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Linear and Nonlinear Time Series Model Selection for Stationary Data Structure: Application to Monthly Rainfall Data in Nigeria

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ABSTRACT

Many available techniques for time series modeling assume linear relationship among variables. However in some situations, variations in data do not exhibit simple regularities and are difficult to model accurately. Linear relationship and their arrangements for describing the behaviour of such data are often found to be inadequate. Since many real life data are nonlinear, there is need to investigate which models can best captured data that are linear as well as those that are nonlinear. This paper examined the performances of the following nonlinear time series model: Self Exiting Threshold Autoregressive (SETAR), Smooth Transition Autoregressive (STAR) and Logistic Smooth Transition Autoregressive (LSTAR) models in fitting general classes of linear and nonlinear autoregressive cases at different sample sizes. The relative performances of the models were examined, within the context of stationarity, and compared with linear Autoregressive (AR). The LSTAR was the best as sample size was increased for different nonlinear autoregressive functions except in polynomial function where SETAR models out-performed others. The performances of the four fitted models increased when sample size was increased. Finally, we demonstrated the application of the models stated earlier on data of monthly rainfall in Nigeria between 1973-2013. SETAR model fitted best to the Rainfall data and LSTAR was the best when the data was transformed to nonlinear.

Keywords: *SETAR, STAR, LSTAR, AIC, MSE*

1. Introduction

A time series is a sequence of data points, measured typically at successive points at uniform time intervals. Time series data is an array of time and numbers. Data obtained from observations collected sequentially over time are extremely common. In business, we observe weekly interest rates, daily closing stock prices, monthly price indices, yearly sales figures, and so forth. In meteorology, we observe daily high and low temperatures, annual precipitation and hourly wind speeds. In agriculture, we record annual figures for crop and livestock production, soilerosion, and export sales. In the biological sciences, we observe the electrical activity of the heart at millisecond intervals. In ecology, we record the abundance of an animal species. The list of areas in which time series are studied is virtually endless. The purpose of time series analysis is generally of twofold: to understand or model the stochastic mechanism that gives rise to an observed series and to predict or forecast the future values of a series based on the history of that series and, possibly, other related series or factors.

Traditional time series analysis is based on assumptions of linearity and stationarity. However, there has been a growing interest in studying nonlinear and nonstationary time series models in many practical problems. The first and the simplest reason for this is that many real world problems do not satisfy the assumptions of linearity and/or stationarity. For example, the financial markets are one of the areas where there is a greater need to explain behaviours that are far from being even approximately linear. Therefore, the need for the further development of the theory and applications for nonlinear models is essential.

In general time series analysis, it is known that there are a large number of nonlinear features such as cycles, asymmetries, bursts, jumps, chaos, thresholds, heteroskedasticity etc. Types of models that can be cast into this form have been presented in my last seminar. See also Tong (1990), Granger and Ter¨asvirta (1993) and Franse, and van Dijk (2000) and Tsay(2010). Kim and Nelson (1999) provides a comprehensive account of different Markov switching models that have been used in economic and financial research.

In this study, we considered some linear and nonlinear time series models and investigate the performance of these models in fitting linear, trigonometry, exponential and polynomial forms of autoregressive function. The goodness of fit for each model with information criteria was considered in detail. A simulation study was carried out to verify the finite sample properties of the models for stationary data. The relative performance of each model were examined based on mean square error (MSE) and AIkaike Information Criteria (AIC).

1.1. Self-exciting threshold autoregressive (SETAR) model

The Threshold Autoregressive model can be considered as an extension of autoregressive models, allowing for the parameters changing in the model according to the value of an exogenous threshold variable S_{t-d}. If it is substituted by the past value of which means $S_{t-d} = Y_{t-d}$ then we call it Self-Exciting Threshold Autoregressive model (SETAR). Some simple cases that are considered in this study are shown as follows:

TAR Model

$$y_t = \begin{cases} \emptyset_0^1 + \emptyset_1^1 Y_{t-1} + \emptyset_2^1 Y_{t-2} + e_t^1 if S_{t-d} \le r \\ \emptyset_0^2 + \emptyset_1^2 Y_{t-1} + \emptyset_2^2 Y_{t-2} + e_t^2 if S_{t-d} > r \end{cases} \tag{5}$$

