Synopsis of the Ph.D. thesis entitled

Study of Some Fluid Dynamics Problems with or without Magnetic Field

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Chapter 1

Introduction

The thesis has been devoted to study of some fluid dynamical problems with or without magnetic field. The effects of Hall current, suction or blowing or slip condition on momentum and heat transfer characteristics in some of these flows have been studied in detail. An investigation is also made on the flow of viscous electrically conducting incompressible fluid past on porous flat plate or disk in rotating environment.

The study of magnetohydrodynamics, the interaction between the magnetic field and the flow of an electrically conducting fluid, received its first importance in connection with astrophysical problems. There are two serious technological applications of MHD, that may both become very important in the future. First, strong magnetic fields may be used to confine rings or columns of hot plasma (A plasma is a hot, ionized gas containing electrons and ions) that will be held in place long enough for thermonuclear fusion to occur and for net power to be generated. In the second application, which is directed toward a similar goal, liquid metals are driven through a magnetic field in order to generate electricity. The study of magnetohydrodynamics is also motivated by its widespread application to the description of space (within the solar system) and astrophysical plasmas (beyond the solar system). However, the major use of MHD is in plasma physics. Later it has found numerous applications in problems of geophysics, power-generation, space-research, thermonuclear fusion, aeronautic and any other engineering fields. Hall effect is likely to be important in many astrophysical situations as well as in the flows of laboratory plasma and in MHD power generation. Hartmann [1] initiated the study of the subject in the name Hg-dynamics in his efforts to pump mercury by exploitation of hydrodynamical pressure and electromagnetic fields. The systematic study under the present name began with discovery of transverse waves by Alven [2] while he was engaged in the theoretical investigations of sunspots. The study of magnetohydrodynamics in astrophysical and geophysical problem have been made by many authors, Cowling [3], Chandrasekhar [4], Bullard [5] and many others.

The study of the MHD flow of viscous incompressible fluid in a rotating frame of reference has drawn considerable interest in recent years due to its frequent applications in designing thermo syphon tube, in cooling turbine blades etc.. The effects of the magnetic field and the rotation on the unsteady hydromagnetic flow due to a rotating disks is an important related to many practical applications, such as boundary layer flow control. Several investigations have been carried out on various types of MHD flow in a rotating frame of reference. Seth et al. [6] studied the effect of rotations on the unsteady MHD Couette flow when the velocity of the moving plate varies as t^n, t and n being the time and any positive number respectively. Mazummder [7] studied an oscillatory Couette flow bounded by two infinite horizontal plates, one of which is oscillating about a non-zero constant mean velocity in its own plane and the other at rest. Recently Guria et al. [9] studied the start-up Couette flow in a rotating frame of reference for small as well as large time where the frictional layer of the upper plate is suddenly set into the motion with uniform velocity. Das et al.[10] has analyzed the unsteady MHD Couette flow in a rotating system. To study of a MHD rotating fluid it is a decisive importance to a flow pattern as studied by Hayat et. al [11]. Guria et al. [12] have studied the unsteady Couette flow in a rotating system. Mohanty [13] has studied the magnetohydrodynamic flow between eccentric rotating disks with the same angular velocity, assuming that the induced magnetic field is smaller than the applied magnetic field. The unsteady viscous flow between eccentric disks has been investigated by Erdogan [14]. Unsteady flow due to concentric rotation of eccentric rotating disks has been studied by Ersoy [15]. Rao and Kasiviswanathan [16] have consider the flow of an incompressible viscous fluid between two eccentric rotating disks. Rajagopal [17]has studied the flow of visco-elastic fluids between rotating disks. Ghosh and Pop [18] give an analytical approach on MHD plasma bahaivor of a rotating enviroment in the presence of an inclined magnetic field as compared to excitation frequency. The effects of wall conductance on MHD fully developed flow with asymmetric heating of the wall has been studied by Guria et al. [19].

