2014

M.Sc. Part–II Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—VII

Full Marks : 100

Time : 4 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Write the answer to questions of each group in Separate answer booklet.

Group—A

[Marks : 25]

Answer *Q. No. 1* and any *two* from the rest

1. Give difference between conduction current and displacement current.

1

2. (a) Prove the following Maxwell's equation :

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

where symbols used have their usual meanings. 8

(b) Deduce equation of continuity from the low of conservation of charge. 4

3. (a) A magnetized sphere, of radius R is placed in uniform external magnetic field $\vec{H}_0$. Find out the potential and field intensity inside and outside the sphere. 6

(b) Obtain the expression for the surface charge density as the interface between two conducting media with different conductivities in terms of the normal components of the current density in medium '2'. 6

4. What is electrical image ? Find the image of a single point charge e at a distance f from a conducting infinite plane sheet kept at zero potential. Deduce the expression for the surface density of induced charge in this case. 2+4+6

Group—B

[Marks : 25]

(Fussy Sets and its application in OR)

Answer Q. No. 5 andany *three* from Q. No. 6 to 10.

5. Define height of a fuzzy set with an example. 1

6. (a) If $\tilde{A} = [a_1, b_1, c_1]$ and $\tilde{B} = [a_2, b_2, c_2]$ then prove that $\tilde{A} - \tilde{B} = [a_1 - c_2, b_1 - b_2, c_1 - a_2]$. 4

(b) What are the causes of uncertainty ? Explain the traditional and modern view of uncertainty with examples. 2+2

7. A company produces four items A, B, C and D. The inputs for the production are man-weeks, material X and material Y. The availability of the resources and profits corresponding to the items A, B and C are shown in the following table.

Items	Man Weeks	Material X	Material Y	Profit/Item
A	2	5	3	5
B	1	6	4	4
C	3	8	6	2
D	4	4	5	6
Availability	12 to 15	60	30 to 40	Maximize

Using Verdegay's approach determine the equivalent parametric programming problem. 8

8. The membership function of the fuzzy set $\underset{\sim}{A}$ and $\underset{\sim}{B}$ are as follows.

$$\mu_{\underset{\sim}{A}}(x) = 1 \quad , x \leq 30$$
$$= (60 - x)/30 \quad , 30 < x < 60$$
$$= 0 \quad , x \geq 60$$

$\mu_{\underset{\sim}{B}}(x) = 0 \quad , x \leq 30$

$= (x - 30)/30 \quad , 30 < x \leq 60$

$= 1 \quad , x > 60$

Determine the membership functions of $\underset{\sim}{A} \cup \underset{\sim}{B}$ and $\underset{\sim}{A} \cap \underset{\sim}{B}$. Represent the membership function of $\underset{\sim}{A}$, $\underset{\sim}{B}$, $\underset{\sim}{A} \cup \underset{\sim}{B}$ and $\underset{\sim}{A} \cap \underset{\sim}{B}$ graphically. 2+2+1+1+1+1

9. Describe Zammermann's method to convert a fuzzy LPP into a crisp LPP. 8

10. Analytically prove the distributive law

$$\underset{\sim}{A} \cup (\underset{\sim}{B} \cap \underset{\sim}{C}) = (\underset{\sim}{A} \cup \underset{\sim}{B}) \cap (\underset{\sim}{A} \cup \underset{\sim}{C})$$

using definition of union and intersection of fuzzy sets. 8

Group—C

[Fluid Dynamics]

[Marks : 30]

Answer any *two* questions.

11. (a) Within a circular boundary of radius 'a' there is a two dimensional irrotational motion due to a source producing liquid at the rate m, at a distance f from the centre and an equal sink at the centre. Find the velocity potential and show that the resultant of the pressure on the boundary is

$$\rho m^2 f^3 / \left[2\pi a^2 \left(a^2 - f^2\right)\right],$$

Where ρ is the liquid density.

(b) Consider two parallel rows of vortices one below the other such that the upper vortices are located at (0, 0), (±a, 0) (±2a, 0), ... and the lower vortices are located at (0, – b), (±a, – b), (± 2a, – b), ... If each vortex in the upper row has strength k and each vortex in the lower row is of strength –k, show that the vortex system moves with uniform velocity

$$\frac{k}{2a} \cot h \frac{\pi b}{a}.$$

12. (a) A circular cylinder is placed in a uniform steam U with a circulation k round the cylinder. Show that the maximum velocity in a liquid is $2\cup+(k/2\pi a)$ where a is the radius of the cylinder. Find the force acting on the cylinder. 8

(b) In the two-dimensional irrotational motion of a liquid streaning past a fixed elliptic disc $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, the velocity at infinity being parallel to the major axis and equal to U, prove that if

$$x + \hat{1}y = C\cos^n(\xi + in)$$

$a^2 - b^2 = c^2$ and $a = \text{Cos}^n \alpha$, $b = c$ Sinh α the velocity at any point is given by

$$q^2 = U^2 \frac{a+b}{a-b} \frac{Sinh^2(\xi - \alpha) + Sin^2\eta}{Sinh^2\xi - Sin^2\eta}.$$

7

13. (a) Assuming the necessary stress-strain rate relations, deduce Navier–Stokes' equations of motion of an incompressible viscous fluid.

(b) Deduce Prandtl's boundary layer equations in two-dimensional flow. 8+7

Group—D

[Magneto hydrodynamics]

[Marks : 20]

14. Answer any *two* questions. 10×2

(a) State and prove Ferroro's low of isorotation.

(b) Write down the basic equations of magneto hydro dynamics. Derive the equation for the magnetic induction in MHD flows and explain the significance of flows at high and low magnetic Reynolds number.

(c) Obtain the Velocity and induced magnetic field components for Hartmann flow problem between two parallel non-conducting plates separated by a distance "(2d)". 10

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