# 2014

## M.Sc. Part-II Examination

## APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

### PAPER—X (OR/OM)

*Full Marks* : 100

*Time* : 4 Hours

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

*Illustrate the answers wherever necessary.*

### SPECIAL PAPER : OR

### (Advanced Optimization and Operations Research—II)

Answer *Q. No. 11* and any *six* from the rest.

**1.** (a) Describe dynamic programming method to solve the following problem :

Maximize $Z = f_1(y_1) + f_2(y_2) + \ldots\ldots f_n(y_n)$

Subject to $y_1\ y_2 \ldots\ldots y_n \geq p$, $p > 0$, $y_j > 0$ for all j.

Using this method find the values of $y_1, y_2, y_3$, for the following problem.

Maximize $Z = y_1 + y_2 + y_3$

Subject to $y_1\ y_2\ y_3 \geq 8$. 3+5

*(Turn Over)*

(b) Solve the following LP problem :

Maximize $z = 8x_1 + 7x_2$

Subject to $2x_1 + x_2 \leq 8$

$5x_1 + 2x_2 \leq 15,$

$x_1, x_2, \leq 0$

Using dynamic programming technique. 8

2. (a) What is inventory ? What are the costs associated with inventory ? Discuss them briefly. Discuss different types of costs.

(b) Find the optimum order level for a product for which the price breaks are as follows :

| Range of quantity to be purchased | Unit purchase cost |
|---|---|
| $b_0 \leq Q < b_1$ | $p_1$ |
| $b_1 \leq Q < b_2$ | $p_2$ |
| $b_2 \leq Q < b_3$ | $p_3$ |

Here

(i) demand rate in known and uniform;

(ii) shortages are not permitted;

(iii) production for supply of commodities is instantaneous;

(iv) lead time is zero.

8+8





3. (a) Find the optimum order quantity for a product for which the price breaks are as follows :

| *Quantity* | *Purchasing Cost (per unit)* |
|---|---|
| $0 \leq Q_1 < 100$ | Rs. 20 |
| $100 \leq Q_2 < 200$ | Rs. 18 |
| $200 \leq Q_3$ | Rs. 16 |

The monthly demand for the product is 400 units. The storage cost is 20% of the unit cost of the product and the cost of ordering is Rs. 25 per month.

(b) What is simulation? Describe its advantages in solving the problems. Give its main limitations with suitable examples. Explain the different mathematical steps in a Monte Carlo simulation method.

8+8

4. (a) What is network analysis ? What are the advantages of it ?

What do you mean by time cost trade off ? What do you mean by crashing ? What is its advantage ? Define cost slope.

(b) A small project consists of seven activities the details of which are given below :

| *Activity* | *Time estimates* | | | *Predecessor* |
|---|---|---|---|---|
| | $t_o$ | $t_m$ | $t_p$ | |
| A | 3 | 6 | 9 | None |
| B | 2 | 5 | 8 | None |
| C | 2 | 4 | 6 | A |
| D | 2 | 3 | 10 | B |
| E | 1 | 3 | 11 | B |
| F | 4 | 6 | 8 | C, D |
| G | 1 | 5 | 15 | E |

Find the critical path. What is the probability that the project will be computed by 18 weeks? 8+8

**5.** (a) What is queuing theory? What information can be obtained by analyzing a queuing system? Explain four important parameters which are associated with the queuing theory.

(b) Obtain the expected queue length for the queuing model $(M/M/C) : (\infty/FCFS/\infty)$. 6+10

**6.** (a) What do you mean by information? Explain with an example. Draw a general structure of a communication system and explain it. 4

(b) Define entropy. Prove that the entropy function $H(p_1, p_2, \ldots, p_n)$ is continuous for each $p_k$, $0 \le p_k \le 1$. 4





(c) Define joint and conditional entropies. Prove that $H(X, Y) \le H(X) + H(Y)$, with equality iff X and Y are independent. Also, prove that $H(X) - H(X/Y) \ge 0$. 8

**7.** (a) Minimize the following function by geometric programming.

$$f(x) = x_1 x_2 x_3^{-2} + 2x_1^{-1} x_2^{-1} x_3 + 5x_2 + 3x_1 x_2^{-2}.$$

(b) Define the reliability of a system. What are MTBF and MTTF? Show that

$$R(t) = \exp\left[-\int_o^t \lambda(t)dt\right], \text{ where } R(t)$$

is the reliability function and $\lambda(t)$ represents the failure rate. 1+3+4

**8.** (a) The maintenance cost increases with time and the money value remains constant during a period. Then formulate the replacement policy and verify the following :

(i) If time is measured continuously then the average annual cost will be minimized by replacing the machine when the average cost till date becomes equal to the current maintenance cost.

(ii) If time is measured in discrete units, then the average annual cost will be minimized by replacing the machine when the next period's maintenance cost becomes greater than the current average cost.

