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Constant- Stress Partially Accelerated Life Tests on Masked Series Systems under Progressive Type II Censoring

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ABSTRACT

In this paper, we discuss the parametric estimation of the constant-stress partially accelerated life test on masked series system, where complementary exponential distributed lifetimes are assumed for the components. Based on the progressive type II censored and masked system life data, the maximum-likelihood estimates (MLEs) of the unknown parameters and acceleration factor are derived. Also, the Bayesian estimates (BEs) of the unknown parameters and the acceleration factor are obtained by using Gibbs sample algorithm and adaptive rejection sampling method under independent symmetrical triangular priors and Gamma prior respectively. The effectiveness of MLEs and BEs are compared through the Monte Carlo simulation under different masking levels and censoring schemes.

Keywords: Constant-stress partially accelerated life test, series systems, masked data, complementary exponential distribution, progressive Type-Ⅱ censoring, parametric estimation, Gibbs sampling

1. Introduction

The failure data from multi-component systems are often used to analysis the reliability of component. Usually, the system failure data contain the failure time and the information on the exact component causing the system failure. In some cases, however, due to lack of proper diagnostic equipment or cost and time constraints, the exact component causing the system failure is not identified, and the failure cause is isolated to a subset of the system components. Such type of data is called masked data. Recently, the statistical analysis for masked data has been studied by several authors. Usher&Hodgson [1] considered parameter estimation for the series system with exponential components under masked data. Sarhan [2] studied Bayesian estimates of component reliabilities under the condition that system components have constant failure rates. Sarhan &Kundu [3] introduced Bayesian estimators for the reliability measures of masked system, and two-sided probability intervals of the parameters are derived. Related work can also be found in [4]-[6].

With the development of modern manufacturing technology, some products have become long lifetime and highly reliable. It is difficult to obtain the lifetime data of such products under normal operating conditions. However, accelerated life testing (ALT) can provide information quickly onthe lifetime of the products by testing them at higher than nominal levels of stress. Some studies of masked data under ALT have been developed in the literature (e.g., [7]-[11]). Based on the ALT to analyze the life characteristics of products, we need to use the relationship between product life and stress levels, that is, acceleration model. But in engineering practice, the accelerated model is not always known, such as the newly developed products. In order to solve this problem, the partially accelerated life test (PALT) is more suitable to be performed [12]. Under the PALT, Abd el al.[13] and Ismail[14][15]derived the maximum likelihood estimates(MLE) and Bayesian estimates (BE) of the acceleration factor and unknown parameters for different lifetime distribution. But the above research does not involve masked system. Thus, this paper presents constant-stress PALT (CSPALT) on masked series systems under progressive Type II censoring.

The rest of the paper is organized as follows. Model description and basic assumptions are given in Section 2. The likelihood function, MLEs and BEs of the unknown parameters and acceleration factor are derived in section 3 and Section 4, respectively. The effectiveness of MLEs and BEs are compared through the Monte Carlo simulation under different masking levels and censoring schemes in Section 5. Section 6 provides some brief concluding for the paper.

2. Some assumptions and model description

Assume that n series systems are placed on the CSPLAT, and each system has J components. Among the n systems, n_k are tested under the stress level S_k, $k=0,1$, where S_0 is use stress level and S_1 is accelerated stress level, $n = n_0 + n_1$. Progressive type-II censoring is applied as follows: For the life test subjected to the stress level S_k, at the first failure time $X_{(k1)}$, R_{k1} systems are randomly removed from the remaining $n_k - 1$ systems. Similarly, at the second failure time $X_{(k2)}$, R_{k2} systems from the remaining $n_k - 2 - R_{k1}$ systems are randomly removed. The test continues until the m_k th failure time $X_{(km_k)}$, all remaining $R_{km_k} = n_k - m_k - \sum_{R_{ki}}^{m_k - 1} R_{ki}$ systems are removed and the test terminates, where m_k, R_{ki}, $k=0,1$ and $i = 1,2,\cdots,m_k$ are pre-fixed numbers. From the test, we can obtain the life data $X_{(k1)} < X_{(k2)} < \ldots < X_{(km_k)}$, $k=0,1$. It is clear that the complete sample and type-II censored samples are special cases of this scheme. In the masked system lifetime data, the exact failure causes are often unknown. Instead, a subset of components is responsible for the failures. Let S_{ki} denote the minimum random set that possibly cause the system i to fail. Thus, the observed data can be expressed as $(X_{(k1)}, S_{k1})$, $(X_{(k2)}, S_{k2}), \ldots, (X_{(km_k)}, S_{km_k})$, $k=0,1$. For the aforementioned test the following assumptions are made.