Beside these, the study of hydromagnetic viscous flows with Hall currents has important engineering applications in problems of magnetohydrodynamic generators and of Hall accelerators as well as in flight magnetohydrodynamics. In an ionized gas where the density is low and/or the magnetic field is very strong, the conductivity normal to the magnetic field is reduced due to the free spiraling of electrons and ions about the magnetic lines of force before suffering collisions and a current is induced in a direction normal to both the electric and the magnetic fields. This phenomenon, well known in the literature, is called the Hall effect. The effects of Hall current is likely to be important in many astrophysical situations as well as the flows of plasma through MHD power generator. Hall effects on the hydromagnetic flow have been studied by Sato [20], Yamanishi [21], Sherman and Sutton [22], Datta and Jana [23]. Katagiri [24] discussed the effect of Hall currents on the boundary layer flow past a semi-infinite flat plate. Gupta [25] studied the effect of Hall currents on the steady magnetohydrodynamic flow of an electrically conducting fluid past an infinite porous flat plate. Hall effects on the unsteady Couette flow have been studied by Jana and Datta [26]. Maji et al.[27] studied the Hall effects on hydromagnetic flow on an oscillatory porous plate. Murthy and Ram [28] studied the magnetohydrodynamic flow due to eccentric rotations of a porous disk and a fluid at infinity. Hall effects on unsteady flow of a viscous fluid due to non-coaxial rotation of a porous disk and a fluid at infinity has been discussed by Guria et al.[29] . The unsteady flow due to eccentric rotations of a disk and a fluid at infinity which are started impulsively has been studied by Pop[30]. The unsteady flow of a viscous fluid due to non-coaxial rotations of a disk and a fluid at infinity have been studied by Kasiviswanathan and Rao [31] and Erdogan [32, 33, 34, 35]. Combined effect of Hall currents and bouyancy forces on the MHD forced convective flows have not received much attention. Hall effects on the free convective flow of an electrically conducting fluid in a vertical channel have been studied by Datta and Jana [36]. Effects of wall conductance on MHD fully developed flow with asymmetric heating of the wall has been studied by Guria et al. [37]. The effect of the slip condition on unsteady flow due to non-coaxial rotations of disk and a fluid at infinity has been studied by Ashar et al.[38]. Ghosh and Pop [39, 40] have discussed the Hall effects on MHD Couette flow in a rotating environment.

Heat transfer is the energy interaction due to a temperature difference in a medium or between media. Heat is not a storable quantity and is defined as energy in transit due to a temperature difference. The applications of heat transfer are diverse, both in nature and industry. Climatic changes, formation of rain and snow, heating and cooling of the earth's surface, spreading of forest fires are some of the natural phenomena wherein heat transfer plays a dominant role. The temperature control applications of heat transfer include cooling of electron equipments such as personal computers and supercomputers, cooling of nuclear reactor cores, electronic chips in a tightly packaged set of electronic circuits and the outer surface of space vehicles during re-entry. Heat transfer from a heated moving or non-moving surface to a quiescent (a fluid at rest) ambient medium occur in may manufacturing processes such as hot rolling, wire drawing and crystal growing. The heat treatment of materials travelling between a feed roll and a wind-up roll or on conveyor belts, the lamination and melt-spinning processes in the extraction of polymers possess the characteristics of moving continuous surfaces. The term heat transfer encompasses all phenomena occurring in the transport of quantity of heat from one point in space to another. It has wide range application in high speed air craft, re-entry vehicles and cooling of rotating turbine blades.

This transport can take place in three different modes - conduction, convection and radiation. Conduction of heat is the process of heat transfer through direct constant of individual particles of a solid or fluid at different temperature. Convection is not a separate mode of heat transfer. It describes a fluid system in motion, and heat transfer occurs by the mechanism of conduction alone. Obviously, we must allow for the motion of the fluid system in witting an energy balance, but there is no new basic mechanism of heat transfer involved. If the fluid is at rest, the problem reduces to simple conduction where there are temperature gradients normal to the interface. However, if the fluid is in motion, heat is transported both by simple conduction and by the movement of the fluid itself. This complex transport process is referred to as convection. A heat transfer occurring in fluid in motion, in which the diffusion of thermal energy is affected by relative motion within the fluid is called convection. It is evident that the study of the phenomenon of thermal convection between a solid body and a fluid in motion involves a consideration of the Science of fluid mechanics. Heat transfer by thermal convection may be subdivided into two groups (i) Forced convection (ii) Free convection.

