Synopsis of the Ph.D. thesis entitled

A Study of Some Hydromagnetic Problems with or without Hall Currents

submitted to the
Vidyasagar University, Midnapore, India
for the award of the degree of

Doctor of Philosophy

(Science)

in

Applied Mathematics

by

Bhaskar Chandra Sarkar

Department of Applied Mathematics
with Oceanology and Computer Programming
Vidyasagar University, Midnapore-721102
West Bengal, India

May 2013

SYNOPSIS

Magnetohydrodynamics (MHD) is an academic discipline which studies the dynamics of electrically conducting fluids in the presence of the electric and magnetic fields. MHD is the combination of three disciplines, viz, Electrodynamics, Fluid dynamics and Thermodynamics. There is an interaction between the flow field and the electromagnetic field. An induced electric current is generated due to the flow of the conducting fluid in the presence of magnetic field which modifies the electromagnetic field. These currents experience a mechanical force, called the Lorentz force. This force tends to modify the initial motion of the conductor. Moreover, the induced currents generate their own magnetic field which is added to the primitive magnetic field. Thus there is an interlocking between the fluid motion and the electromagnetic fields. Applications of magnetohydrodynamics are very broad ranging from astrophysics to plasma physics. There are two serious technological applications of MHD, that may both become very important in the future. Firstly, strong magnetic fields are used to confine rings or columns of hot plasma that will be held in place long enough for the thermonuclear fusion to occur and for the net power to be generated. In the second application, the liquid metals are driven through a magnetic field in order to generate electricity. However, the major use of MHD is in plasma physics. It has found wide applications in problems of geophysics, power-generation, space-research, thermonuclear fusion, aeronautic, liquid-metal cooling of nuclear reactors, electromagnetic casting and many other engineering fields. In aeronautical engineering MHD finds its application in MHD generators, ion propulsion, MHD bearing, MHD pumps, MHD boundary layer control of reentry vehicles etc. It is also used in reducing drag and hence enhancing the vehicle power by a substantial amount, designing thermosyphon tube, cooling turbine blades etc. The MHD principle also finds its application in medicine and biology. Application in the biomedical engineering includes cardiac MRI, ECG etc. The principle of MHD is also used in stabilizing a flow against the transition from laminar to turbulent flow. An important task in cancer research is developing more precise methods for delivery of medicine to affected areas. One method involves the binding of medicine to the biologically compatible magnetic particles (e.g. ferrofluids), which are guided to the target via careful placement of the permanent magnets on the external body. MHD, in its present form is due to the pioneer contribution of several notable authors. Hartmann [1] has initiated the study of the subject in the name Hg-dynamics in his efforts to pump mercury by exploitation of hydrodynamical pressure and electromagnetic fields. The systematic study under the present name begins with discovery of transverse waves by Alfv́en [2] while he is engaged in the theoretical investigations of sunspots. The study of magnetohydrodynamics problems has been made by many reputed authors - Bullard [3], Cowling [4], Chandrasekhar [5], Shercliff [6], Ferraro and Plumpton [7], Cramer and Pai [8], Davidson [9] and many others.

The thesis is devoted to the study of some magnetohydrodynamic problems with or without Hall currents. The thesis consists of **ten** chapters. The basic equations of magnetohydrodynamics and a brief literature review are described in **Chapter 1**. The problems related to the present work are studied in **Chapter 2 to 9** and **Chapter 10** deals with the conclusions and suggestions for future works.

CHAPTER 2

Radiation effects on free convection MHD Couette flow started exponentially with variable wall temperature in presence of heat generation[1]

Consider the unsteady free convective MHD Couette flow of a viscous incompressible electrically conducting radiative heat generating fluid between two infinitely long vertical parallel walls separated by a distance h. The flow is set up by the buoyancy force arising from the temperature gradient occurring as a result of asymmetric heating of the parallel walls as well as exponentially accelerated motion of one of the walls. Choose a Cartesian co-ordinates system with the x-axis along one of the walls in the vertically upward direction and the y-axis normal to the walls [See Fig.1]. Initially, at time $t \leq 0$, both the walls and the fluid are assumed to be at the same temperature T_h and stationary. At time $t > 0$, the wall at $y = 0$ starts to move in its own plane with a velocity $u_0\, e^{a' t}$ and the temperature of the plate rises to $T_h + (T_0 - T_h)\, \frac{t}{t_0}$, T_0 being the temperature of the wall at $y = 0$ and t_0 being constant whereas the wall at $y = h$ is stationary and maintained at a constant temperature T_h, where u_0 and a' are constants. A uniform transverse magnetic field of strength B_0 is imposed perpendicular to the walls. It is also assumed that the radiative heat flux in the x-direction is negligible as compared to that in the y-direction. As the walls are infinitely long, the velocity and temperature distributions are functions of y and t only.