A1. Masking is S-independent of the failure cause.

A2. Under the stress level S_k, the lifetime of component j in system i is denoted by X_{kij}, $k=0,1$, $i=1,2,\cdots,n_k$, $j=1,2,\cdots,J$, and $X_{ki} = \min\{X_{ki1}, X_{ki2}, \cdots, X_{kiJ}\}$ is the lifetime of system i.

A3. Under the use stress level, the lifetime of component j in each system follows the complementary exponential distribution(CED) with the probability distribution function (PDF), the survival function (SF) and hazard rate function (HRF):

$f_{0j}(x)=\lambda_j x^{-2}\exp\{-\lambda_j/x\}$, $\overline{F_{0j}}(x)=1-\exp\{-\lambda_j/x\}$, $h_{0j}(x)=\lambda_j x^{-2}\exp\{-\lambda_j/x\}\left(1-\exp\{-\lambda_j/x\}\right)^{-1}$,

where $x>0, \lambda_j>0, \ j=1,\cdots,J$

A4. The tampered failure rate (TFR) model holds: the hazard rate of the component j under accelerated stress level S_1 is $h_{1j}(x)=\beta h_{0j}(x)$, where β is the acceleration factor, $\beta>1$. Thus, under the accelerated condition the PDF, SF and HRFof the component j are obtained, for $j=1,2,\cdots,J$, respectively, by

$$f_{1j}(x)=\beta\lambda_j x^{-2}\exp\{-\lambda_j/x\}\left(1-\exp\{-\lambda_j/x\}\right)^{\beta-1}, \overline{F_{1j}}(x)=\left(1-\exp\{-\lambda_j/x\}\right)^{\beta},$$

$$h_{1j}(x)=\beta\lambda_j x^{-2}\exp\{-\lambda_j/x\}\left(1-\exp\{-\lambda_j/x\}\right)^{-1}, \quad \text{where } x>0, \lambda_j>0, j=1,2,\cdots,J.$$

3. Maximum likelihood estimation

For convenience, we let $X_{ki}=X_{(ki)}$, and x_{ki} denote the observe value of X_{ki}, $k=0,1$ and $i=1,2,...,n_k$. Based on the observed data from CSPALT with type II progressive censoring, we obtain the likelihood function as follows

$$L(\lambda_1,\lambda_2,\lambda_3,\beta)\propto\prod_{k=0}^{1}\left\{\prod_{i=1}^{m_k}\left[\sum_{j\in s_{ki}}\left[f_{kj}(x_{ki})\prod_{l=1,l\neq j}^{J}\overline{F_{kl}}(x_{ki})\right]\right]\cdot\left[\prod_{j=1}^{J}\overline{F_{kj}}(x_{ki})\right]^{R_{ki}}\right\} \tag{1}$$

In this paper, we only consider J=3. Let $\theta=(\lambda_1,\lambda_2,\lambda_3,\beta)$, and $|\psi|$ be the number of the elements for any set ψ, and K_{ki} denote the index of the component actually causing the i-th system to fail under the stress level S_k, Set $A_{kj}=\{i\,\|s_{ki}|=1,K_{ki}=j\}$ as the set of the failed systems without masked under the stress level S_k, $i=1,2,...,m_k$ and $k=0,1$, $j=1,2,3$ .Let $T^1=\{1,2\},T^2=\{1,3\}$, $T^3=\{2,3\}$, $T^4=\{1,2,3\}$,where the element m in set T^l represent the component m ($m=1,2,3$). We let $M_k^l=\{i\,\|s_{ki}|>1,s_{ki}=T^l\},k=0,1,l=1,2,3,4$ .Then likelihood function can be written as

$$L(\lambda_1,\lambda_2,\lambda_3,\beta)\propto\prod_{k=0}^{1}\left\{\prod_{j=1}^{3}\prod_{i\in A_{kj}}h_{kj}(x_{ki})\cdot\prod_{l=1}^{4}\prod_{i\in M_k^l}[\sum_{j\in s_{ki}}h_{kj}(x_{ki})]\cdot\prod_{i=1}^{m_k}[\prod_{j=1}^{J}\overline{F_{kj}}(x_{ki})]^{R_{ki}+1}\right\} \tag{2}$$

