2014

M.Sc. Part-II Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—IX (OR/OM)

Full Marks : 100

Time : 4 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

SPECIAL PAPER : OR

(Advanced Optimization and Operations Research—I)

Answer *Q. No. 11* and any *six* from the rest

1. (a) Apply Wolfe's method to solve the following quadratic programming problem :

$$\text{Maximize } Z = 2x_1 + 3x_2 - 2x_1^2$$

$$\text{Subject to } \quad x_1 + 4x_2 \le 4$$

$$x_1 + 2x_2 \le 2$$

$$x_1, x_2 \ge 0$$ 8

(Turn Over)

(b) State and prove Fritz John saddle point sufficient optimality theorem. What are the basic differences between the necessary criteria and sufficient criteria of Fritz John saddle point problem. 8

2. (a) What is decomposition principle due to Dantzig and Wolfe ? Discuss the various steps of solution procedure using decomposition principle for a large problem of linear programming problem (to be considered by you). 8

(b) State and prove Motzkin's theorem of the alternative in non-linear programming. 5

(c) State and prove weak duality theorem in non-linear programming. 3

3. (a) Using modified duel simplex method, solve the LPP.

Maximize $Z = -10x_1 - 6x_2 - 2x3$

Subject to $-x_1 + x_2 + x_3 \geq 1$

$3x_1 + x_2 - x_3 \geq 2$

and $x_1, x_2, x_3 \geq 0$ 8

(b) State Farkas' theorem. Give the geometrical interpretation of it. 5

(c) What do you mean by differetiable conven and concave functions. 3

4. (a) Solve the following quadratic progrmming problem using Beale's method :

Maximize $z = 10x_1 + 25x_2 - 10x_1^2 - x_2^2 - 4x_1x_2$

Subject to $x_1 + 2x_2 \leq 10,\ x_1 + x_2 \leq 9,\ x_1, x_2 \geq 0.$ 8

(b) State and prove Kuhn-Tueker saddle point necessary optimality theorem. 5

(c) (i) the (primal) minimization problem (MP).

(ii) the dual (maximization) problem (DP). 3

5. (a) Use the artificial constraint method to find the initial basic solution of the following problem and then apply the dual Simplex algorithm to solve it.

Maximize $z = x_1 - 3x_2 - 2x_3$

Subject to $x_2 - 2x_3 \geq 2$

$x_1 - 4x_2 - 6x_3 = 8$

$2x_2 + x_3 \leq 5$

$x_1, x_2\ x_3 \geq 0.$ 8

(b) Give an iterative procedure for solving on LPP by dual Simplex's algorithm. 8

6. (a) Discuss the effect of discrete change in the profit vector C to the LPP.

Maximize $z = cx$

Subject to $Ax = b,\ x \geq 0$

where $c, x^T \in R^n$, $b^T \in R^m$ and A is an $m \times n$ real matrix. 8

(b) Use revised simplex method to solve the following LPP:

$$\text{Maximize} \quad z = x_1 + 2x_2$$
$$\text{Subject to} \quad x_1 + x_2 \le 3$$
$$x_1 + 2x_2 \le 5$$
$$3x_1 + x_2 \le 6$$
$$\text{and} \quad x_1, x_2 \ge 0.$$ 8

7. (a) (i) Define the terms : Quadratically Convergent Method, conjugate directions of a matrix. 2+2

(ii) Find the conjugate directions for the matrix $\begin{pmatrix} 4 & 5 \\ 5 & 4 \end{pmatrix}$. 4

(b) Solve the following problem by using Gomory's constraint method 8

$$\text{Maximize} \quad z = x_1 + 2x_2 + x_3$$
$$\text{Subject to} \quad 2x_1 + 3x_2 + 3x_3 \le 11$$
$$x_1, x_2, x_3 \ge 0$$
and integers.

8. (a) Define goal programming Problem. Formullate the following goal programming problem.