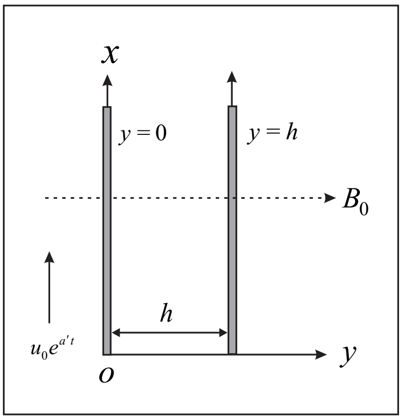

Fig.1: Geometry of the problem

Results: The dimensionless governing equations are solved by the usual Laplace transform technique. The results obtained are presented either graphically or in the tabular form. It is observed that the fluid velocity u_1 decreases with an increase in either squared-Hartmann number M^2 or radiation parameter R or Prandtl number Pr whereas the fluid velocity u_1 increases with an increase in either heat generation parameter ϕ or Grashof number Gr or accelerated parameter a or time τ. An increase in either radiation parameter R or Prandtl number Pr leads to fall in the fluid temperature θ. The fluid temperature θ increases with an increase in either heat generation param-

[1] *Published in* **Open Journal of Fluid Dynamics** *2 (2012) 14-28*

eter ϕ or time τ. Further, it is observed that the shear stress τ_x at the wall $\eta = 0$ decreases with an increase in either squared-Hartmann number M^2 or radiation parameter R while it increases with an increase in either heat generation parameter ϕ or Prandtl number Pr. The rate of heat transfer $-\theta_\eta(0,\tau)$ at the wall $\eta = 0$ increases with an increase in either radiation parameter R or Prandtl number Pr or time τ whereas it decreases with an increase in heat generation parameter ϕ.

CHAPTER 3

MHD natural convection between vertical parallel plates with oscillatory wall temperature[2]

Consider the unsteady MHD natural convective flow of a viscous incompressible electrically conducting fluid between two infinitely long vertical parallel plates separated by a distance h in the presence of a transverse magnetic field. The x-axis is taken along one of the plates in the vertically upward direction and the y-axis is taken normal to the plates [See Fig.2]. The temperature of the plate at $y = 0$ is T_1 and the temperature of the plate at $y = h$ is $T_2 + \epsilon(T_2 - T_1)\cos\omega^* t$ where ω^* is the frequency of oscillations of the plate temperature and ϵ a small real constant quantity $\ll 1$. A uniform transverse magnetic field of strength B_0 is imposed perpendicular to the plates. The magnetic Reynolds number is assumed to be very small, so that the induced magnetic field is negligible. Also, it is assumed that there is no applied voltage so that the electric field is absent.

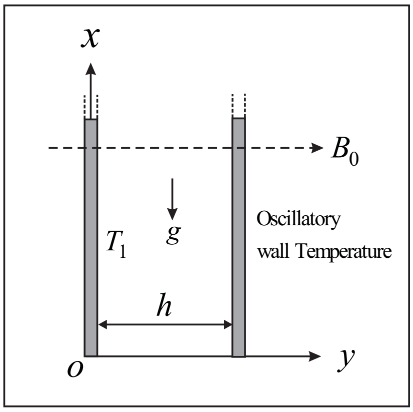

Fig.2: Geometry of the problem

Results: The governing non-linear partial differential equations are solved analytically. The velocity and temperature distributions are presented graphically whereas the shear stress, the rate of heat transfer and their amplitudes and tangent of phase are given in tabular form. It is observed that the fluid velocity u decreases with an increase in either squared-Hartmann number M^2 or frequency parameter ω or phase angle $\omega\tau$ or Prandtl number Pr. A decrease in the fluid temperature θ occurs due to an increase in either frequency parameter ω or phase angle $\omega\tau$ or Prandtl number