The logarithm of the likelihood function is given by

$$\begin{aligned} l\propto\sum_{k=0}^{1}\Bigg\{&\sum_{j=1}^{3}[\sum_{i\in A_{kj}}\phi_k+\ln\lambda_j-\lambda_j/x_{ki}-\ln\left(1-\exp(-\lambda_j/x_{ki})\right)] \\ &+\sum_{l=1}^{4}\sum_{i\in M_k^l}[\phi_k+\ln[\sum_{j\in s_{ki}}(\lambda_j/x_{ki}^2)\exp(-\lambda_j/x_{ki})\left(1-\exp(-\lambda_j/x_{ki})\right)^{-1}]] \\ &+\exp(\phi_k)\cdot\sum_{i=1}^{m_k}(1+R_{ki})\cdot\sum_{j=1}^{3}\ln\left(1-\exp(-\lambda_j/x_{ki})\right)\Bigg\}, \end{aligned} \tag{3}$$

where $\phi_k=0, as\ k=0, \phi_k=\ln\beta, as\ k=1$.

The likelihood equation of the parameters $\lambda_j, j=1,2,3.$ and β are given by

$$\frac{\partial l}{\partial \lambda_j}=\sum_{k=0}^{1}\left\{\sum_{i\in A_{kj}}1/\lambda_j-1/x_{ki}-\exp(-\lambda_j/x_{ki})/x_{ki}\left(1-\exp(-\lambda_j/x_{ki})\right)+\sum_{l\in l_{(j)}}\sum_{k\in M_k^l}h_j^1\left(x_{ki}\right)/\sum_{p\in s_{ki}}h_{kp}\left(x_{ki}\right)\right\}$$
$$+\sum_{i=1}^{m_0}\left(1+R_{0i}\right)\exp(-\lambda_j/x_{0i})/x_{0i}\left(1-\exp(-\lambda_j/x_{0i})\right)+\beta\sum_{i=1}^{m_1}\left(1+R_{1i}\right)\exp(-\lambda_j/x_{1i})/x_{1i}\left(1-\exp(-\lambda_j/x_{1i})\right)=0, \tag{4}$$

where $h_j\left(x_{ki}\right)=(\lambda_j/x_{ki}^2)\exp(-\lambda_j/x_{ki})\left(1-\exp(-\lambda_j/x_{ki})\right)^{-1}$, $l_{(j)}=\{l\,|\,j\in T^l\}$, $l=1,2,3,4.$ and

$$h_j^1\left(x_{ki}\right)=\frac{\partial h_j\left(x_{ki}\right)}{\partial \lambda_j}=(1/x_{ki}^2)\exp(-\lambda_j/x_{ki})\left(1-\exp(-\lambda_j/x_{ki})\right)^{-2}[1-\exp(-\lambda_j/x_{ki})-\lambda_j/x_{ki}] \tag{5}$$

$$\begin{aligned}\frac{\partial l}{\partial \beta}&=\sum_{j=1}^{3}\sum_{i\in A_{1j}}\frac{1}{\beta}+\sum_{l=1}^{4}\sum_{i\in M_1^l}\frac{1}{\beta}+\sum_{i=1}^{m_1}\left(1+R_{1i}\right)\sum_{j=1}^{3}\ln\left(1-\exp(-\lambda_j/x_{1i})\right)\\&=\frac{m_1}{\beta}+\sum_{i=1}^{m_1}\left(1+R_{1i}\right)\sum_{j=1}^{3}\ln\left(1-\exp(-\lambda_j/x_{1i})\right)=0\end{aligned} \tag{6}$$

From Eq. (4), the MLE $\hat{\lambda}_j(j=1,2,3)$ can be obtained by using the Newton–Raphson methods. From Eq. (6), we can obtain the MLE $\hat{\beta}$ as follows

$$\hat{\beta}=-m_1/[\sum_{i=1}^{m_1}\left(1+R_{1i}\right)\sum_{j=1}^{3}\ln(1-\exp\{-\hat{\lambda}_j/x_{1i}\})]. \tag{7}$$

