2013

DDE

M.Sc. Part–I Examination

PHYSICS

PAPER—II

Full Marks : 75

Time : 3 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Write the answers Questions of each group in separate books.

Group—A

[*Marks : 50*]

Answer *Q. No. 1* and *2* and any *two* from the rest.

1. Answer any *six* bits : 2×6

(a) Prove that $[f(x), \hat{p}] = i\hbar \frac{df(x)}{dx}$.

(b) Prove that the eigenfunctions corresponding to different eigenvalues of a Hermitian operator are orthogonal.

(c) An operator satisfies the quadratic equation $\hat{A}^2 - 3\hat{A} + 2 = 0$. Is $\hat{A}$ an observable? Explain.

(d) Show that the transformation matrix which transforms from one basis set to the other is unitary.

(e) In a 3-dimensional quantum mechanical harmonic oscillator has an energy 3.5 $\hbar\omega$ in a particular state. Find the degree of degeneracy.

(f) In a wave function $\psi(r) = \frac{e^{ik.r}}{r}$

Show that radial probability current density $J_r = \frac{v}{r^2}$.

(g) A certain state $|\psi\rangle$ is an eigenstate of $\hat{L}^2$ and $\hat{L}_z$. Then calculate $\langle \hat{L}_k \rangle$ and $\langle \hat{L}_x^2 \rangle$.

2. Answer any *three* bits : 4×3

(a) Prove that if an orthonormal set of kets $\{ |u_i\rangle, i = 1,2,\ldots \}$ from a basis, then $\sum_i |u_i\rangle \langle u_i| = \hat{I}$.

(b) If $\psi(r) = \sqrt{\frac{1}{\pi a^3}}\, e^{-r/a}$, find $\langle r^2 \rangle$.

(c) Prove that $\hat{L}_+ \hat{L}_- = \hat{L}_x^2 + \hat{L}_y^2 - i\left[\hat{L}_x, \hat{L}_y\right]$.

(d) Show that the product of uncertainties associated with the measurement of variables represented through the two non-commutating Hermitian operators is :

$$\Delta\hat{A}.\Delta\hat{B} \geq \tfrac{1}{2}\left|<\left[\hat{A},\hat{B}\right]>\right|.$$

3. (a) If we introduce the operators

$$\hat{a} = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{q} + \frac{i\hat{p}}{m\omega}\right)$$

$$\hat{a}^+ = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{q} - \frac{i\hat{p}}{m\omega}\right)$$

Then prove that :

(i) $[a^+a, a] = a$;

(ii) $[a^+a, a^+] = a^+$;

(iii) $[a, a^+] = 1$. 3+3+1

(b) Prove that $\left(Y_1^1 - Y_1^{-1}\right)$ is eigen function of $\hat{L}_x$ and $\hat{L}^2$. 3

(c) Write the eigenvalues and eigenfunctions for the two lowest states of simple Harmonic Oscillator. Also draw these eigenfunctions. 3

4. (a) Discuss the equation of motion for both wave function and the operator in interaction picture. 4

(b) The wave function of H- atom in a definite state is given by :

$$\psi(r, \theta, \phi) = Ar \cos\theta \, e^{-r/2a_0}$$

Find the most probable radius of the electron in this state. 4

(c) $\psi(x,a) = Ae^{-ax^2}$; use variational method to find the ground state energy of simple harmonic oscillator. 5

5. (a) Establish the stationary perturbation theory for a doubly degenerate state. 6

(b) An Unperturbed system has a doubly degenerate energy eigen value E_1 and eigen functions $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$. When perturbed, its Hamiltonian is represented by $\begin{pmatrix}E_1 & A\\ A^* & E_1\end{pmatrix}$. Find the first order correction to E_1. 4

(c) Prove that

$$[J_z, J_+] = \hbar J_+$$

where J_+, J_- are raising and lowering operators of angular momentum. 3

Group—B

[Marks : 25]

Answer *Q. No. 1* and any *two* from the rest.

1. Answer any *three* of the following : 3×3

(i) Explain what is meant by hot band.

(ii) What is Fortrat Parabole ?

(iii) The values of ω_e and $\omega_e x$ are 1580·36 and 12·073 cm^{-1} respectively for the ground state of molecular oxygen. Calculate the zero point energy.

(iv) What is centrifugal distortion ? Explain the effect of Centrifugal distortion on the energy of a diatomic molecule. 3

(v) The observed rotational spectrum of HF shows decrease in the line separation on the high frequency side. Why ?

(vi) Why does a spherical symmetric molecule is microwave inactive ? Give an example of such type of molecule.

2. (a) Assuming Molecule as a vibrating rotator implying Born-Oppenheimer approximation, find the transition corresponding P Branch and R Branch.

(b) What is band origin ? 7+1

3. (a) Show the variation of intensity in a progression when the internuclear distance in the upper state is greater than the ground state.

(b) Clearly distinguish the features of vibrational spectrum of harmonic oscillator and anharmonic oscillator assuming only vibration in a molecule. 3+5

4. The rotational constant for $H^{35}cl$ is observed to be 10.5909 cm^{-1}. What are the values of B for $H^{37}cl$?

A space probe was designed to seek CO in the atmosphere of Saturn by looking for lines in its rotational spectrum. If the bond length of CO is 11·28 nm, at what wavenumbers do the first three rotational transitions appear?

$h = 6{\cdot}626 \times 10^{-34}$ JS ; $C = 2{\cdot}998 \times 10^{8}$ mS^{-1} ;

Atomic masses :

$H = 1{\cdot}673 \times 10^{-27}$ Kg ; $^{35}Cl = 58{\cdot}06 \times 10^{-27}$ Kg ;

$^{37}Cl = 61{\cdot}38 \times 10^{-27}$ Kg ; $C = 19{\cdot}93 \times 10^{-27}$ Kg ;

$O = 26{\cdot}56 \times 10^{-27}$ Kg. 4+4