SETAR Model

$$y_t = \begin{cases} \emptyset_0^1 + \emptyset_1^1 Y_{t-1} + \emptyset_2^1 Y_{t-2} + e_t^1 if Y_{t-d} \le r \\ \emptyset_0^2 + \emptyset_1^2 Y_{t-1} + \emptyset_2^2 Y_{t-2} + e_t^2 if Y_{t-d} > r \end{cases} \tag{6}$$

where d is the delay parameter and r is threshold value , triggering the changes between two different regimes. These models can be applied to the time series data which has a regime switching behavior The threshold parameters satisfy the innovation within the i^{th} regime $e_t^i$is a sequence of identically independent normal random variables with zero mean and constant variance$\sigma_i^2 < \infty (i = 1, 2.)$. The overall process Y_t, is non-linear when there are at least two regimes with different linear models. The simplest class of TAR models is the Self Exciting Threshold Autoregressive (SETAR) models of order p introduced by Tong(1983) and specified to order 2 by equation 3 in this work. The popularity of SETAR models is due to their being relatively simple to specify, estimate, and interpret as compared to many other nonlinear time series models.

1.2. Smooth transition AR (STAR) model

A criticism of the SETAR model is that its conditional mean equation is not continuous. The thresholds (r_j) are the discontinuity points of the conditional mean function μ_t. In response to this criticism, smooth TAR models have been proposed; see Chan and Tong (1986) and Ter¨asvirta (1994) and the references therein. A time series Y_t follows a 2-regime STAR(p)model of the form

$$Y_t = c_0 + \sum_{i=1}^{p} \emptyset_{0.i} Y_{t-i} + F\left(\frac{Y_{t-d} - \Delta}{s}\right)\left(c_1 + \sum_{i=1}^{p} \emptyset_{1.i} Y_{t-i}\right) + e_t \qquad (7)$$

Where d is the delay parameter Δand s are parameters representing the location and scale of model transition, and F(·) is a smooth transition function. In practice, F(·) often assumes one of three forms—namely, logistic, exponential, or a cumulative distribution function. The conditional mean of a STAR model is a weighted linear combination between the following two equations:

$$\mu_{1t} = c_0 + \sum_{i=1}^{p} \emptyset_{0.i} Y_{t-i}$$

$$\mu_{2t} = (c_0 + c_1) + \sum_{i=1}^{p} (\emptyset_{0.i} + \emptyset_{1.i}) Y_{t-i}$$

The weights are determined in a continuous manner by F[($\frac{Y_{t-d} - \Delta}{s}$]. The prior two equations above also determine properties of a STAR model. For instance, a prerequisite for the stationary of a STAR model is that all zeros of both AR polynomials are outside the unit circle. An advantage of the STAR model over the TAR model is that the conditional mean function is differentiable. However, experience shows that the transition parameters Δand s of a STAR model are hard to estimate. In particular, most empirical studies show that standard errors of the estimates of Δ and s are often quite large, resulting in t ratios of about 1.0; see Terasvirta (1994). This uncertainty leads to various complications in interpreting an estimated STAR model.

1.3. Logistic smooth transition AR (LSTAR) model

A more general model of a logistic smooth transition autoregressive model of order p (LSTAR(P) model) is:

$$Y_t = F(\gamma, c; Y_{t-d}) = (1 + exp - \{\gamma(Y_{t-d} - d)\})^{-1} \quad (8)$$

The coefficient $\gamma, \gamma > 0$ is the smoothness parameter and the scalar c is the location parameter and d is known as the delay parameter, the variable Y_{t-d} is then called the transition variable for some d > 0 in model.

The main aim of this study, therefore, is to suggest simple linear and nonlinear models stated earlier that can be fitted to data generated from general classes of linear and nonlinear second order autoregressive model. Its performance in finite sample cases was evaluated by simulation.

2. Materials and methods

Simulation studies were conducted to investigate the performance of autoregressive, self exciting threshold autoregressive, Smooth transition autoregressive models and logistic Smooth transition autoregressive models for fitting different general classes of linear and nonlinear autoregressive time series earlier stated. Effect of sample size and the stationarity of the models were examined on each of the general linear and nonlinear data simulated. Each model is subjected to 1000 replication simulation at different sample sizes for stationary data structure.

