affected the owners of the bonded labourers drastically ($O_1 < 0.5$ has no impact on owners).

$O_2 = \frac{1}{1+\frac{C}{R}}$; where C =Total cost and R =Total revenue.

$O_2 = 0.5$ implies no profit or no loss ($O_2 < 0.5$ implies loss).

$O_3 = \frac{1}{n+1}$; n =Number of shortage item.

$O_3 \geq 0.5$ implies availability of raw materials. ($O_3 < 0.5$ implies shortage of raw materials).

$O_4 = \frac{D}{D+S}$; where D = Demand for finished goods and S = Produced goods.

$O_4 = 0.5$ implies just they can meet both ends i.e. demand for finished goods and produced goods balance. ($O_4 < 0.5$ implies demand is less than production).

4. Uses of t-norms

For case 1: The opinion of a group of labourers who are bonded for the two generations aged in sixties and their owners is given vital importance and their (labourers and owners) opinion is transformed into the FRE, we obtain

$$O_1 = 0.80 \quad O_2 = 0.41 \quad O_3 = 0.50 \quad O_4 = 0.50$$

$$P = \begin{matrix} B_1 = 0.50 \\ B_2 = 0.70 \\ B_3 = 0.50 \\ B_4 = 0.48 \\ B_5 = 0.70 \\ B_6 = 0.90 \end{matrix} \begin{pmatrix} 0.5 & 0.41 & 0.5 & 0.5 \\ 0.7 & 0.41 & 0.5 & 0.5 \\ 0.5 & 0.41 & 0.5 & 0.5 \\ 0.48 & 0.41 & 0.48 & 0.48 \\ 0.7 & 0.41 & 0.5 & 0.5 \\ 0.8 & 0.41 & 0.5 & 0.5 \end{pmatrix}$$

By considering the profit, suppose the owners give values for Q where $Q^T = (0.6, 0.5, 0.7, 0.5)$. Now P and Q are known in the fuzzy relational equation $P \circ Q = R$. In order to find the result of these compositions we introduce Mathematica program code as follows:

```
In[75]:= maxmincomposition[A_, x_] :=
          Module[{AA, BB, p}, AA = A; BB = x;
           n = Dimensions[AA][[1]]; m = Dimensions[BB][[2]];
           For[i = 1, i ≤ n, i++,
            For[j = 1, j ≤ m, j++, p = {};
             Do[{d = Min[AA[[i, k]], BB[[k, j]]];
               g = Join[p, {d}]; p = {g}; t[i, j] = Max[g]},
              {k, 1, m}];
             tt = Table[t[i, j], {i, 1, n}, {j, 1, m}]]];
           Print["PoQ=", MatrixForm[tt]]]
```

```
In[76]:= P = {{.5, .41, .5, .5}, {.7, .41, .5, .5},
    {.5, .41, .5, .5}, {.48, .41, .48, .48},
    {.7, .41, .5, .5}, {.8, .41, .5, .5}};
  Q = {{.6}, {.5}, {.7}, {.5}};
  maxmincomposition[P, Q]
```

$$P \circ Q = \begin{pmatrix} 0.5 \\ 0.6 \\ 0.5 \\ 0.48 \\ 0.6 \\ 0.6 \end{pmatrix}$$

We get $R^T = (0.5 \quad 0.6 \quad 0.5 \quad 0.48 \quad 0.6 \quad 0.6)$

For the same data, we use the t-norm algebraic product and get

$$Q_1 = 0.80 \quad Q_2 = 0.41 \quad Q_3 = 0.50 \quad Q_4 = 0.50$$

$$P = \begin{matrix} B_1 = 0.50 \\ B_2 = 0.70 \\ B_3 = 0.50 \\ B_4 = 0.48 \\ B_5 = 0.70 \\ B_6 = 0.90 \end{matrix} \begin{pmatrix} 0.4 & 0.21 & 0.25 & 0.25 \\ 0.56 & 0.29 & 0.35 & 0.35 \\ 0.4 & 0.21 & 0.25 & 0.25 \\ 0.38 & 0.20 & 0.23 & 0.23 \\ 0.56 & 0.29 & 0.35 & 0.35 \\ 0.72 & 0.37 & 0.45 & 0.45 \end{pmatrix}$$