8

(b) Define separable game. Find the value of the game for separable game whose payoff function is

M [x, y] = 3 Cos 7x Cos 8y + 5 Cos 7x Sin 8y + 2 Sin 7x Cos 8y + Sin 7x Sin 8y.

2+6

**9.** (a) A computer has 20,000 resistors. When any of the resistors fail, it is replaced. The cost of replacing a resistor individually is Rs. 1. If all the resistors are replaced at the same time the cost per resistor is reduced to Rs. 0.40. The percentage surviving at the end of month t, and the probability of failure during the month, are given below :

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| Percentage surviving of the end of t | 100 | 96 | 90 | 65 | 35 | 20 | 0 |
| Probability of failure during month t | – | 0·04 | 0·06 | 0·25 | 0·30 | 0·15 | 0·20 |

What is the optimum replacement plan ?

8

(b) Find the stationary path x = x(t) for the functional

$$J = \frac{1}{2}\int_0^2 (\ddot{x})^2 dt$$

Subject to the boundary conditions

$x(0) = 1,\ \dot{x}(0) = 1,\ x(2) = 0,\ \dot{x}(2) = 0$ 5

(c) What do you mean by optimal control ? 3





**10.** (a) Find an optimal sequence for the following sequencing problem of five jobs and four machines, when passing is not allowed. Its processing time (in hours) is given below :

| Machine | Job A | Job B | Job C | Job D | Job E |
|---|---|---|---|---|---|
| $M_1$ | 11 | 13 | 9 | 16 | 17 |
| $M_2$ | 4 | 3 | 5 | 2 | 6 |
| $M_3$ | 6 | 7 | 5 | 8 | 4 |
| $M_4$ | 15 | 8 | 13 | 9 | 11 |

8

(b) In a system, there are n number of components connected in series with reliability $R_i(t) = n$, i = 1,2, ..., n. Find reliability of the system. If $R_1(t) = R_2(t) = \ldots = R_n(t) = e^{-\lambda t}$ then find the liability of the system. 8

The system connected in series consist of three independence parts A, B and C which have MTBF of 100, 400 and 800 hours respectively. Find MTBF of the system and reliability of the systems for 30 hours. How much MTBF of the parts A has to be increased to get and improvement of MTBF of the system by 30% ? 8

**11.** Answer any *one* question : 4

(a) State Bellman's principle of optimality.

(b) Write a brief note on geometric programming.

## SPECIAL PAPER : OM

[Marks : 75]

Time : 3 hours

Answer any *five* questions. 5×15

1. (a) What is the purpose of aerological diagram ? Give the criteria of this diagram. Derive the area equivalence of tephigram and discuss its important features. 1+2+8

   (b) Derive the expression of the pressure gradient force in the atmosphere. 4

2. (a) Derive the horizontal equation of motion of an air parcel in the atmosphere in natural co-ordinate system. 8

   (b) Define virtual temperature and show that if $T_v$ is the virtual temperature, then $T_v = T(1 + 0.61r)$ where r is the mixing ration. 3

   (c) What is Rossby Wave ? Find the variation of Coriolis parameter to create Rossby Wave. 4

3. (a) Derive the meridional temperature gradient due to Global Circulation and also find meridional temperature and temperature gradient at 45°N latitude at sea level. 6

   (b) Derive the hypsometric equation in the atmosphere 4

   (c) Derive the angle between the frontal surface and earth's surface. 5





4. (a) What is thermal wind in the atmosphere ? Derive the equation of thermal wind. 2+6

   (b) Derive an expression for the density $\rho$ of an air parcel at pressure $p$ if it is adiabatically expands from a level where pressure and density of $p_s$ and $\rho_s$ respectively. 4

   (c) What is the concept of coriolis force in the atmosphere ? 3

5. (a) Define specific entropy and establish the relationship between the specific entropy and the potential temperature. 6

   (b) Derive the pressure tendency below a frontal surface. 5

   (c) Derive the specific heat constant with respect to pressure of moist air in terms of dry air. 4

6. (a) Define homogeneous atmosphere. Show that the height of the homogeneous atmosphere depends entirely on the temperature at the bottom. Also prove that the pressure at the top of the homogeneous isothermal atmosphere is equal to $\frac{1}{e}$ time that at the sea level. 1+3+2

   (b) Obtain the atmospheric energy equation and interpret each term. 9

7. (a) Deduce the linearized equations of two – dimensional internal gravity waves propagating in the x – t plane neglecting the Coriolis force.

8

(b) Show that in a geostropic wind field, an ideal front is necessarily stationary.

2

(c) What is humidity variable ? Discuss different types of humidity variables.

1+4

8. (a) Derive the adiabatic lapse rate of unsaturated moist air.

4

(b) "The pressure is taken as a vertical co-ordinate in the atmosphere" — Justify this statement.

3

(c) Discuss about the phase change of an ideal gas and derive the relation of dependency of latent heart of evaporation with respect to temperature during the phase change of an air parcel.

8

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