2013

DDE

M.Sc. Part–I Examination

PHYSICS

PAPER—I

Full Marks : 75

Time : 3 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Write the answers Questions of each group in separate books.

Group—A

[*Marks : 30*]

1. Answer any *four* of the following : 4×2

(a) In Rutherford's α-particles scattering experiment, 10^3–α-particles are scattered at an angle 4°. Calculate the number of α-particles, scattered at an angle 14°.

(b) Deduce the principle of least action in the form as $\Delta \int_{t_1}^{t_2} T\,dt = 0$, (where T represent kinetic energy of a conservative system).

(c) Prove that if a generalised co-ordinate is cyclic in the Lagrangian it should also be cyclic in the Hamiltonian.

(d) Prove that Poisson bracket

$$[p_\alpha, q_\beta] = \delta_{\alpha\beta} \quad = 1 \quad \text{if } \alpha = \beta$$
$$= 0 \quad \text{if } \alpha \neq \beta$$

(e) The Lagrangian of an anharmonic oscillator is given as :

$$L(q,\dot{q}) = \tfrac{1}{2}\dot{q}^2 - \tfrac{1}{2}\omega^2 q^2 - \alpha q^3 + \beta q\dot{q}^2$$

where α, β and ω are constant. Obtain the corresponding Hamiltonian.

(f) Derive modified Hamilton's Principle from Hamilton's Principle. Write the basic difference between these two principles.

(g) Deduce Lagrange's equation in presence of non-conservative forces, in the form as :

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_K}\right) - \frac{\partial L}{\partial q_K} + \frac{\partial R}{\partial \dot{q}_K} = 0$$

Where, $R = \frac{1}{2}\sum_{v_i} K_i v_i^2$ represent the Rayleigh dissipation function.

(h) The Potential energy of a system of two particles depends on their mutual distance (x) as :

$$\varphi = \frac{C_1}{x^2} - \frac{C_2}{x}$$

(where C_1, C_2 are constant)

Prove that the system may be in stable equilibrium when C_1 and C_2 are both positive constant, and the system will be in unstable equilibrium when C_1, C_2 are both negative.

2. Answer any *two* of the following : 2×3

(a) Using Variational Principle, prove that the shortest distance between any two points in a plane is a straight line.

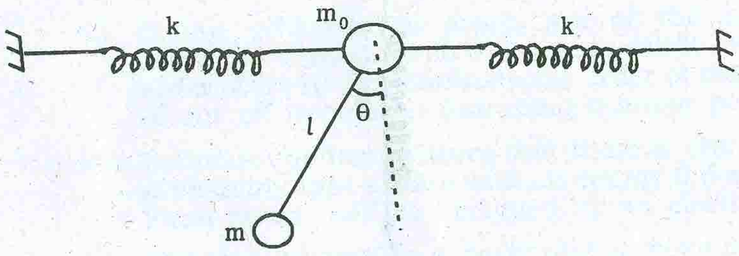
(b) A particle moves in a plane under a central force of magnitude :

$$F = \frac{1}{r^2}\left[1 - \frac{\dot{r}^2 - 2r\ddot{r}}{C^2}\right]$$

where r is the distance of the particle from the centre of force. Show that the expression for generalised potential in this case is :

$$U = \frac{1}{r}\left[1 + \frac{\dot{r}^2}{C^2}\right]$$

(c) Find the normal modes of small oscillations for the dynamical system as shown below by using method of small oscillation :



where K represent the spring constant.

3. Answer any *two* of the following questions : 2×8

(a) (i) Derive the expression of relativistic Lagrangian and relativistic Hamiltonian of a particle in relativistic mechanics.

(ii) Discuss the force-free motion of a symmetrical top. Explain what you understood by precessional motion. Calculate the precessional angular frequency. 3+(2+1+2)

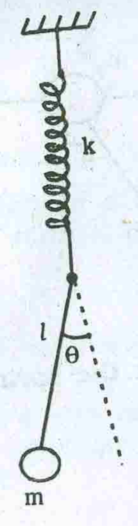
(b) Write down the Hamiltons-Jacobi equation in terms of Hamilton's principal function.

Use Hamilton-Jacobi method for solving the motion of a mechanical system with the Hamiltonian :

$$H = \frac{p^2}{2m} + \frac{1}{2}Kq^2$$

What is the physical significance of Hamilton's principal function ? 1+5+2

(c) A simple pendulum is suspended from a massless spring of spring constant K, as shown in figure below :



(Continued)

The spring has only vertical motion.

Find th Lagrangian of the system and also fixed the Lagrange's equation of motion of the system. 5

(ii) Set up the Hamilton's equation of motion for the following Lagrangian :

$$L(q, \dot{q}, t) = \frac{1}{2}M\left[q^2\omega^2 + \dot{q}^2 \sin^2 \omega t + q\dot{q}\omega \sin(2\omega t)\right]$$

(The symbols have their usual meaning) 3

Group—B

[Marks : 45]

Answer *Q. No. 1* and any *three* from the rest.

1. Answer any *three* of the following : 3×3

(a) Find the minimum radius of the institutional sphere that can just fit into the void at $\left(\frac{1}{2}\ \frac{1}{2}\ \frac{1}{2}\right)$ between the body centred atoms of b.c.c. structure.

(b) Find the Brillouin Zone of a f.c.c. lattice.

(c) Sound velocities in solids are of the order of $3\times10^3 ms^{-1}$. Interatomic distance in solids are of the order of 3×10^{-10}m. Estimate the order of magnitude of cut off frequency, assuming a linear lattice.

(d) Estimate the temperature that there is one percent probability that a state with an energy 0.5 eV above Fumi energy will be occupied by an electron.

(e) The E–K relation in a particular semiconductor is given by $E(k) = Ak^2 + 8k^3$ where A & B are positive constants. Find the wavevectors for which the electron-group velocity is zero and also determine the effective mass of electron at those wavevector values.

(f) Express the symmetry elements which are associated with point group in a solid.

2. (a) Derive Lane equations assuming scattering of X-ray from a crystal.

(b) Find the condition of systematic absence derivening structure factor in a B.C.C. lattice. 9+3

3. (a) Derive the dispersion relation for one dimensional monoatomic lattice vibration in a solid.

(b) Prove the equivalence between vibrational mode and a simple Harmonic Oscillator. 6+6

4. (a) Assuming Sommerfield model find the number of allowed states in the range between E and E + dE.

(b) Find an expression of Fermi Energy in a metal at T = 0K.

(c) An electron is confined in a one dimensional wall of width 0.3 nm. Find the kinetic energy of the electron when it is in the ground state.

8+2+2

5. (a) Explain 'what is the physical origin of energy gap'? Show that, the band gap is determined by the magnitude of periodic potential existing in a lattice.

(b) What is meant by 'Extended Zone Scheme' & 'Reduced Zone Scheme'?

(c) Clearly distinguish metal, insulator and semiconductor on the basis of bond structure.

7+3+2

6. (a) Find the local field in a dielectric medium according to Lorentz.

(b) Find the dipolar polarizability of a dipolar system at temperatures that are not too low and at field that are not too large. 6+6

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