[2] *Published in* **Journal of Computer and Mathematical Science** *(2012).*

Pr. Further, the magnitude of the shear stress at the plate $\eta = 1$ increases with an increase in either frequency parameter ω or Eckert number Ec or Prandtl number Pr while it decreases with an increase in either squared-Hartmann number M^2 or phase angle $\omega\tau$. It is also seen that the rate of heat transfer $-\theta_\eta(1,\tau)$ at the plate $\eta = 1$ increases with an increase in either squared-Hartmann number M^2 or frequency parameter ω or Prandtl number Pr whereas it decreases with an increase in either phase angle $\omega\tau$ or Eckert number Ec.

CHAPTER 4

Transient MHD natural convection between two vertical walls heated/cooled asymmetrically[3]

Consider an unsteady MHD natural convective flow of a viscous incompressible electrically conducting fluid between two infinitely long vertical parallel walls separated by a distance h in the presence of a uniform transverse magnetic field. Choose a Cartesian co-ordinates system with the x-axis along one of the walls in the vertically upward direction and the y-axis normal to the walls [See Fig.3]. Initially, at time $t \leq 0$, the two walls and the fluid are assumed to be at the same temperature T_m and stationary. At time $t > 0$, the wall at $y = 0$ starts to move in its own plane with a velocity $\lambda\, u_0$ and it is heated with temperature T_0 whereas the temperature at the wall $y = h$ is rised to T_h, λ being a constant. A uniform transverse magnetic field of strength B_0 is imposed perpendicular to the walls. As the walls are infinitely long, the velocity field and the temperature distribution are functions of y and t only.

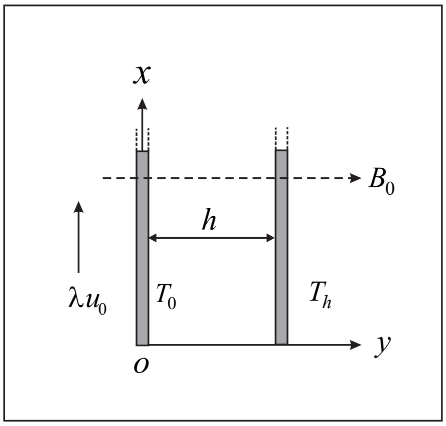

Fig.3: Geometry of the problem

Results: The governing equations are solved analytically using the Laplace transform technique. It is observed that the fluid velocity u_1 decreases for both the stationary walls as well as for the impulsive motion of one of the walls with an increase in either squared-Hartmann number M^2 or Prandtl number Pr. It is also observed that the fluid velocity u_1 increases with an increase in either

[3] *Published in* **International Journal of Computer Applications** *52(3) (2012) 27-34*

Grashof number Gr or temperature difference ratio r_T or time τ for both the stationary walls as well as for the impulsive motion of one of the walls. An increase in the fluid temperature $\theta(\eta, \tau)$ occurs due to an increase in either time τ or temperature difference ratio r_T. Further, the shear stress τ_x at the wall $\eta = 0$ increases with an increase in either Grashof number Gr or temperature difference ratio r_T whereas it decreases with an increase in Prandtl number Pr for both the stationary walls as well as for the impulsive motion of one of the walls. The rate of heat transfer $-\theta_\eta(0, \tau)$ at the wall $\eta = 0$ increases with an increase in Prandtl number Pr whereas it decreases with an increase in either temperature difference ratio r_T or time τ.