Considering $Q^T = (0.6, 0.5, 0.7, 0.5)$, R^T will be $(0.24, 0.34, 0.24, 0.23, .34, 0.43)$.
The Mathematica programming code for the t-norm "algebraic product" are as follows:

Mathematica Program 2:

Similarly, we have used the t-norms "bounded difference" and "drastic product" consequently, we get $(0, 0.11, 0, 0, 0.11, 0.30)^T$ and $(0, 0, 0, 0, 0, 0)^T$ respectively as R.In the fuzzy relational equation $P \circ Q = R$, Q is the profit, the owners expect and R is the resultant giving the status of the bonded labourers. When we assume the owners are badly affected by globalizations but wants to carry out his business with no profit or no loss, with moderate or good availability of the raw material and they have enough demand or demand andsupply balance. We obtain the following attributes related with the bonded labourers. The bonded labourers life in acute poverty as they have no other knowledge of any other work. The power loom has made their live from bad to worst but the earning is medium with no savings and little debts. They do not receive any help from government,

but they have to labor almost ten hours which is given by $(0.5 \quad 0.6 \quad 0.5 \quad 0.48 \quad 0.6 \quad 0.6)^T$ $(0.24, 0.34, 0.24, 0.23, 0.34, 0.43)^T$, $(0, 0.11, 0, 0, 0.11, 0.30)^T$ and $(0, 0, 0, 0, 0, 0)^T$ respectively.

Using the same matrix P and taking the expected views of the bonded labourers $R = (0.6 \quad 0.4 \quad 0.5 \quad 0.5 \quad 0.2 \quad 0.8)^T$, we obtain $Q = (0.8 \quad 0.41 \quad 0.5 \quad 0.5)^T$, $(0.31, 0.16, 0.19, 0.19)^T$, $(0.5, 0.11, 0.2, 0.2)^T$, $(0, 0, 0, 0)^T$ using FRE $P^T \circ R = Q$.

The value of Q states the owners are affected by globalization. They have no profit but loss. They do not get enough raw materials to give to the bonded labor as the market prefers machine woven saris to handmade ones, so the demand for the finished goods declines. Thus the main reason for their poverty is due to globalization. That is, the advent of power looms has not only affected them drastically as they do not have the knowledge of any other trade but it has also affected the lives of their owners.

```
Maxprod[A_, x] :=
 Module[{AA, BB, P}, AA = A; BB = x;
  n = Dimensions [AA] [[1]];
  m = Dimensions [BB] [[2]]
    For[i = 1, i ≤ n, i++,
     For[j = 1, j ≤ m, j++, p = {};
      Do[{d = AA[[i, k]] * BB[[k, j]];
        g = join[p, {d}]; p = {g}; t[i, j] = Max[g]},
       {k, 1, m}];
      tt = Table[t[i, j], {i, i, n}, {j, i, m}]]];
  print["P∘Q=", MatrixForm[tt]]]
```

For case 2: A small owner's (who own around ten bonded labourers families) opinion is taken by the matrix P:

$$Q_1 = 0.85 \quad Q_2 = 0.40 \quad Q_3 = 0.40 \quad Q_4 = 0.45$$

$$P = \begin{array}{r|cccc} B_1 = 0.67 & 0.67 & 0.40 & 0.40 & 0.45 \\ B_2 = 0.80 & 0.80 & 0.40 & 0.40 & 0.45 \\ B_3 = 0.44 & 0.44 & 0.40 & 0.40 & 0.44 \\ B_4 = 0.47 & 0.47 & 0.40 & 0.40 & 0.45 \\ B_5 = 0.80 & 0.80 & 0.40 & 0.40 & 0.45 \\ B_6 = 0.1 & 0.85 & 0.40 & 0.40 & 0.45 \end{array}$$