4. Bayesian estimation

In this subsection, the BE of the parameters $\lambda_1,\lambda_2,\lambda_3$ and β under squared error loss (SEL) is derived by their posterior expectation, respectively. Assume that $\lambda_1,\lambda_2,\lambda_3$, β independent of each other. The prior distribution of $\hat{\lambda}_j(j=1,2,3)$ is taken as the symmetrical triangle distribution on a positive interval $B_j=[a_j,b_j]$ and the PDF of λ_j can be written as

$$\pi_j(\lambda_j)=\varepsilon_j^{-2}[\varepsilon_j-|\lambda_j-\mu_j|],\qquad \lambda_j\in B_j, \tag{8}$$

where $\mu_j=(b_j+a_j)/2,\varepsilon_j=(b_j-a_j)/2,\quad j=1,2,3$. The prior distribution of β is taken as Gamma distribution with the parameters (a, b), that is

$$\pi\left(\beta\right)=[b^a\Gamma\left(a\right)]^{-1}\beta^{a-1}\exp(-\beta/b),\beta>1 \tag{9}$$

Hence, the joint prior distribution of $(\lambda_1,\lambda_2,\lambda_3,\beta)$ is

$$\pi(\lambda_1,\lambda_2,\lambda_3,\beta)=[b^a\Gamma\left(a\right)]^{-1}\beta^{a-1}\exp(-\beta/b)\prod_{j=1}^{3}\varepsilon_j^{-2}[\varepsilon_j-|\lambda_j-\mu_j|],(\lambda_1,\lambda_2,\lambda_3)\in B_1\times B_2\times B_3,\beta>1. \tag{10}$$

For the convenience to study the full conditional distribution, we introduce the latent variables $Z_{i^{(k)}j}^l$. Let $Z_{i^{(k)}j}^l=\begin{cases}1, & if\ K_{ki}=j,j\in T^l\\0, & otherwise\end{cases}$, where $i^{(k)}=1,2,...,m_k,l=1,2,3,4,j=1,2,3.$

According to the assumption A1 and the definition for the $Z_{i^{(k)}j}^l$, under stress level S_k, the conditional probability of system i fails caused by component j is

$$\mathrm{P}_{i^{(k)}j}^l(\theta)=h_{kj}\left(x_{ki}\right)/\sum_{j\in T^l}h_{kj}\left(x_{ki}\right),\qquad j\in T^l,i\in M_k^l,l=1,2,3,4. \tag{11}$$

Hence for system $i\in M_k^l$, the latent random vector $Z_k=\left(Z_{i^{(k)}1}^l,Z_{i^{(k)}2}^l,Z_{i^{(k)}3}^l\right)$, $k=0,1,$ follows the multinomial distribution, it can be written as

$$(Z_{i^{(k)}1}^l,Z_{i^{(k)}2}^l,Z_{i^{(k)}3}^l)\sim M(1,\mathrm{P}_{i^{(k)}1}^l(\theta),\mathrm{P}_{i^{(k)}2}^l(\theta),\mathrm{P}_{i^{(k)}3}^l(\theta)),i\in M_k^l$$

Let $Z=(Z_0,Z_1)$, then the likelihood function in Eq. (1) can be rewritten as

$$L\left(data\,|\,\theta,\mathrm{Z}\right)\propto\prod_{k=0}^{1}\{\prod_{j=1}^{3}\prod_{i\in A_{kj}}\varphi_k(\lambda_j/x_{ki}^2)\exp(-\lambda_j/x_{ki})(1-\exp(-\lambda_j/x_{ki}))^{-1}$$
$$\cdot\prod_{l=1}^{4}\prod_{i\in M_k^l}\prod_{j\in s_{ki}}\left[\varphi_k(\lambda_j/x_{ki}^2)\exp(-\lambda_j/x_{ki})(1-\exp(-\lambda_j/x_{ki}))^{-1}\right]^{Z^l_{i(k)j}}\}$$
$$\cdot\prod_{i=1}^{m_0}\left[\prod_{j=1}^{3}(1-\exp(-\lambda_j/x_{ki}))\right]^{R_{0i}+1}\cdot\prod_{i=1}^{m_1}\left[\prod_{j=1}^{3}(1-\exp(-\lambda_j/x_{ki}))^{\beta}\right]^{R_{1i}+1},\qquad(12)$$

where $data=\{(X_{k1},S_{k1}),\ (X_{k2},S_{k2}),\dots,(X_{km_k},S_{km_k})\}$, $\phi_k=1, if\ k=0, \phi_k=\beta, if\ k=1$ .The joint posterior PDF of the parameters $\theta=(\lambda_1,\lambda_2,\lambda_3,\beta)$ can be expressed as