A literature survey reveals that the natural convection boundary layer flows past a hot vertical wall have been studied by several authors. An account must be taken to the study of Ghosh and Nandi[41], Sparrow and Cess [42], Riley [43], Kuiken [44] and Weidman [45]. The study of Aung and Worku [46] on mixed convection flow through the vertical channel with asymmetric heating of the wall have a great interest of subject based on similarity solutions. Aung [47] analyzed the fully developed laminar convection between vertical plates heated asymmetrically. Batchelor [48] has studied the heat transfer by free convection across a closed cavity between vertical boundaries at different temperatures. Ghosh and Nandi [49] have discussed magnetohydrodynamic fully developed combined convection flow between vertical plates heated asymmetrically.

The fundamental equations of MHD describe the motion of a conducting fluid in a magnetic field. This fluid is usually either a liquid metal or a plasma. These fundamental equations of MHD are the modified electrodynamics equations together with the modified hydrodynamic equations. The electrodynamic Maxwell's equations are unchanged whereas the Ohm's law which relates the electric current to the electric field has to be modified to include the induced current. The hydrodynamic momentum equation has to be modified to include the Lorentz force and the modified energy equation has to include the Jule dissipation. We summaries below the basic equations for the flow of an electrically conducting viscous incompressible fluid.

The equation of continuity is

$$\nabla \cdot \vec{q} = 0, \tag{1}$$

where $\vec{q}$ is the fluid velocity vector. The equation of continuity is a statement about the conservation of mass of fluid.

The momentum equation of MHD is

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla)\vec{q} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \overrightarrow{q} + \frac{1}{\rho}(\vec{j} \times \vec{B}), \tag{2}$$

where $\vec{q}$, ρ, ν, p, $\vec{j}$ and $\vec{B}$ are respectively the fluid velocity vector, the fluid density, the kinematic coefficient of viscosity, the modified fluid pressure including centrifugal force, the currents density vector and magnetic induction vector. The term $(\vec{q} \cdot \nabla)\vec{q}$ represents convective acceleration, ∇p the pressure gradient which represents the stress in fluid and $\nu \nabla^2 \vec{q}$ the viscous force. The equation (2) is the well-known Navier-Stokes equation.

The Maxwell's equations are

$$\nabla \times \vec{B} = \mu_e \vec{j} \ \text{ (Ampere's law)}, \tag{3}$$

$$\nabla \times \vec{E} = 0 \ \text{ (Faraday's law of induction)}, \tag{4}$$

$$\nabla \cdot \vec{B} = -\frac{\partial \vec{B}}{\partial t} \ \text{ (Gauss'law for magnetism)}, \tag{5}$$

$$\nabla \cdot \vec{j} = 0 \ \text{ (Conservation of electric charge)}. \tag{6}$$

Equation (3) is Amperes circuital law relating the magnetic field to its basic source, the electric current, displacement current being neglected and equation (5) expresses the fact that the magnetic field is solenoidal. Equation (4) is Faradays law of induction in its differential form. Equation (6) relates the electric field to the volume density of electric charge.

The magnetic induction equation is

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{q} \times \vec{B}) + \nu_m \nabla^2 \vec{B}, \tag{7}$$

where $\nu_m = \frac{1}{\sigma \mu_e}$ is the magnetic diffusivity (or resistivity) of the fluid. Equation (7) has a marvellous generality; it holds quite independently of the particular dynamical forces generating the flow (e.g. whether these are of thermal or compositional origin, whether the Lorentz force is or is not important, whether Coriolis forces are present or not); it is holds also whether the fluid is incompressible or not. The set of equations which describe MHD effects are a combination of the momentum equations of fluid dynamics and the Maxwell's equations of electromagnetism. These differential equations are to be solved simultaneously.