2.1. Criteria for assessment of the study

The goodness of fit for each model was assessed using common two criteria in time series, mean square error and AIC. The model with lowest criteria is the best among the models for the simulated data.

Alkaike Information Criteria

There are several information criteria available to determine the best model of auto regressive process. All of them are likelihood based. For example, the well-known Akaike information criterion (AIC) (Akaike, 1973 cited by Tsay, 2010) is defined as

$$\text{AIC} = -\frac{2}{n}\ln\text{ (likelihood)} + \frac{2}{n}\text{(number of parameters)}$$

where the likelihood function is evaluated at the maximum-likelihood estimates and nis the sample size.

Mean Squared Error

The mean squared error (MSE) of an estimator measures the average of the squares of the "errors", that is, the difference between the estimator and what is estimated. If $\hat{Y}$is a vector of estimated series, and Y is the vector of the true values, then the (estimated) MSE

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}(\hat{Y}_i - Y_i)^2.$$

2.2. Selection rule

We computes MSE, Residual Variance, AIC and MAPE for n=50, 70, 100, 130, 150, 180, 200, 250, 300 and 400 for each case model, and select the model that hasthe minimum criteria values as the best. Note that only second order of autoregressive were considered in each case and situation.

2.3. Models selected for simulation

Data is generated from several linear and nonlinear second orders of general classes of autoregressive models as given below:

Data is generated from several linear and nonlinear second orders of general classes of autoregressive models given below:

Model 1.AR(2): $Y_{ti} = 0.3Y_{ti-1} - 0.6Y_{ti-2} + e_t$

Model 2.TR(2): Y_{ti} = 0.3sin(Y_{ti-1}) - 0.6cos(Y_{ti-2})+ e_t

Model 3:EX(2): Y_{ti} = 0.3Y_{ti-2}+ exp(-0.6Y_{ti-2}) + e_t

Model 4: PL(2): $Y_t = 0.3Y_{t-1}^2 - 0.6Y_{t-2} + e_t$

$$Y_{ti} \sim N(0,1) \text{ and } e_{ti} \sim N(0,1) \text{ for stationary series and } Y_{ti} \sim N(2000,20)$$

$$\text{and } e_{ti} \sim N(1000,10),$$

$$t = 1,2, \ldots 50, 150 \; and \; 300. \; i = 1,2, \ldots, 1000$$

The model 1, 2, 3 and 4 are linear, trigonometry, exponential and polynomial autoregressive functions respectively with coefficients of Y_{t-1} being 0.3 and Y_{t-2} being -0.6. Simulation studies were conducted to investigate the performance of self exciting threshold autoregressive, Smooth transition autoregressive models and logistic Smooth transition autoregressive models for fitting different general classes of linear and nonlinear autoregressive time series stated above. Effect of sample size and the stationarity of the models were examined on each of the general linear and nonlinear data simulated.

Note that in autoregressive modeling, the innovation (error), e_t process is often specified as independent and identically normally distributed. The normal error assumption implies that the stationary time series is also a normal process; that is, any finite set of time series observations are jointly normal. For example, the pair (Y_1,Y_2) has a bivariate normal distribution and so does any pair of Y's; the triple (Y_1,Y_2,Y_3) has a trivariate normal distribution and so does any triple of Y's, and so forth. Indeed, this is one of the basic assumptions of stationary data. However, in this study, the data will be generated under white noise assumption of stationarity and when the stationarity assumption is violated for order of past responses and random error terms to see behavior of the models in each case. 1000 replications were used to stabilize models estimations at different combinations of sample size (n) and models. The white noise assumption of the error term was also observed to make the data simulated be stationary. Data simulated were fitted to each of the model as shown in the goodness of fit model in table 1-4. Each of the created data were replicated 1000 times using tsDyn Package in R software.