$$\pi^*(\theta\,|\,data,\mathrm{Z})\propto L(data\,|\,\theta,Z)\cdot\pi(\theta)$$
$$\propto\prod_{j=1}^{3}\left\{\prod_{k=0}^{1}\left[\lambda_j^{|A_{kj}|}\exp\left(-\lambda_j\cdot T_{kj}\right)\prod_{i\in A_{kj}}\left(1-\exp(-\lambda_j/x_{ki})\right)^{-1}\prod_{l\in I_{(j)}}\prod_{i\in M_k^l}\left((1-\exp(-\lambda_j/x_{ki}))^{-1}\right)^{Z^l_{i(k)j}}\right]\right.$$
$$\left.\cdot\prod_{i=1}^{m_0}(1-\exp(-\lambda_j/x_{ki}))^{R_{0i}+1}\cdot\prod_{i=1}^{m_1}(1-\exp(-\lambda_j/x_{ki}))^{\beta(R_{1i}+1)}\cdot\left(\varepsilon_j-\left|\lambda_j-\mu_j\right|\right)\right\}$$
$$\cdot\beta^{m_0+m_1+a-1}\cdot\exp\left\{-\beta\left[\frac{1}{b}-\sum_{i=1}^{m_1}\left(R_{1i}+1\right)\sum_{j=1}^{3}\ln(1-\exp(-\lambda_j/x_{ki}))\right]\right\},\qquad(13)$$

where $T_{kj}=\sum_{i\in A_{kj}}1/x_{ki}$. For $v=1,2,3,4$, we define $\theta_{-v}=\{\theta_\delta:1\le\delta\le4,\delta\ne \mathrm{v}\}$, then the full conditional posterior distribution of λ_j and β can be obtained as

$$\pi_j^*\left(\lambda_j\,|\,\theta_{-j},data,\mathrm{Z}\right)\propto\prod_{k=0}^{1}\left[\lambda_j^{|A_{kj}|}\exp\left(-\lambda_j\cdot T_{kj}\right)\prod_{i\in A_{kj}}(1-\exp(-\lambda_j/x_{ki}))^{-1}\prod_{l\in I_{(j)}}\prod_{k\in M_k^l}\left((1-\exp(-\lambda_j/x_{ki}))^{-1}\right)^{Z^l_{i(k)j}}\right]$$
$$\cdot\prod_{i=1}^{m_0}(1-\exp(-\lambda_j/x_{ki}))^{R_{0i}+1}\cdot\prod_{i=1}^{m_1}(1-\exp(-\lambda_j/x_{ki}))^{\beta(R_{1i}+1)}\cdot(\varepsilon_j-\left|\lambda_j-\mu_j\right|),\quad j=1,2,3.$$

$$\pi_4^*\left(\beta\,|\,\theta_{-4},data\right)\propto\beta^{m_0+m_1+a-1}\cdot\exp\left\{-\beta\left[\frac{1}{b}-\sum_{i=1}^{m_1}\left(R_{1i}+1\right)\sum_{j=1}^{3}\ln(1-\exp(-\lambda_j/x_{ki}))\right]\right\}.$$

We can see that β follows the Gamma distribution. That is,

$$\beta\sim Gamma\left(m_0+m_1+\mathrm{a},\left[\frac{1}{b}-\sum_{i=1}^{m_1}\left(R_{1i}+1\right)\sum_{j=1}^{3}\ln(1-\exp(-\lambda_j/x_{ki}))\right]^{-1}\right).$$

Let $\xi(\theta)=\xi(\lambda_1,\lambda_2,\lambda_3,\beta)$. Under the Squared error loss function, the BE of $\xi(\theta)$ is given by

$$\hat{\xi}(\theta)=E_{\theta|data,\mathrm{Z}}\left[\xi(\theta)\right]$$
$$=\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty L(data\,|\,\theta,Z)\cdot\pi(\theta)\cdot\xi\left(\theta\right)d\lambda_1d\lambda_2d\lambda_3d\beta\Big/\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty L(data\,|\,\theta,Z)\cdot\pi(\theta)d\lambda_1d\lambda_2d\lambda_3d\beta\qquad(14)$$

It can be seen that the Bayes estimation of ξ has complicated integrals which cannot be given in simple closed form. So that numerical methods must be used for the computations. We propose the Gibbs sample algorithm to compute BE of ξ as follows:

Step 1 Start with $\theta^{(0)}=(\lambda_1^{(0)},\lambda_2^{(0)},\lambda_3^{(0)},\beta^{(0)})$.