Let a company produces two products A and B. Product A requires 20 hrs in machine 1 and 10 hrs. in Machine 2. Production time is limited in Machine 1 to 60 hrs and in Machine 2 to 40 hrs. Contribution to profits for the two products are respectively Rs. 40 and Rs. 80. Management's objectives are to maximize the profit and to produce at least two units of each type product. Let the management has established the following goal priorities :

Priority 1 : To maximize the profit

Priority 2 : To meet production goals of two units for each product. 2+6

(b) Use cutting plane method

$$\text{Maximize} \quad z = 5 - 2x_1 - 4x_2$$
$$\text{Subject to} \quad x_1 + 2x_2 \le 5$$
$$x_1^2 + 2x_2^2 - 10x_1 - 8x_2 + 21 \le 0$$
$$2 \le x_1 \le 7 \text{ and } 0 \le x_2 \le 5.$$
8

9. (a) Discuss the revised simplex method to solve the following LPP :

$$\text{Maximize } z = cx$$
Subject to the constraints
$$Ax = 6$$
$$x \ge 0.$$ 8

(b) Discuss the changes in the optimal solution of the following problem :

$$\text{Maximize} \quad z = 3x_1 + 5x_2$$
$$\text{Subject to} \quad 3x_1 + 2x_2 \le 18$$
$$x_1 \le 4$$
$$x_1, x_2 \ge 0$$
after adding a new constraint $x_2 \le 6$ 8

10. (a) What are the limitations of Fibonacci Method. Discuss the Fibonacci method to find the maximum point of an unimodal function define on [a, b]. 2+6

(b) Using steepest descent method, minimize $f(x) = x_1^2 + x_2^2 + 2gx_1 + 2fx_2 + c$ (g, f, c being constants) starting from the point $\begin{pmatrix}\alpha \\ \beta\end{pmatrix}$. 8

11. Answer any one :

(a) (i) What are difference between Classical optimization and Numerical optimization ? 2

(ii) Necessity of post optimal analysis in optimization. 2

(b) Write a short note on any one of the following : Constraint qualification. 4

SPECIAL PAPER : OM

Answer Q. No. *11* and any *six* questions from the rest.

1. (a) Establish Gibb's relation of thermodynamics. Deduce Gibb's-Buhem relation. 8

(b) Considering sea-water as a two components mixture of salt and pure water, deduce for sea-water the relations mentioned above. 8

2. Show that under usual notations :

$$T = \frac{1}{\lambda},\ \mu_s = -U - \frac{\lambda_s}{\lambda} + \frac{\vec{Q}^2}{2}$$

$$\mu_w = -U - \frac{\lambda_w}{\lambda} + \frac{\vec{q}^2}{2},\ \vec{q} = \frac{\vec{a}}{\lambda} - \frac{1}{\lambda}\left(\vec{b} \times \vec{\pi}\right).$$

are the necessary conditions of thermodynamical equilibrium of a finite volume of sea-water. Hence deduce the hydrostatic pressure equations (symbols have their usual meanings) 13+3

3. Derive Fridman's equation for diffusion of absolute vorticity in a viscous flow in terms of motion relative to the earth. Define potential vorticity of a fluid particle. Deduce Erteliz formula for the evolution of potential vorticity and hence find out an approximate adiabatic invariant. 10+2+2+2

4. What are the assumption of Boussinequ approximation ? Derive the approximate form of the field equations under these assumptions. 4+12

5. Obtain the solution of the equation of motion for the pure wind drift currents in a finitely deep, plane, homogeneous layer of fluid which rotates uniformly about a vertical axis. Hence deduce the following :

(a) The surface current Us, is directed 450 to the right of the wind strees vector τ in the northern hemisphere.

(b) The ends of the current vectors, projected on a horizontal plane, form an Ekman spiral. 16

6. Assuming two-dimensional model of ocean currents, obtain the solution for a problem of boundary layer flow and show that a weak back flow appears close to the external edge of the boundary western shore. 16

7. Show that the total flow function ψ in the Ekman's theory for a small Ekman number E satisfies the equation.

$$\Omega E\sqrt{\sin\varphi}\ \nabla_h^2\psi + \frac{2\Omega}{a^2}\frac{\partial\psi}{\partial\lambda} = rot_z(\tau),$$

where the symbols have their usual meanings. Show that there is a strong coastal current at western shore of the ocean. 16

8. Deduce the equations of motion of Thermal wind. Hence deduce the Taylor-Proudman theorem. 10+6

9. Establish the Sverdrup's Curl equation of motion of wind driven currents in a baroclinic ocean. 16

10. Show that geostrophic approximation of motion can be expressed as

$$fv = \frac{1}{\rho_s r\ Cos\theta}\cdot\frac{\partial p}{\partial p},$$

$$fu = -\frac{1}{\rho_s r}\cdot\frac{\partial p}{\partial\theta},$$

$$\rho g = -\frac{\partial p}{\partial r},$$

(symbols have their usual meanings). 16

11. Answer any one question :

(i) Calculate the acceleration of a fluid parcel in a rotating frame.

(ii) Ekman layer. 4