3. Data analysis

The performances of the fitted models on the basis of the two criteria of assessment were displayed in table 1-4 as follows:

Table 1. Performances of the Fitted Models on the Basis of Mean Square Error and AIC Criteria for model 1: AR(2): $Y_{ti} = 0.3Y_{ti-1} - 0.6Y_{ti-2} + e_t$

Sample Size(n)	MSE				AIC			
	AR	SETAR	STAR	LSTAR	AR	SETAR	STAR	LSTAR
50	1.0034	1.0704	1.0950	1.1162	1.2435	2.2551	14.9486	15.3815
80	1.0007	1.0632	1.0743	1.0769	1.0823	1.7752	10.3405	15.4199
100	0.9781	0.9482	0.9844	1.0341	0.6519	0.5120	5.3006	13.4578
130	0.9000	0.9326	0.9128	1.0160	-1.9453	0.0118	-0.3022	13.1352
150	0.8841	0.9265	0.9036	1.0067	-2.0358	-0.1791	-1.5080	12.5703
180	0.8378	0.9124	0.9223	1.0010	-3.0098	-0.3771	-1.6924	11.2028
200	0.8316	0.9021	0.8507	0.8999	-3.5045	-0.4642	-2.3415	9.0534
250	0.8127	0.8788	0.8568	0.8820	-4.0992	-0.7922	-3.1600	8.6308
300	0.8108	0.8502	0.8299	0.8439	-4.1467	-0.8058	-3.6287	8.5844
400	0.8076	0.7704	0.8081	0.8238	-5.1515	-1.0361	-4.9978	2.4269

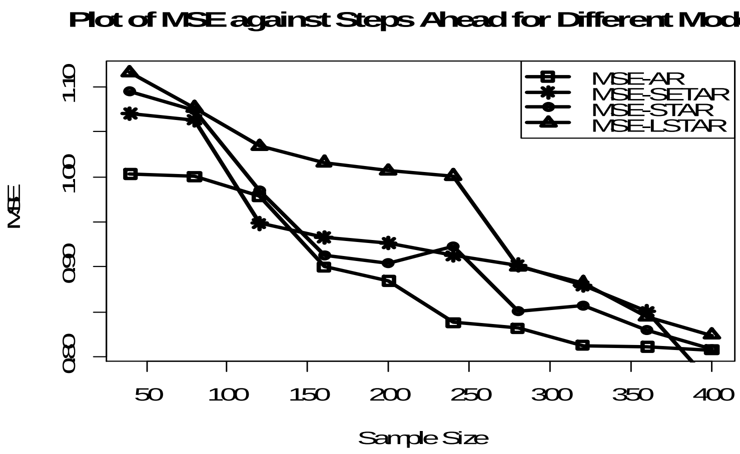


Figure 1(a). MSEof the Fitted Models on Model 1

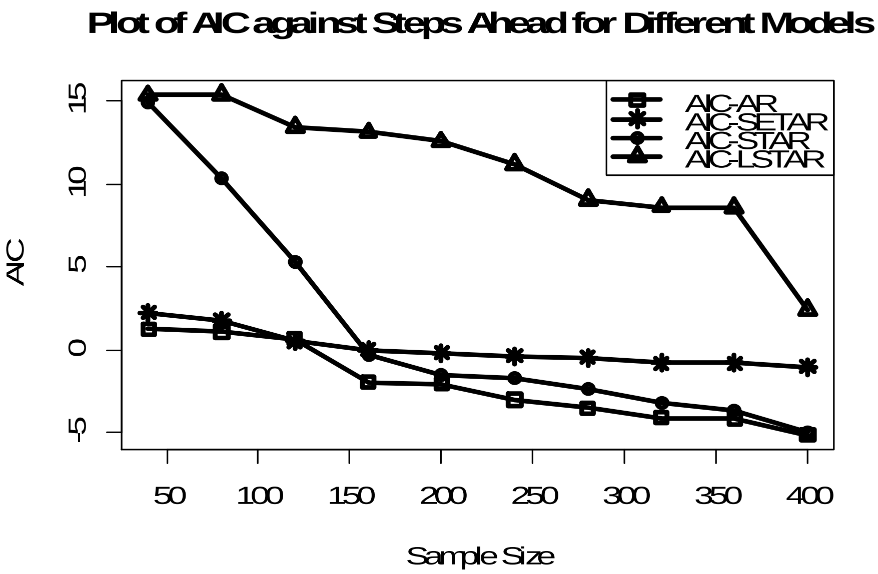


Figure 1(b). AICof the Fitted Models on Model 1

Table 1 shows the goodness of fit test for the four models to model 1 with the average values of mean square error and AIC of 1000 replication simulated from each model at various sample sizes. The results obtained were plotted on the graphs as shown in figure 1a and 1b respectively. The best fit to model 1 is AR based on both MSE and AIC followed by STAR model. However, with increase in sample size the performance of the four fitted models increase.