Step 2 Set p=1, generate $\beta^{(i)}$ from Gamma distribution $\pi_4^*(\beta\,|\,\lambda_1^{(p-1)},\lambda_2^{(p-1)},\lambda_3^{(p-1)},data)$.

Step 3 Generate $Z^{(p)}=(Z_0^{(p)},Z_1^{(p)})$ from the multinomial distribution of

$M(1, P^{l}_{i^{(k)}1}{}^{(p)}(\theta), P^{l}_{i^{(k)}2}{}^{(p)}(\theta),$
$P^{l}_{i^{(k)}3}{}^{(p)}(\theta)), k=0,1, i \in M_k^l$. The $P^{l}_{i^{(k)}j}(\theta)$ is calculated by the Eq. (11) using values of $\lambda_1^{(p-1)}, \lambda_1^{(p-1)}, \lambda_1^{(p-1)}, \beta^{(p)}$.

Step 4 Generate $\lambda_j^{(p)}$ $(j=1,2,3)$ by the adaptive rejection sampling method proposed by Gilks and Wild [16]

1) Generate $\lambda_1^{(p)}$ from the function $\pi_1^*\left(\lambda_1 \mid \lambda_2^{(p-1)}, \lambda_3^{(p-1)}, \beta^{(p)}, data, Z^{(p)}\right)$.

2) Generate $\lambda_2^{(p)}$ from the function $\pi_2^*\left(\lambda_2 \mid \lambda_1^{(p)}, \lambda_3^{(p-1)}, \beta^{(p)}, data, Z^{(p)}\right)$.

3) Generate $\lambda_3^{(p)}$ from the function $\pi_3^*\left(\lambda_3 \mid \lambda_1^{(p)}, \lambda_2^{(p)}, \beta^{(p)}, data, Z^{(p)}\right)$.

Step 5 Set p=p+1

Step 6 Repeat step3-5 N times

Finally, we obtain the approximate mean of $\xi(\theta)$ as regards posterior distribution is

$$E(\xi \mid data) = \frac{1}{N-M}\sum_{p=M+1}^{N} \xi\left(\lambda_1^{(p)}, \lambda_2^{(p)}, \lambda_3^{(p)}, \beta^{(p)}\right), \tag{15}$$

where M is the burn-in period, which is the number of iterations before the stationary distribution is achieved. When $\xi(\theta)=\lambda_j, j=1,2,3,$ and $\xi(\theta)=\beta$, the BE of these parameters can be obtained from Eq. (15) respectively.

5. Simulation study

In this section, a Monte Carlo simulation study is conducted to illustrate the performance of the estimates. Assume that 50 series systems are put on the CSPALT and each containing three-components from the CED with the parameters $\lambda_1=1, \lambda_2=1.2, \lambda_3=0.7$ respectively with $\beta=1.5, n_0=20, m_0=12, n_1=30, m_1=20$. We consider the following three different progressive censored schemes:

CS [1]: $R_{0i}=\begin{cases}1, & i=3,4,\dots,10\\ 0, & otherwise\end{cases}$, $R_{1v}=\begin{cases}1, & v=6,7,\dots,15\\ 0, & therwise\end{cases}$; CS [2]: $\begin{cases}R_{01}=8\\ R_{0i}=0, i\neq 1\end{cases}$, $\begin{cases}R_{11}=10\\ R_{1v}=0, v\neq 1\end{cases}$;

CS [3]: $\begin{cases}R_{0,12}=8\\ R_{0i}=0, i\neq 12\end{cases}$, $\begin{cases}R_{1,20}=10\\ R_{1v}=0, v\neq 20\end{cases}$, where $i=1,2,\dots,m_0, v=1,2,\dots,m_1$.