CHAPTER 5

Effects of radiation on MHD free convective Couette flow in a rotating system[4]

Consider the unsteady free convective MHD Couette flow of a viscous incompressible electrically conducting fluid between two infinitely long vertical parallel walls separated by a distance h. Choose a Cartesian co-ordinates system with the x-axis along one of the walls in the vertically upward direction and the z-axis normal to the walls and the y-axis is perpendicular to the xz-plane [See Fig.4]. The fluid and the walls rotate in unison with a uniform angular velocity Ω about z-axis. Initially, at time $t \leq 0$, both the walls and the fluid are assumed to be at the same temperature T_h and stationary. At time $t > 0$, the wall at $z = 0$ starts to move in its own plane with a velocity $U(t)$ and is heated with temperature $T_h + (T_0 - T_h)\,\frac{t}{t_0}$, T_0 being the temperature of the wall at $z = 0$ and t_0 being a constant. The wall at $z = h$ is stationary and maintained at a constant temperature T_h. A uniform transverse magnetic field of strength B_0 is imposed perpendicular to the walls. It is also assumed that the radiative heat flux in the x-direction is negligible as compared to that in the z-direction. As the walls are infinitely long, the velocity and temperature fields are functions of z and t only.

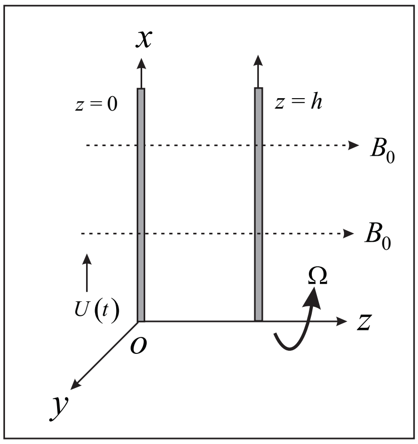

Fig.4: Geometry of the problem

Results: The governing equations are solved analytically using the Laplace transform technique.

[4] *Published in* **International Journal of Engineering Research and Applications** *2(4) (2012) 2346-2359*

The variations of fluid velocity components and fluid temperature are presented graphically. It is observed that the magnitude of the primary velocity u_1 decreases whereas the magnitude of the secondary velocity v_1 increases for both the impulsive as well as the accelerated motion of one of the walls with an increase in either squared-Hartmann number M^2 or radiation parameter R. The fluid temperature decreases with an increase in either radiation parameter R or Prandtl number Pr whereas it increases with an increase in time τ. The absolute value of the shear stress τ_{x0} at the wall $\eta = 0$ due to the primary flow and the shear stress τ_{y0} at the wall $\eta = 0$ due to the secondary flow for both the impulsive as well as the accelerated motion of one of the walls increase with an increase in either radiation parameter R or rotation parameter K^2. Further, the rate of heat transfer $-\theta_\eta(0,\tau)$ at the wall $\eta = 0$ increases whereas the rate of heat transfer $-\theta_\eta(1,\tau)$ at the wall $\eta = 1$ decreases with an increase in radiation parameter R.

CHAPTER 6

Hall effects on MHD Couette flow in rotating system[5]

Consider the viscous incompressible electrically conducting fluid bounded by two infinitely long horizontal parallel porous plates separated by a distance d. Choose a Cartesian co-ordinates system with the x-axis along the lower stationary plate in the direction of the flow, the y-axis is normal to the plates and the z-axis perpendicular to the xy-plane [see Fig.5]. The upper plate moves with a uniform velocity U in the x-direction. The plates and the fluid are in a state of rigid body rotation with a uniform angular velocity Ω about the y-axis. A uniform transverse magnetic field B_0 is applied perpendicular to the plates. The velocity components are u, v and w relative to a frame of reference rotating with the fluid. Since the plates are infinitely long, all physical variables, except pressure, depend on y only. The equation of continuity then gives $v = -v_0$ everywhere in the fluid where v_0 is the suction velocity at the plates.

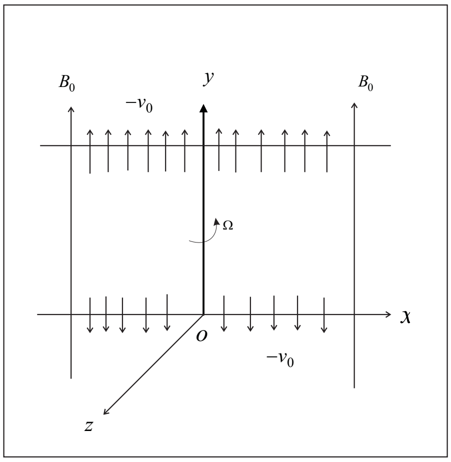

Fig.5: Geometry of the problem

[5] *Published in* **International Journal of Computer Applications** *35(13)(2011) 22-30*

