2014

M.Sc. Part–I Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—III

Full Marks : 100

Time : 4 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Write the answer to questions of each group in Separate answer booklet.

Group—A

(*Probability and Statistics*)

[Marks : 30]

Answer any *two* questions : 15×2

1. (a) For the pure birth and death process, show that the interval T_K between the Kth and (K+1)th birth has an exponential distribution with parameter λ_K. Also find the mean population size under this process. 8

(b) In the usual notations, prove that :

$$R^2_{1.23} = \frac{r^2_{12}+r^2_{13}-2r_{12}r_{23}r_{31}}{1-r^2_{23}} \leq r^2_{12}$$

If $R_{1.23} = 1$, prove that $r_{2.13}$ is also equal to 1. If $R_{1.23} = 0$, does it necessarily mean that $R_{2.13}$ is also zero ? 4

(c) Explain partial correlation and multiple correlation coefficients. 3

2. (a) Define Wiener process. Deduce differential equations (forward and backward diffusion equations) for a Wiener process. 8

(b) Let $\{X_n, n \geq 0\}$ be a Markov chain having state space $S = \{1, 2, 3, 4\}$ and corresponding transition probability matrix :

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1/3 & 2/3 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 \end{bmatrix} \end{array}$$

Show that the state three and four are transient, the state two is persistent, and the state one is ergodic. 7

3. (a) What is the concept of probability distribution associated with Markov chain ? Let $\{X_n, n \geq 0\}$ be a Markov chain with three states 0, 1, 2 with transition matrix :

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 0 \\ 0 & 3/4 & 1/4 \end{bmatrix} \end{array}$$

and the initial distribution $P[X_0 = i] = 1/3$, $i = 0, 1, 2$. Find the value of $P[X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2]$. 4

(b) What do you mean by extinction probability ? Describe the linear growth process involving birth and death process and determine extinction probability of this process. 7

(c) Deduce the equation of the plane of regression of x_1 on x_2 and x_3. 4

Group—B

(*Numerical Analysis*)

[Marks : 40]

Answer *Q. No. 4* and any *three* from the rest.

4. Prove the following relations : 4

(a) Prove that :

$$\Delta\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\Delta f(x) - f(x)\Delta g(x)}{g(x+h)g(x)}, g(x) \neq 0.$$

(b) Let $f_i = f(x_i)$, where $x_i = x_0 + ih$, $i = 1, 2, \ldots$

Prove that $f_i = E^i f_0 = \sum_{j=0}^{i} \binom{i}{j} \Delta^j f_0$.

5. (a) Deduce Cubic spline interpolation formula, assuming that the spline is periodic. 8

(b) The value of the Elliptic integral :

$$E(\alpha) = \int_0^{\frac{\pi}{2}} \left(1 - \alpha \sin^2 \theta\right)^{-\frac{1}{2}} d\theta$$

for certain equidistance values of α are given below. Use Bessel's formula to evaluate E(0.25) : 4

α :	0.20	0.22	0.24	0.26	0.28	0.30
E(α) :	1.659624	1.669850	1.680373	1.691208	1.702374	1.713889

6. (a) Describe a suitable method to solve a system of tri-diagonal linear equations. 6

(b) For the matrix :

$$A = \begin{bmatrix} 4 & 2 & 1 \\ 2 & 5 & -2 \\ 1 & -2 & 7 \end{bmatrix}$$

Find L and U. Where A = LU and L is lower triangular matrix and V is upper triangular matrix.

7. (a) Describe a suitable method to solve the second order boundary value problem :

$$y'' + p(x)y' + q(x)y = r(x),$$

$a < x < b$, with the boundary conditions $y(a) = \gamma_1$ and $y(b) = \gamma_2$.

(b) Solve the following boundary value problem

$$y'' + xy' + 1 = 0$$

with boundary conditions $y(0) = 0$, $y(1) = 0$. 7+5

8. (a) Solve the following parabolic equation :

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$$

subject to the boundary conditions $u(0, t) = f_1(t)$, $u(1, t) = f_2(t)$ and initial condition $u(x, 0) = g(x)$.

(b) Discuss the stability of second order Runge-Kutta method.

6+6

9. (a) Define Chebyshev polynomial. What is its advantage to approximate a function ? 2

(b) Approximate the function $f(x) = e^x$ upto third degree using Chebyshev polynomial. 4

(c) Deduce three-point Gauss-Legendre quadratic formula. What is the truncation error of this formula ? 6

Group—C

(*Introduction to Computing*)

[Marks : 30]

10. Answer any *six* questions : 5×6

(a) Using Karnaugh map, simplify the following Boolean function :

$$f(A,B,C,D) = \sum(1,3,5,6,8,11,13) + \sum_{\phi}(0,2,4)$$

(b) What is an escape sequence ? What is its purpose ? Can escape sequences be included in a string constant ? Explain, how do string constants differ from character constants ?

(c) The value of π can be calculated from the infinite series :

$$\pi = 4 - 4|3 + 4|5 - 4|7 + 4|9 - 4|11 + \cdots$$

Write a program in C to find out the number of terms which have to be used before first getting the value 3.1415.

(d) What is structure ? How does a structure differ from an array ? How can structure variables be declared ? How are the members of structure variable assigned initial values ? How is a structure member accessed ?

(e) Write program segments that will read the value of x and evaluate the following function :

$$y = \begin{cases} 4x^3 + 100 & \text{for } x < 40 \\ 250 & \text{for } x = 40 \\ 5x^{10} + 2e^{x^2} & \text{for } x > 40 \end{cases}$$

using (i) nested if statements, (ii) conditional operator.

(f) Explain the different types of storage class in C language.

(g) Explain the use of the following functions with proper arguments :

strlen( ), strcpy( ), strrev( ), strclr( ), strset( ).

(h) Write a C program to accept a data in the year 2014 in the format dd/mm/yyyy and print the number of days starting from January 01, 2013.

(i) Subtract $(1001000)_2$ from $(0001110)_2$ using 2's complement method of binary subtraction. What do you mean by Carry and overflow during arithmetic operation ?

(j) Write a program in C to find the sum of all digits in a number using a function.

———