The simulation study is performed according to the following steps:

1) Based on the values of n_k , generate random samples from $F_{0j}(x)=1-\overline{F}_{0j}(x)$ and $F_{1j}(x)=1-\overline{F}_{1j}(x)$, $(X_{ki1}, X_{ki2}, X_{ki3})$, $j=1,2,3,$ $k=0,1, i=1,2,\dots,n_k$. Set $X_{ki}=\min(X_{ki1}, X_{ki2}, X_{ki3})$.
2) For the values of m_k , according to different masking level p and progressive censoring schemes, generate the ordered samples $(X_{(k1)}, S_{k1}), (X_{(k2)}, S_{k2}), \dots, (X_{(km_k)}, S_{km_k})$, $k=0,1$, which represent two type-II progressive censored samples from CED under constant PALT.
3) Using the method presented in section 3, compute the MLEs of parameters $\lambda_1, \lambda_2, \lambda_3$ and β respectively.
4) Assume that $\lambda_1, \lambda_2, \lambda_3$ have symmetrical triangular prior density functions on the intervals $B_1=[0.5, 2.5]$, $B_2=[0.2, 2.0]$, $B_3=[0, 1.2]$ respectively, and β has the prior

distribution $Gamma(2, 0.27)$, the BE of $\lambda_1, \lambda_2, \lambda_3$ and β can be obtained by using (15). We set the masking level $p = 0, 0.4, 0.8$, respectively, the steps 1)–4) are repeated 2000 times, and compute the average estimates (AE)of the MLEs and BEs of the unknown parameters and mean squared errors (MSE) for these estimates, see Table 1.

Par	Cs	p=0		p=0.4		p=0.8	
		MLE(MSE)	BE(MSE)	MLE(MSE)	BE(MSE)	MLE(MSE)	BE(MSE)
λ_1	[1]	1.1244(0.0165)	1.1677(0.0980)	1.0367 (0.0646)	1.1482 (0.0575)	1.2219 (0.1091)	1.1158 (0.1045)
	[2]	0.9082(0.0218)	1.1340(0.0194)	1.2235 (0.0704)	1.2019 (0.0656)	1.1809 (0.0975)	1.1022 (0.0923)
	[3]	0.9398 (0.0237)	1.1742(0.0195)	1.0867 (0.0732)	1.1771 (0.0603)	1.2165 (0.0918)	1.1794 (0.0876)
λ_2	[1]	0.9350(0.0587)	0.9688(0.0451)	1.1342 (0.0765)	0.9886 (0.0649)	1.2269(0.1140)	0.9575 (0.0974)
	[2]	1.0853(0.0462)	1.0412(0.0357)	0.9578 (0.0820)	0.9902 (0.0780)	1.0544 (0.0984)	0.9527 (0.0889)
	[3]	1.1837 (0.0520)	0.9533(0.0397)	1.2870 (0.0809)	1.0000 (0.0758)	1.3917(0.1021)	0.9792 (0.0987)
λ_3	[1]	0.7476(0.0162)	0.6244(0.0134)	0.6364 (0.0697)	0.5980 (0.0764)	0.8977(0.1048)	0.5903 (0.1002)
	[2]	0.7446(0.0099)	0.6557(0.0091)	0.6073 (0.0791)	0.5935 (0.0727)	0.6667 (0.1086)	0.5763 (0.1015)
	[3]	0.6418 (0.0107)	0.6167(0.0085)	0.7584 (0.0812)	0.5929 (0.0785)	0.8235(0.1085)	0.5916 (0.0959)
β	[1]	1.3233(0.0169)	1.8376(0.0147)	1.3828 (0.0738)	1.7761 (0.7016)	1.3621(0.1263)	1.8493 (0.1098)
	[2]	1.4926(0.0279)	1.5997(0.0214)	1.5149 (0.0829)	1.8623 (0.0791)	1.5802 (0.1145)	1.8127 (0.1050)
	[3]	1.5831 (0.0347)	1.5596(0.0301)	1.5776 (0.0985)	1.6989 (0.0894)	1.4214(0.1098)	1.7700 (0.1021)

Table 1. MLEs,BEs and MSEs for the parameters $(\lambda_1, \lambda_2, \lambda_3, \beta)$ at $\lambda_1 = 1, \lambda_2 = 1.2, \lambda_3 = 0.7, \beta = 1.5$

From table 1, it can be observed that the MSEs of the MLE and BE approach are smaller under different masking levels and censoring schemes. In addition, the Bayesian estimates yields smaller MSEs than the MLE approach. However, as the masking level increases the accurate of the both estimation methods are less poor in terms of the MSEs since less information among components is available.

5. Conclusions

In this article, we consider the constant-stress partially accelerated life test on series system with masked data under progressive Type II censoring. Both the MLEs and BEs of the unknown parameters and acceleration factor are derived. The effectiveness of two methods are compared through the Monte Carlo simulation under different masking levels and censoring schemes. The results show that BEs has more accurate results than the MLE approach.

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