Consider the profit expectation of the owner $Q^T = (0.6, 0.5, 0.7, 0.5)$. From FRE

$P \circ R = Q$, we obtain $R = (0.6, 0.6, 0.44, 0.47, 0.6, 0.6)^T$.Bonded labourers live in below poverty, as they have no other trade but the earning is below average with no savings and new debts. They do not get any help from government, but they have to work almost ten hours a day which is given by $(0.6, 0.6, 0.44, 0.47, 0.6, 0.6)^T$.If we consider a satisfactory norms for bounded labourers is to be $R = (0.6, 0.4, 0.5, 0.5, 0.2, 0.8)^T$, then using the relational equation $P^T \circ R = Q$, we get $Q = (0.8, 0.4, 0.4, 0.45)^T$, which states the owners are badly affected by globalization. They have no profit but loss, they do not get enough raw materials and demand for the finished goods declines.

For case 3: We have chosen very poor bonded labourers. Then the fuzzy relational matrix P takes the form

$$Q_1=1.00 \quad Q_2=0.38 \quad Q_3=0.39 \quad Q_4=0.43$$

$$P= \begin{matrix} B_1=0.71 \\ B_2=0.90 \\ B_3=0.38 \\ B_4=0.44 \\ B_5=0.85 \\ B_6=1.00 \end{matrix} \begin{pmatrix} 0.71 & 0.38 & 0.39 & 0.43 \\ 0.90 & 0.38 & 0.39 & 0.43 \\ 0.38 & 0.38 & 0.38 & 0.38 \\ 0.44 & 0.38 & 0.39 & 0.43 \\ 0.85 & 0.38 & 0.39 & 0.43 \\ 1.00 & 0.38 & 0.39 & 0.43 \end{pmatrix}$$

By considering the owner expectation of profit, $Q = (0.6, 0.5, 0.7, 0.5)^T$, from $P \circ Q = R$, we find $R = (0.6, 0.6, 0.38, 0.44, 0.6, 0.6)^T$. This reveals that the bonded labourers standard of living is in a very pathetic condition. They do not have any other source of income or job. Their earning is bare minimum with no savings. Neither the government comes forward to help them nor redeem them from their bondage. In their work spot, they have to slog for 10 hours per day.

Again if we consider a satisfactory norms for bonded labourers, $R = (0.6, 0.4, 0.5, 0.5, 0.2, 0.8)^T$, we get $Q = (0.8, 0.38, 0.39, 0.43)^T$ from the relation equation $P^T \circ R = Q$. The value of Q states due to the impact of globalization (modern textile machinery), the owners are badly affected. They are not able to purchase enough raw materials and thus the output from the industry declines. The owners do not get any profit but eventually end up in a great loss.

5. A comparative study among the result after applying the t-norms used here

Charts for the Comparison among the t-norms are given below:

