2013

DDE

M.Sc. Part-I Examination

PHYSICS

PAPER—V

Full Marks : 75

Time : 3 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Write the answers Questions of each group in separate books.

Group—A

[*Marks : 35*]

1. Answer any *three* questions of the following : 3×2

(a) What is the function of CPU ?

(b) Define 'array' in FORTRAN.

(c) Define IF – THEN – ELSE statement in FORTRAN.

(d) What is interpolation ?

(e) What is eigen value and eigen vector ?

2. Answer any *three* questions of the following : 3×3

(a) Write a FORTRAN program to find the mean and standard deviations of N numbers.

(b) Write a subroutine subprogram to choose the biggest of two numbers A and B.

(c) Write the function subprogram to find the factorial of a number.

(d) Write a FORTRAN program to print all the Fibonacci numbers less than 50.

(e) $f(x) = x^2 + \sin 2x$ if $n < 3$

$= 10.3$ if $n = 3$

$= x^3 - \cos 3x$ if $n > 3$

Write a FORTRAN program using 'IF – THEN – ELSE – END IF' statement.

3. Answer any *four* questions : 5×4

(a) Find the greatest eigen value and the corresponding eigen vector of the matrix by power method.

$$\begin{pmatrix} 4 & 0 & 2 \\ 0 & -1 & 0 \\ 2 & 0 & 4 \end{pmatrix}$$

(b) Solve $\sin x - x \cos x = 0$ by bisection method.

(c) Find the least-square parabolic fit curve

$y = Ax^2 + Bx + C$ for the data set :

x_i :	– 3	0	2	4
y_i :	3	1	1	3

(d) Solve by Runge-Kutta method :

$$\frac{dx^2}{dt^2} + x = 6\cos t \text{ with } x(0) = 2,\ x'(0) = 3.$$

(e) Find the table find the value of f(12) :

x	10	15	20	25	30
f(x)	0·10	0·15	0·20	0·25	0·03

(f) Solve the diffusion equation :

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2},\ 0 < x < 1 \text{ and } 0 \le t \le 0{\cdot}1$$

with initial conditions —

$u(0, t) = 0,\ u(1, t) = 0$ for $0 \le t \le 0{\cdot}1$

and $u(x, 0) = \phi(x) = \sin \pi x$ for $0 \le x \le 1$

(Take $\Delta x = 0{\cdot}2,\ \Delta t = 0{\cdot}02$)

Group—B

[Marks : 40]

Answer Q. No. 1 and any *three* from the rest.

1. Answer any *five* bits : 2×5

(a) Find a basis of R^3 containing vectors (1, 1, 2) and (3, 5, 2).

(b) If A and B are Hermitian matrices, then show that i [A, B] is also a Hermitian matrix.

(c) Evaluate : $\int_0^\infty \frac{x^a}{a^x} dx$.

(d) The Laplace transform of a function

F (t) is $f(S) = \frac{1}{S(S-a)}$.

Find the equation.

(e) Find the Fourier transform of the function :

$f(x) = e^{-x^2/2}$.

(f) Write down the generating function of Hermite and Laguerre polynomials.

(g) Show that the transformations

$x' = ax + b, a \neq 0$ from a Lie group.

2. (a) Construct an orthonormal basis for R^3 out of the vectors :

$\psi_1 = (1, 2, 2)$; $\psi_2 = (1, -1, 2)$; $\psi_3 = (1, 0, 1)$.

(b) If $\hat{A}(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

Then prove that

$\hat{A}(\theta)\hat{A}(\phi) = \hat{A}(\theta + \phi)$.

(c) Prove that $\int_0^\infty \frac{x^{p-1}}{1+x} dx = \frac{\Pi}{\sin p\Pi}$

By Cauchy's residue theorem. 3+3+4

3. (a) If F(S) be the F.T. of f(x), then F.T. of f(n) cos bx is

$F_1(S) = \frac{1}{2}[F(S+b) + F(S-b)]$.

(b) Find the Laplace transform of the function

$F(t) = \text{Sint}$, for $0 < t < \pi$

$= 0$, for $t > \pi$.

(c) Using the Convolution theorem, show that

$\beta(m,n) = \frac{T(m)\,T(n)}{T(m+n)}$. 3+3+4

4. (a) Find the Green's function for the differential equation

$\frac{d^2y}{dx^2} - \omega^2 y = 0$; $0 \le x < a$

with boundary conditions

$y(0) = 0$ and $\frac{1}{y}\frac{dy}{dx}\Big|_{x=a} = -\omega$.

(b) Expand the function

$f(x) = e^{-x}$ in terms of Laguerre's polynomials. 5+5

5. (a) Determine the number and the dimensions of the inequivalent irreducible representations of D_4, the symmetry group of the square.

(b) Construct the character table of D_4. 5+5

6. (a) Find the particular integral of

$$u_{xx} - 3u_{xy} + 2u_{yy} = e^{x+y} + \sin(2x + y).$$

(b) Solve :

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{du}{dr}\right) + \left(k^2 - \frac{n(n+1)}{r^2}\right)^{u=0}$$

where u is a radial function. 5+5