Table 2. Performances of the Fitted Models on the Basis of Mean Square Error and AIC Criteria for model 2: TR(2): Y_{ti} = 0.3sin(Y_{ti-1}) - 0.6cos(Y_{ti-2})+ e_t

Sample Size(n)	MSE				AIC			
	AR	SETAR	STAR	LSTAR	AR	SETAR	STAR	LSTAR
50	1.8778	1.0856	1.2555	1.1032	39.3347	25.1926	34.9504	27.7112
80	1.2878	1.0419	1.1979	1.1038	32.0152	22.1599	25.8792	25.0232
100	1.2410	1.0201	1.0322	1.1032	24.5778	17.8963	20.0044	19.6159
130	1.1213	1.0013	1.0253	1.0252	20.7725	13.6752	16.3612	14.3609
150	1.0942	0.9954	1.0195	0.9935	20.5946	11.9736	12.7373	12.8144
180	1.0770	0.9837	1.0016	0.9838	13.8477	11.8879	11.9715	11.7373

200	1.0211	0.9683	0.9935	0.9008	9.9183	10.6524	10.9698	6.6653
250	1.0123	0.9399	0.9871	0.8618	9.7873	8.5465	8.8975	6.4006
300	0.9988	0.9341	0.9838	0.8469	9.6550	6.5681	5.6446	3.0428
400	0.9119	0.9106	0.8472	0.8401	7.5899	4.6985	5.5609	1.1264

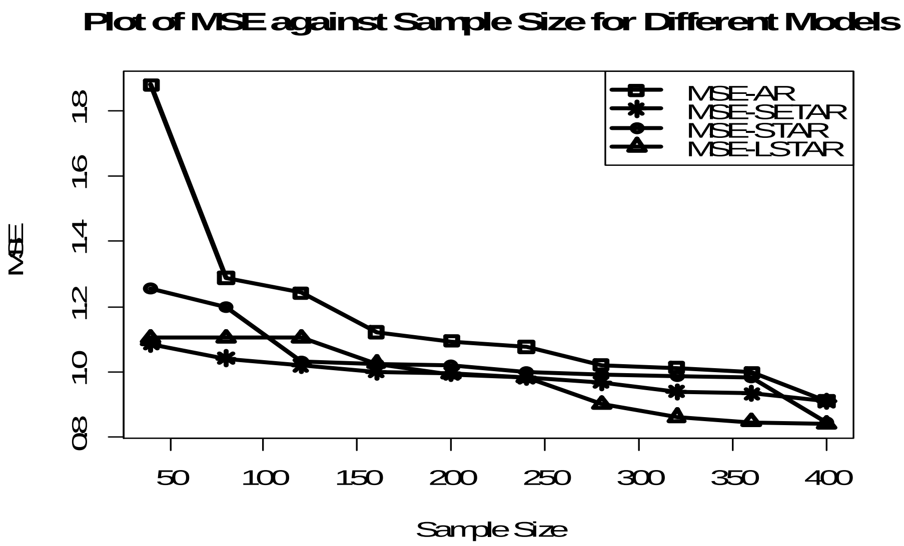


Figure 2(a). MSEof the Fitted Models on Model 2

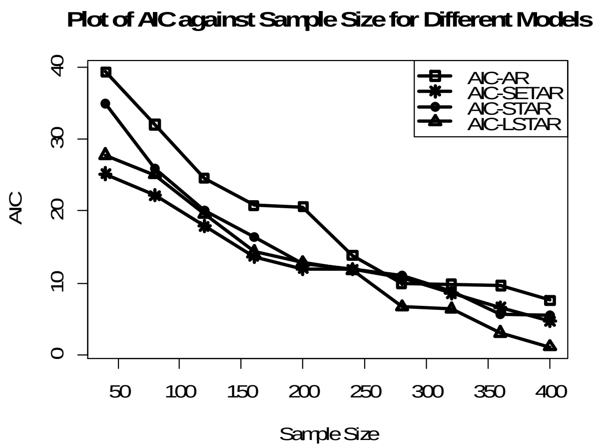


Figure 2(b). AIC of the Fitted Models on Model 2

From the above results, in figure 2a and 2b, it was observed that LSTAR is fitted best to trigonometric function at sample sizes below 200 based but LSTAR is the best at sample size above 200 based on the two criteria. Meanwhile STAR compete well with SETAR as sample size increases