The generalized Ohm's law including Hall currents into account for a moving conductor is

$$\vec{j} + \frac{\omega_e \tau_e}{H_0}(\vec{j} \times \vec{H}) = \sigma(\vec{E} + \mu_e \vec{q} \times \vec{H}), \tag{8}$$

where $\vec{j}$, $\vec{H}, \omega_e, \tau_e$ and σ denote the current density, the magnetic field vector, the cyclotron frequency, the electron collision time and the electrical conductivity of the fluid respectively. The term $\frac{\omega_e \tau_e}{H_0}(\vec{j} \times \vec{H})$ in equation (8) is known as Hall current.

The constitutive field equations are

$$\vec{B} = \mu_e \vec{H} \text{ and } \vec{D} = \epsilon \vec{E}, \tag{9}$$

$$\vec{j} = \sigma\left(\vec{E} + \vec{q} \times \vec{B}\right) \text{ (Ohm's law)}, \tag{10}$$

where $\vec{E}, \vec{D}$, μ_e, ρ, σ and ϵ are the the electric field vector, the displacement vector, the kinematic coefficient of viscosity, the magnetic permeability, the electrical conductivity and dielectric constant respectively.

According to Boussinesq

$$\rho = \rho_0 \left[1 - \beta(T - T_0)\right], \tag{11}$$

where β is the coefficient of thermal expansion of the fluid , T_0 the reference temperature and ρ_0 the density at temperature T_0. In well known Boussinesq approximation, the variation of ρ is taken into account only in the buoyancy term $\rho\, g$.

The energy equation is

$$\frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla)T = \frac{k}{\rho c_p} \nabla^2 T + \frac{\nu}{c_p} \Phi + \frac{\vec{j}^2}{\rho c_p \sigma}, \tag{12}$$

where c_p is the specific heat at constant pressure, k the thermal conductivity of the fluid. The second and third terms on the right-hand side of (12) represent the viscous and Joule dissipations respectively.

This thesis consists of the following problems:

Chapter 2
Unsteady MHD Couette flow in a rotating system[1]

The unsteady hydromagnetic Couette flow of a viscous incompressible electrically conducting fluid in a rotating system has been studied. Consider the unsteady flow of a viscous incompressible electrically conducting fluid between two infinite parallel plates when the fluid and the plates rotate in unison with uniform angular velocity Ω about an axis normal to the plates. Let d be the distance between the two plates, where d is small in comparison with the characteristic length of the plates. The upper plate moves with a uniform velocity U in its own plane in the x-direction, where x-axis is taken along the lower stationary plate. The z-axis is taken normal

[1] *Published in Mathematical and Computer Modelling 50 (2009) 1211-1217*

to the x-axis and the y-axis is taken normal to the xz-plane, lying in the plane of lower plate which is also assumed that the flow is fully developed as shown in Fig.2. Further, there is no applied pressure gradient as the flow is due to the motion of the upper plate. Since the plates are infinitely long along x and y- directions, all physical quantities will be functions of z and t only.

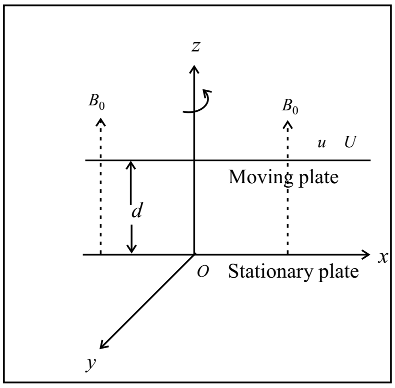

Fig.2: Geometry of the problem

The solutions for the velocity distributions as well as shear stresses have been obtained for small times as well as for large times using Laplace transform technique. It is found that for large times the primary velocity u_1 decreases with an increase in rotation parameter K^2 while it increases with an increase in magnetic parameter M^2. It is also found that with an increase in K^2, the secondary velocity v_1 decreases near the stationary plate while it increases near the moving plate. On the other hand, the secondary velocity decreases with an increase in M^2. Further, it is found that the series solution obtained for small times converges more rapidly than the general solution.