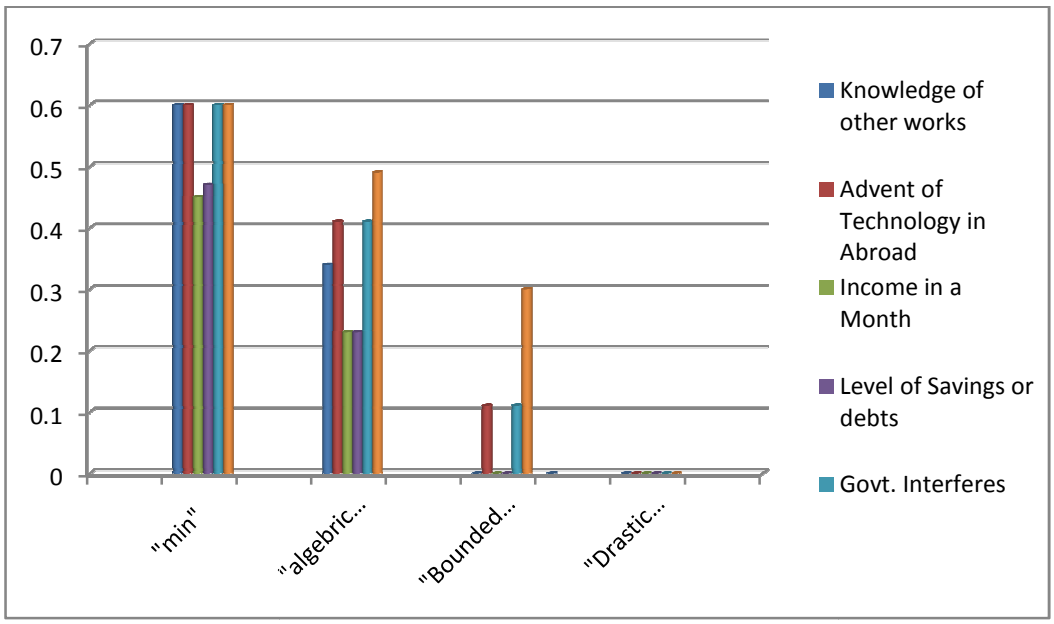

Figure 1. Status of labor's for t-norms "min", "algebraic product", "bounded difference" and "drastic product"

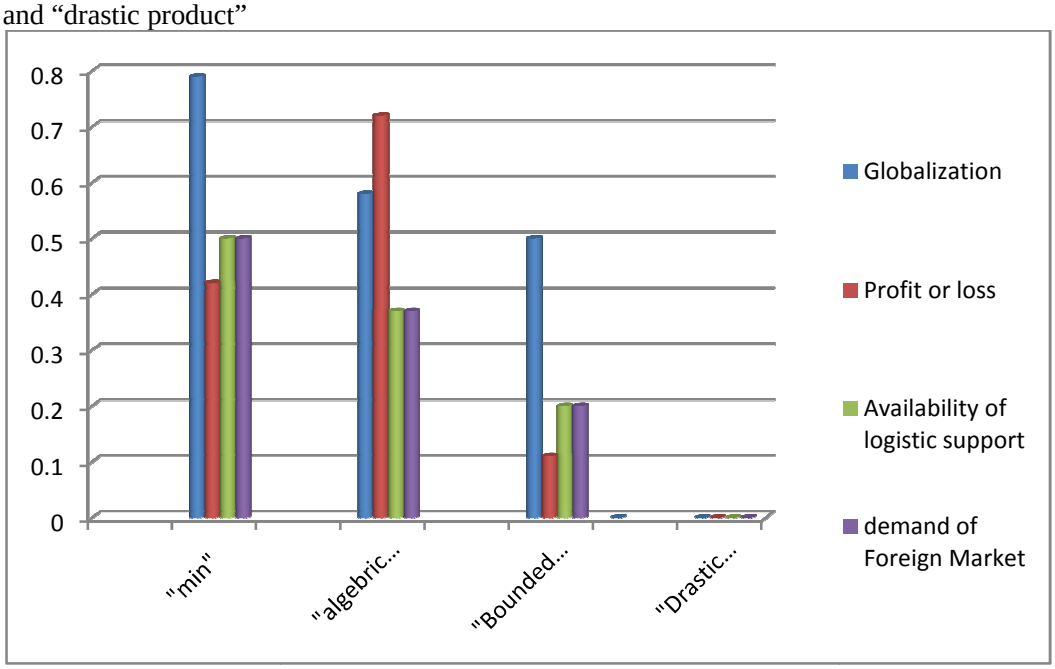

Figure 2. Status of the owner for t-norms "min", "algebraic product", "bounded difference" and "drastic product

6. Conclusion

A fuzzy model of relational equation based on Triangular norms has been proposed to find the status of Bangladeshi weavers. The procedure to form fuzzy relational matrices is more scientific here i.e. our technique is significantly more accurate than the methods which depend on expert's or reviewer's opinions. We are currently working to reduce the system error to improve the accuracy of the model.

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