Table 3a: Performances of the Fitted Models on the Basis of Mean Square Error and AIC Criteria for model 3: EX(2): $Y_{ti} = 0.3Y_{ti-2} + \exp(-0.6Y_{ti-2}) + e_t$

Sample Size (n)	MSE				AIC			
	AR	SETAR	STAR	LSTAR	AR	SETAR	STAR	LSTAR
50	1.5195	1.2044	1.5001	1.0843	29.7012	29.8014	15.1502	17.2593
80	1.3350	1.0192	1.0173	1.0045	23.133	20.2449	10.7939	10.7939
100	1.2870	1.0074	1.0044	0.9794	18.4658	14.855	10.7571	10.6242
130	1.2570	0.9973	0.9968	0.9735	17.1489	11.9369	7.8047	7.7358
150	1.1998	0.9802	0.9794	0.9479	15.8781	8.5940	7.7897	7.7009
180	1.1340	0.9744	0.9741	0.9053	15.518	8.0272	7.3729	4.9094
200	1.0932	0.9645	0.9732	0.8705	14.4622	6.4697	5.6874	0.3905
250	1.0390	0.9593	0.9391	0.8555	13.0971	5.8417	-4.3047	-6.8908
300	1.0206	0.9488	0.9238	0.8345	12.901	4.9595	-5.3248	-7.5132
400	0.8925	0.9259	0.9034	0.8054	12.5988	3.8493	-9.3038	-10.6848

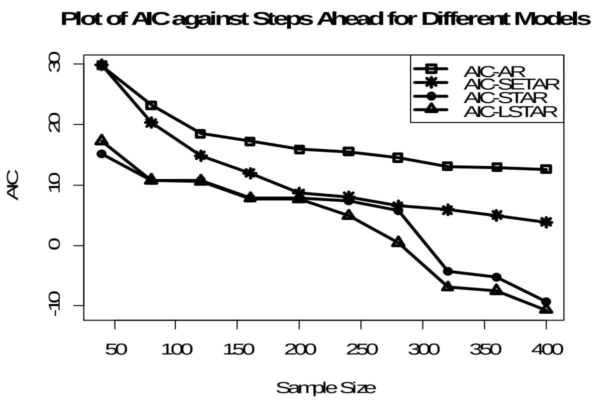


Figure 3(a). MSEof the Fitted Models on Model 3

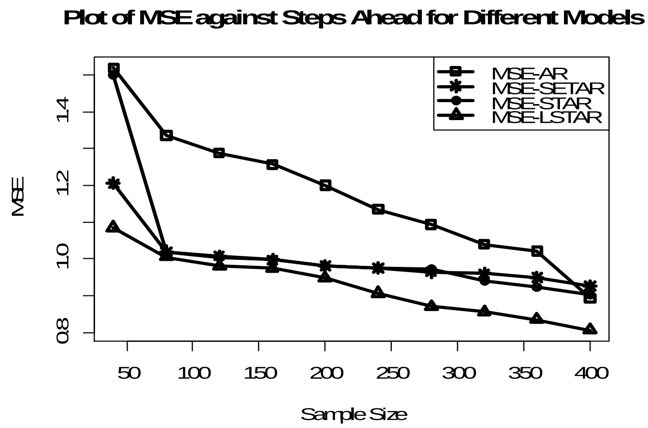
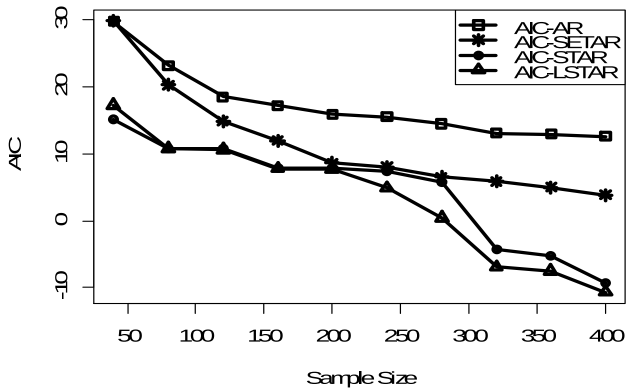