Chapter 3

Oscillatory Couette flow in the presence of an inclined magnetic field [2]

An investigation is made on the oscillatory MHD Couette flow of electrically conducting fluid between two parallel plates, of which the upper plate is held at rest and the lower plate oscillates non-torsionally in a rotating system in the presence of an inclined magnetic field. We consider the unsteady MHD Couette flow of a viscous incompressible electrically conducting fluid confined between two infinite parallel plates at a distance d apart, rotate in unison with uniform

[2] *Published in Meccanica 44(2009) 555-564*

angular velocity Ω about an axis perpendicular to the plates under the influence of an inclined magnetic field with the positive direction of the axis of rotation. The plates and fluid rotate in unison with reference to a rigid body rotation. We choose the Cartesian coordinate system in such a way that x- axis along the lower plate and z- axis perpendicular to it and y- axis is normal to xz plane [see Fig.3]. A uniform magnetic field of strength B_0 is applied at an angle θ with the axis of rotation. The flow is induced due to the non-torsional oscillations of the lower plate. Since the plates are infinite along x and y- directions, all physical quantities will be the functions of z and t only.

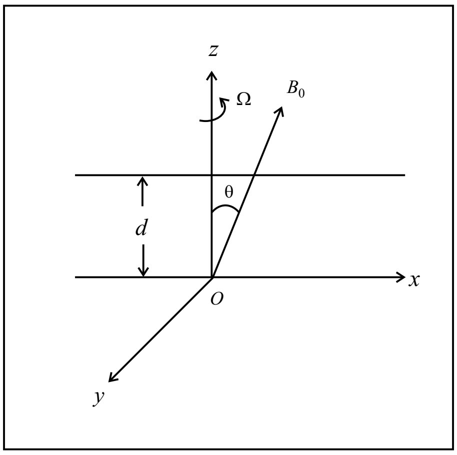

Fig.3: Geometry of the problem

The velocity field and the shear stress components at the plates are found exactly by using Laplace transform technique. It is found that both the velocities u_1 and v_1 increase with an increase in angle of inclination θ of the magnetic field. The main velocity u_1 decreases where as the cross velocity v_1 increases with an increase in the rotation parameter K^2. It is also found that the coriolis force acts as a constraint in its motion. It is evident that the effects of magnetic field tends to retarding the flow. The main and cross flows decrease with an increase in $\omega\,\tau$ for $\omega\tau \neq 0$. Asymptotic behavior of the solution is analyzed for square of Hartmann number $M^2 \ll 1$, $K^2 \ll 1$ and $\omega \ll 1$ and for large values of M^2, K^2 and frequency parameter ω. It is found that a thin boundary layer is formed near the lower plate, for large values of K^2, M^2 and ω. The thickness of this boundary layer increases with an increase in inclination of the magnetic field with the axis of rotation.

Chapter 4

Hall effects on unsteady flow of a viscous fluid due to non-coaxial rotation of a porous disk and a fluid at infinity[3]

The effects of Hall current on the unsteady hydromagnetic flow due to non-coaxial rotations of a porous disk and a fluid at infinity has been analyzed. We consider the unsteady flow of a viscous incompressible conducting fluid occupying the space $z > 0$ and is bounded by an infinite porous disk at $z = 0$. The axes of rotation of both the disk and that of the fluid at infinity to be in the plane $x = 0$. The distance between the axes of rotation is l. Initially, at $t = 0$, the disk and the fluid at infinity rotate about z'-axis with the same uniform angular velocity Ω. At time $t > 0$, the disk suddenly starts to rotate about z- axis with uniform angular velocity Ω while the fluid at infinity continues to rotating about z'- axis with the same angular velocity as that of the disk [see Fig.4]. A uniform magnetic field B_0 is applied perpendicular to the disk.

