2013

M.Sc. Part-II Examination

PHYSICS

PAPER—VIII

Full Marks : 75

Time : 3 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Use separate Answer scripts for Gr. A & Gr. B.

Group—A

(Advanced Quantum Mechanics)

[Marks—40]

Answer *Q. No. 1, 2, 3* and *two* from the rest.

1. Answer any *five* bits : 2×5

(i) Find the eigenvalues and corresponding eigenfunctions of the Spin matrix S_x for S = 1/2.

(ii) Show that the Dirac matrices should be square matrices of even order.

(iii) Simplify the term $(\vec{\sigma}.\vec{A})(\vec{\sigma}.\vec{B})$, where $\vec{\sigma}$ represent the Pauli matrices.

(iv) Define differential scattering cross section and give the relation between the differential scattering cross-section and scattering amplitude for scattering from a spherically symmetric potential.

(v) Taking into account the spin orbit interaction find the different energy states for an excited alkali atom wi... the outermost electron in the p state and find th... energy difference between these states.

(vi) For an excited alkali atom with the outermost electron in the p state find the shift in the energy levels due to application of a magnetic field such that magnetic interaction term is stronger than the spin orbit interaction term in the Hamiltonian (Paschen-Back effect).

(vii) Calculate total scattering cross-section for s-wave scattering on a completely impenetrable sphere i.e., for

$$V(r) = \begin{cases} \infty \ for \ r < \alpha \\ o \ for \ r < \alpha \end{cases}$$

(viii) Write the equation of continuity for a Dirac particle and thus find velocity operator.

2. Answer any *two* bits : 3×2

(i) Consider a system of two identical particles each with spin $^1/_2$. The Hamilitonian of the system is given as $H = A + B\,\overrightarrow{S_1}.\,\overrightarrow{S_2} + C(S_{1z} + S_{2z})$. Find the eigenvalues of the system.

(ii) Discuss Thomas Fermi approximation for finding the central potential for an electron in a many electron atom.

(iii) Write the wave function for a Helium atom in its first excited state ($IS^1\ 2S^1$) and find the energy difference between the triplet and the singlet state.

3. Answer any *one* bit : 4

(i) Expand the plane wave e^{ikz} in spherical harmonics.

(ii) Show that the spin orbit interaction appears naturally in the nonrelativistic limit of the Dirac equation.

4. Write the canonical momentum in terms of kinetic momentum and field momentum for a charged particle in an electromagnetic field and hence write the Hamiltonian for an atomic electron in an electromagnetic field. Use the semiclassical treatment to find the coefficient for induced emission of an atomic electron in presence of an electromagnetic field. 10

5. (a) Find the expression for the Greens function G(r) which satisfy the relation

$$\left(\nabla^2 + K^2\right)G(\bar{\gamma}) = \delta(\bar{\gamma})$$

(b) Use the form of Greens function to derive an expression for the scattering amplitude from a spherically symmetric potential.

(c) Using first Born approximation, find an expression for the differential scattering cross-section of a particle from a screened coulomb potential $V(\gamma) = \frac{A}{r} e^{-\alpha r}$.

3+4+3

6. Find the four free particle solutions of Dirac equation and explain their significance. Discuss how the negative energy solutions lead to the concept of antiparticle.

10

Group—B

(Statistical Mechanics)

[Marks—35]

Answer *Q. No. 1* and *two* from the rest.

1. Answer any *five* bits : 3×5

(a) Distinguish between ensemble average and quantum mechanical average.

(b) If the grand partition function is given by ln z = F − AH^2V, where F and A are constants, independent of H. Find the magnetization of the system.

(c) If the density of states between ν and $\nu + d\nu$ is $g(\nu)d\nu = A\nu^2 d\nu$. Find the zero point energy of a 1 − D quantum Harmonic oscillator.

(d) Prove that for pure state density matrix operator $\hat{\rho}$ is a projection operator.

(e) For non-interacting photons radiation pressure is $\frac{1}{3}u$; where u is the energy density. — Why?

(f) If the canonical partition function

$$Q_N(V,T) = \frac{V^N}{N!}\left(\frac{2\pi m K_B T}{h^2}\right)^{\frac{3N}{2}}$$

Find the equation of state.

(g) If $E = \pm\frac{1}{2}\mu_B H$ for a spin $\frac{1}{2}$ particle, prove that entropy gives rise to concept of negative temperature.

(h) How Bragg William approximation predicts MFA ?

2. (a) Deduce B.E. distribution function from grand canonical ensemble partition function.

(b) Show that for a two dimensional ideal B–E gas, number of particles

$$N = \left(\frac{2A\pi m K_B T}{h^2}\right) B_1(\alpha)$$

where $\alpha = -\mu/K_B T$; A is the area of the system. Other symbols have usual meanings. Can it undergo B–E condensation ?

5+5

3. (a) What is Landau diamagnetism ? Prove that magnetic moment of the gas $M = -\langle N\rangle \mu_{eff} L(x)$ where L(x) is the Langevin function if the particle has energy

$$E = \frac{e\hbar B}{m}\left(j + \frac{1}{2}\right) + \frac{p_z^2}{2m}$$

Under strong magnetic field (B) and at very low temperature.

(b) Prove that in one-dimensional Ising model, spontaneous magnetism does not exist at finite temperature.

6+4

4. (a) Prove that $i\hbar\dot{\hat{p}} = [\hat{H}, \hat{p}]$, where $\hat{p}$ is the density matrix and $\hat{H}$ is the Hamiltonian.

(b) A spin $\frac{1}{2}$ particle with 50% in $|x\rangle_+$ state and 50% in $|x\rangle_-$ state in a magnetic field.

Find the density matrix and the corresponding state of polarization.

5+5

5. (a) Find out an expression for carrier statistics for two dimensional Fermi gas.

(b) From Planck's radiation law formulate Wien's law.

(c) Prove that B–E condensation is 1st order phase transition (according to Ehrenfest) and 3rd order phase transition (according to Landau).

5+2+3

———