Figure 3(b). AIC of the Fitted Models on Model 3



In figure 3a and 3b, we observed that STAR and LSTAR perform equally at sample size below 200 but LSTAR supersede the other three models as sample size increases and fitted best to the exponential function at large sample sizes sample sizes.

Table 4(a). Performances of the Fitted Models on the Basis of Mean Square Error and Residual Variance Criterion for model 4: PL(2): $Y_t = 0.3Y_{t-1}^2 - 0.6Y_{t-2} + e_t$

Sample Size(n)	MSE				AIC			
	AR	SETAR	STAR	LSTAR	AR	SETAR	STAR	LSTAR
50	1.5399	1.1077	1.5432	1.7703	161.2003	14.3851	50.9901	85.9526
80	1.5342	1.0389	1.3453	1.5884	159.8064	9.8809	49.8896	84.5517
100	1.5231	0.9812	1.3399	1.5007	158.0353	9.5891	32.481	80.3441
130	1.5134	0.9758	1.2253	1.4914	156.004	9.5891	32.4134	76.6701
150	1.5132	0.9757	1.2252	1.3844	154.5532	8.3157	32.4134	74.5517
180	1.4213	0.9492	1.2252	1.3833	50.8064	6.6123	32.4134	72.3378
200	1.3399	0.9279	1.2252	1.1194	47.0134	-1.6793	32.4134	30.6764

250	1.2350	0.8326	1.2248	1.0646	40.8526	-5.8224	32.4134	30.6619
300	1.1392	0.8202	1.1266	1.0115	30.2822	-9.4363	17.9162	12.2683
400	1.1222	0.7461	1.1062	1.0106	24.7411	-9.8073	14.0722	10.8421