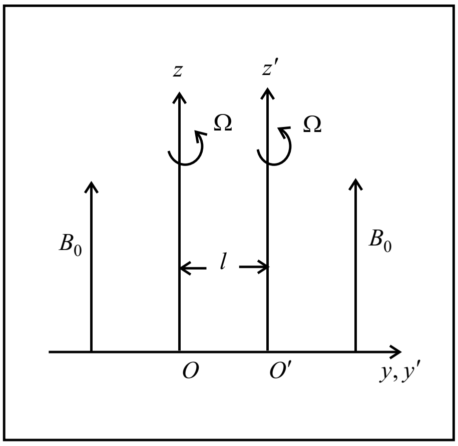

Fig.4: Geometry of the problem

An analytical solution of the governing equations describing the flow is obtained at small and large times after the start by the Laplace transform method. It is found that the primary velocity $\frac{f}{\Omega l}$ decreases while the secondary velocity $\frac{g}{\Omega l}$ increases with an increase in Hall parameter m. It is also found that for small times there is no inertial oscillations while for large times the steady state is reached through inertial oscillations. The frequency of these oscillations first increases, reaches a maximum and then decreases with an increase in m.

[3] *Published in International Journal of Non-Linear Mechanics 42 (2007) 1204-1209*

Chapter 5

Effects of Hall current and slip condition on unsteady flow of a viscous fluid due to non-coaxial rotation of a porous disk and a fluid at infinity[4]

An analysis is made on the effects of Hall current and slip condition on the unsteady hydromagnetic flow due to non-coaxial rotations of a porous disk and a fluid at infinity. We consider the unsteady flow of a viscous incompressible fluid occupying the space $z > 0$ and is bounded by an infinite porous disk at $z = 0$. A uniform magnetic field B_0 is applied perpendicular to the disk. The axes of rotation of both the disk and that of the fluid at infinity are to be in the plane $x = 0$. The distance between the axes of rotation is l [see Fig.5]. Initially, at $t = 0$, the disk and the fluid at infinity rotate about z'-axis with the same uniform angular velocity Ω. At time $t > 0$, the disk suddenly starts to rotate about z-axis with uniform angular velocity Ω while the fluid at infinity continues to rotate about z'-axis with the same angular velocity as that of the disk.

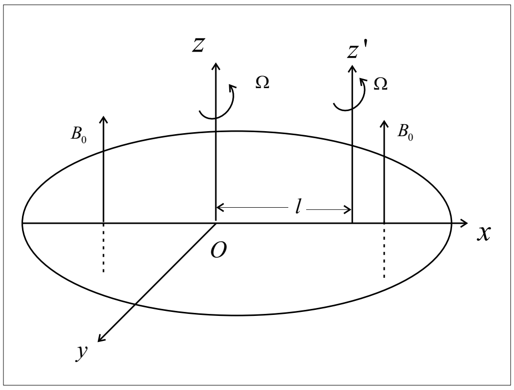

Fig.5: Geometry of the problem

The problem is solved using Laplace transform method. It is obtained that at a given location, the primary velocity $\frac{f}{\Omega l}$ decreases and the secondary velocity $\frac{g}{\Omega l}$ increases with an increase in Hall parameter m. These imply that the Hall current exerts a retarding influence on the primary flow. On the other hand, Hall currents accelerate the secondary flow. It is found that at any point, the primary velocity increases while the secondary velocity decreases with an increase in slip parameter λ. The asymptotic behavior of the flow has been analyzed for large values of time to highlight the transient approach to the steady state flow. It is found that for large values of time there exists a thin boundary layer near the disk. The thickness of this boundary layer

[4] *Published in Meccanica 45(2010) 23-32.*

decreases with an increase in either magnetic parameter M^2 or suction parameter S. It is also found that the time required to attain the steady state is less in the presence of Hall currents than that of without Hall currents. On the other hand, the secondary velocity takes more time to reach the steady state in the presence of Hall currents than in the absence of Hall currents.

Chapter 6

Hall effects on unsteady MHD flow between two disks with non-coincident parallel axes of rotation [5]

The Hall effects on the unsteady flow of a viscous incompressible electrically conducting fluid between two parallel disks, rotating with uniform angular velocity Ω about two different axes at a distance a apart has been studied. We consider the unsteady flow of a viscous incompressible electrically conducting fluid between two parallel disks, rotating with uniform angular velocity Ω about two different axes at a distance a apart. We choose a system of cylindrical polar coordinates (r, θ, z) with the axis normal to the disks as situated symmetrically between two axes of rotation. The axis of rotation of the disk $z = h$ lies to the right and that for the disk $z = 0$ lies to the left of the axis [see Fig.6]. A uniform magnetic field B_0 is applied perpendicular to the disks.