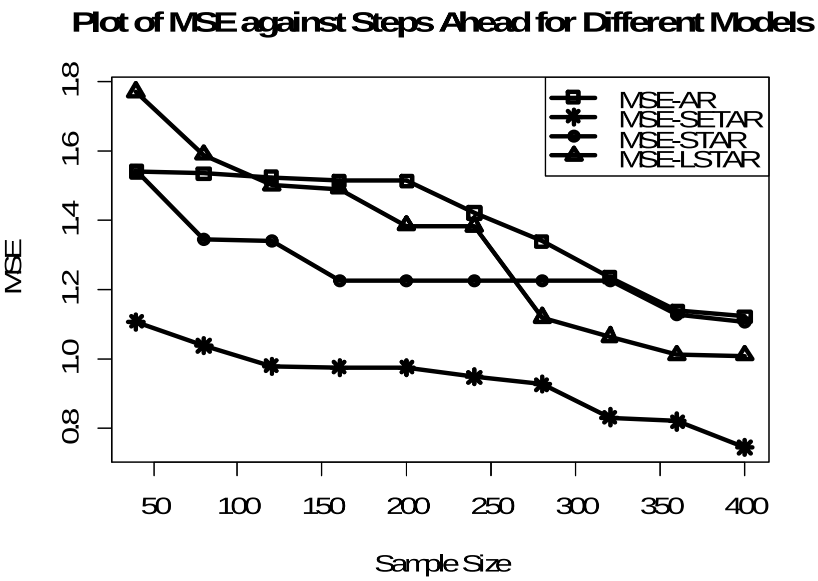


Figure 4(a). MSEof the Fitted Models on Model 4

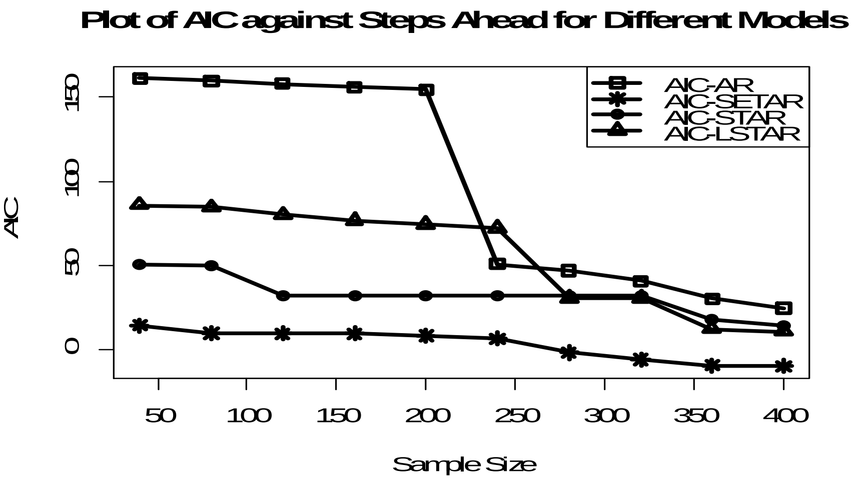


Figure 4(b). AIC of the Fitted Models on Model 4

From fig. 4a and 4b, it can be observed that the best model is SETAR followed by STAR at sample sizes below 300 and LSTAR as sample size increases.

3.1. Application of the fitted models on real life data

The four models was fitted to data on monthly rainfall collected from Nigeria Metallurgical Agency. The data was gathered from 1974 to 2013. Before fitting a nonlinear time series model to a given set of data, it is good if the nonlinearity characteristics of the data can be detected. There are various tests that have been suggested over the past years to distinguish linear from the nonlinear data sets. For example, SubbaRao (1980) et al and Hunnich (1982) used the bispectrum test. They used the fact that the square modulus of normalized bispectrum is constant when the time series is linear. The hypothesis is based on the non-centrality of parameters of the marginal distribution of the square moduli, where n is the sample size. Yuan (2000) modified the Hunnich's test in such a way that the parameter being tested under the null hypothesis is no longer but the location parameters, such as the mean or variance. The above mentioned methods are based on frequency domain approach.

Furthermore, once a model is selected, sufficiently strong evidence need to be found in the data to abandon the linear model. Therefore, good statistical and diagnostic tests are needed to determine the nonlinearity in time series data. However in this work two tests are used to detect whether the rainfall data is nonlinear or linear. The tests are Keenan and Tsay F-tests. Both tests are based on time domain. They have been used in the literature for detection of nonlinearity in time series data (see for example Keenan, 1985 and Tsay, 1986). The data was transformed using logarithmic transformation to ensure nonlinearity and the results are shown in table 5 and 6 respectively.

Table 5. Test of Nonlinearity on Monthly Rainfall in Nigeria between 1974-2013

Nonlinearity Test	Real Data				Transform Data			
	Test-Stat	DF	p-value	Decision	Test-Stat	DF	p-value	Decision
Keenan	8.1645	24	0.0045	reject	1.4837	24	0.2239	accept
Tsay F	1.534	24	0.0027	reject	1.787	24	0.09424	accept

Table 5 shows that the null hypothesis of nonlinearity was rejected for the rainfall data before being transformed but accepted after being transformed using the two statistics.

Table 6. Performances of the Fitted Models on Monthly Rainfall between 1974-2013

Model	Real Data		Transform Data	
	MSE	AIC	MSE	AIC
AR	6022	5546.21	2.777	1858.89
SETAR	5873.27	4179.52	2.4041	435.0518
STAR	5998.32	4181.63	2.3913	434.7764
LSTAR	5873.29	4181.52	2.3839	432.99

Table 26 shows that SETAR is the best to fit the rainfall data followed by LSTAR. However, when the data is transformed to make it nonlinear, LSTAR performs better than others based on the three criteria.

4. Conclusion

The best model to fit linear autoregressive function is AR at different sample sizes. The performance of LSTAR model supersedes other models as number of sample size increases except in polynomial function where SETAR model performs better than others. The three nonlinear models SETAR, LSTAR and STAR have closed performances in exponential autoregressive function as number of sample size increases based on MSE and AIC criteria. The performance of the four fitted models increases as sample size increases. Finally, it was observed that SETAR model fits best to the Rainfall Data and LSTAR was the best when the data is transformed to nonlinear.

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