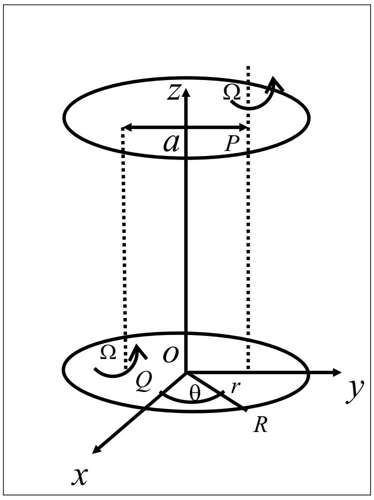

Fig.6: Geometry of the problem

The solutions for the velocity distributions as well as shear stresses are obtained for small time τ by applying Laplace transform technique. It is found that both the primary velocity and

[5] *Published in International Journal of Applied Mechanics and Engineering 15(1)(2010) 5-18.*

the secondary velocity decrease with an increase Hall parameter m, at the left to a disk from the axis of rotation whereas they increase with an increase in m, at the right to a disk from the axis of rotation. It is also found that the effect of Hall currents leads to a decrease in the torque on the disks with an increase in m. In the light of our present problem it is rigorously stated that there arises symmetric motion about the mid plane between the two disks as referred to a rigid body rotation.

Chapter 7

Hall effects on unsteady flow of a viscous fluid due to an accelerated plate

Hall effects on the hydromagnetic flow of a viscous incompressible electrically conducting fluid past a flat plate in the presence of a uniform transverse magnetic field have been analyzed. Let us consider a semi-infinite mass of a viscous incompressible conducting fluid bounded by an infinite flat plate occupying the plane $z = 0$. We introduce a cartesian coordinates as shown in Fig.7. Initially, at time $t \leq 0$, the plate is at rest and at time $t > 0$, the plate is impulsively started from rest and then moves with uniform acceleration in its own plane along x-axis. A uniform magnetic field H_0 acts along the z-axis and the plate is electrically non-conducting. The horizontal homogeneity of the problem shows that the physical quantities are function of z and t only, t being the time variable. The equation of continuity $\nabla \cdot \vec{q} = 0$ gives $w' = 0$ where $\vec{q} \equiv (u', v', w')$, u', v', w' being the velocity components along the coordinates axes.

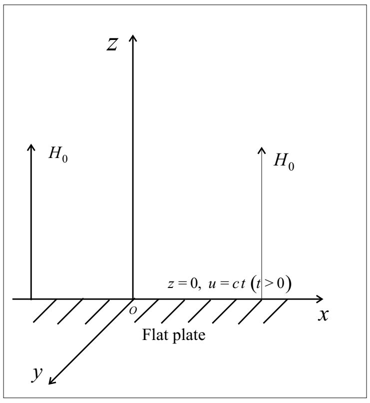

Fig.7: Geometry of the problem

The velocity field and the shear stress components at the plate are found exactly by using Laplace transform technique. The solution is also obtained for small as well as large time. It is seen that both the primary velocity u and magnitude of the secondary velocity v increase with increase in Hall parameter m. The asymptotic behavior of the solution is also analyzed for large times to highlight the transient approach to the final steady state. It is found that for large time there exists an inertial oscillations. The frequency of these oscillations first increases, reaches a maximum and then decreases with increase in m. This oscillatory behavior has not been seen in the absence of Hall currents.

Chapter 8

Combined effects of Hall current and wall conductance on MHD fully developed flow with asymmetric heating of the wall

The combined effects of Hall current and wall conductance on MHD fully developed flow of a viscous incompressible electrically conducting fluid through a vertical channel with asymmetric heating of the walls in the presence of a uniform transverse magnetic field have been studied. Let us consider the fully developed flow of an electrically conducting viscous incompressible fluid between infinitely long vertical conducting walls. Let the distance between the walls be L. The origin being taken at the left wall of the vertical channel, x- axis is along the walls, z-axis perpendicular to it and y- axis is normal to the $z\,x$-plane as shown in Fig.8. A uniform magnetic field B_0 is imposed perpendicular to the walls of the vertical channel. For fully developed steady flow all physical quantities, except pressure, will be function of y only. The flow is generated due to buoyancy force in the presence of a transverse magnetic field. It is assumed that the forced flow entering the channel is directed vertically upwards whereas the pure free convection is motivated by a zero pressure gradient.The equation of continuity $\nabla.\vec{q} = 0$ and the no-slip condition at the walls give $v' = 0$ everywhere in the flow where $\vec{q} \equiv (u', v', w')$. The solenoidal equation $\nabla.\vec{B} = 0$ gives $B_y = \text{constant} = B_0$ everywhere in the flow, where $\vec{B} \equiv (B_x, B_0, B_z)$.

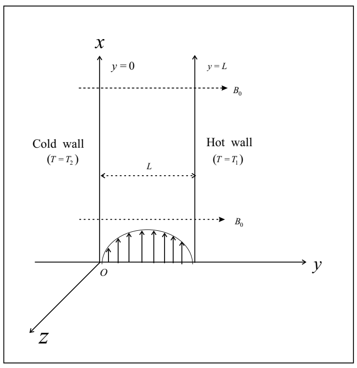

Fig.8: Geometry of the problem

The velocity and induced magnetic field distribution is obtained exactly. It is found that the primary velocity u decreases while the secondary velocity w increases with an increase in either Hall parameter m or wall conductance parameter ϕ. The induced primary magnetic field b_x decreases near the cold wall and increases in magnitude near the hot wall of the vertical channel with an increase in m. The effect of the Hall current on the secondary magnetic field b_z is reversed as that of primary magnetic field. The critical Grashof number at the left wall due to the primary flow decreases while that due to the secondary flow increases in either m or ϕ.

Chapter 9

MHD free convection between vertical walls

The steady MHD free convective flow of a viscous incompressible electrically conducting fluid between vertical walls heated asymmetrically in the presence of a uniform applied magnetic field has been analyzed. We consider a two-dimensional natural convective steady hydromagnetic fully developed flow of a viscous incompressible electrically conducting fluid confined between two vertical walls. The walls are at a distance d apart. We choose a cartesian co-ordinates system with x-axis in the upward direction in the direction of flow and the axis of y is taken perpendicular to it as shown in Fig.9 . A uniform magnetic field of strength H_0 is imposed perpendicular to the walls of the vertical channel. The origin of the axes is such that the channel walls are at positions $y = -\frac{d}{2}$ and $y = \frac{d}{2}$. The velocity components are (u, v) relative to the cartesian frame of reference. We do not model the pressure drop across the end caps and only consider the fully-developed flow far from the end caps.

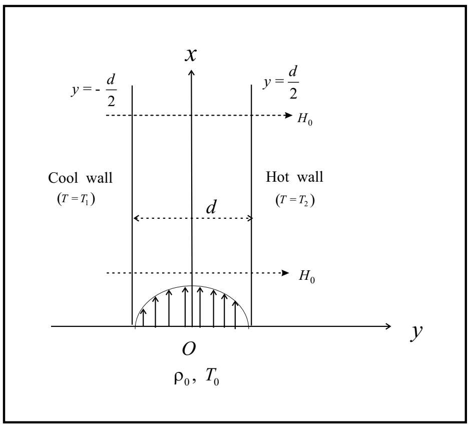

Fig.9: Geometry of the problem

The velocity field, induced magnetic field and the temperature distribution have been obtained in closed forms. It is found that the velocity decreases with increase in either magnetic parameter or the temperature parameter θ_0 whereas it increases with increase in Grashof number. It is perceived that an increase in Hartmann number leads to an increase in the velocity but decrease the temperature of the channel flow. The critical values of the temperature parameter at the cold wall, for which the flow reversal occurs near the cold wall have been obtained. It is observed that the critical values of the temperature parameter increases with increase in either magnetic parameter or Grashof number. Asymptotic behaviors of the solutions are analyzed for $M \ll 1$.

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