Results: An exact solution of the governing equation has been obtained in closed form. The velocity field and the temperature distribution have been presented graphically. It is observed that the primary velocity u_1 decreases whereas the magnitude of the secondary velocity v_1 increases with an increase in Hall parameter m. It is also observed that the primary velocity u_1 increases whereas the magnitude of the secondary velocity v_1 decreases with an increase in either magnetic parameter M^2 or Reynolds number Re. The fluid temperature $\theta(\eta)$ increases with an increase in either Hall parameter m or Reynolds number Re whereas it decreases with an increase in rotation parameter K^2. The resultant shear stresses τ_0 at the plate $\eta = 0$ and τ_1 at the plate $\eta = 1$ increase with an increase in Hall parameter m. Further, the rate of heat transfer $\theta'(0)$ at the plate $\eta = 0$ increases while the rate of heat transfer $\theta'(1)$ at the plate $\eta = 1$ decreases with an increase in either Hall parameter m or Reynolds number Re.

CHAPTER 7

Combined effects of Hall Current and rotation on steady hydromagnetic Couette flow[6]

Consider the viscous incompressible electrically conducting fluid bounded by two infinitely long horizontal parallel plates separated by a distance d. Choose a Cartesian co-ordinates system with the x-axis along the lower stationary plate in the direction of the flow, the z-axis is normal to the plates and the y-axis is perpendicular to the xz-plane [see Fig.6]. The lower plate is perfectly conducting whereas the upper plate is non-conducting. The upper plate is moving with a uniform velocity U_0 while the lower plate is held at rest. The fluid and the plates rotate in unison with a uniform angular velocity Ω about an axis perpendicular to the plates. A uniform transverse magnetic field B_0 is applied perpendicular to the plates in the positive z-direction. Since the plates are infinitely long, all physical variables, except pressure, depend on z only.

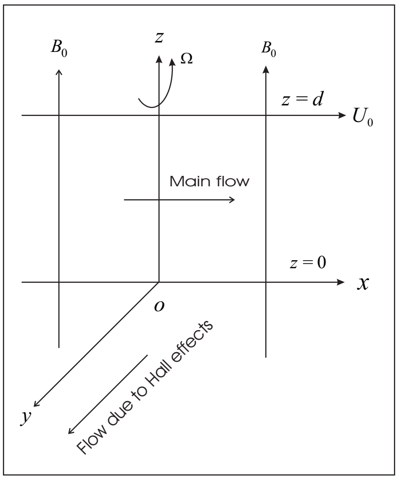

Fig.6: Geometry of the problem

[6] *Published in* **Research Journal of Applied Sciences, Engineering and Technology** *5(6) (2013) 1864-1875*

Results: The governing equations describing the flow are solved analytically. The results obtained are shown either graphically or in the tabular form. It is found that both the primary velocity u_1 and the secondary velocity v_1 decrease with an increase in the squared-Hartmann number M^2. It is also observed that both the velocity components increase with an increase in rotation parameter K^2. It is seen that the primary velocity u_1 increases whereas the secondary velocity v_1 decreases with an increase in Hall parameter m. It is observed that both the primary and secondary induced magnetic field components b_x and b_y decrease with an increase in squared-Hartmann number M^2 whereas they increase with an increase in rotation parameter K^2. It is also seen that the primary induced magnetic field component b_x decreases while the secondary induced magnetic field component b_y increases with an increase in Hall parameter m. It is found that the shear stress at the lower plate due to the primary flow increases while the shear stress at the lower plate due to the secondary flow decreases with an increase in Hall parameter m. The heat transfer characteristics have also been studied on taking viscous and Joule dissipations into account. It is found that the rate of heat transfer at the lower plate decreases while the rate of heat transfer at the upper plate increases with an increase in Hall parameter m.

CHAPTER 8

Hall effect on MHD free convection boundary layer flow past a vertical flat plate [7]

Consider the steady MHD free convective boundary layer flow of a viscous incompressible electrically conducting fluid confined to an infinitely long vertical flat plate of finite dimension. Choose a Cartesian co-ordinates system in such a way that the x- and z-axes are in the plane of the plate with the x-axis along the upward direction while the y-axis is perpendicular to the xz-plane [see Fig.7]. A uniform transverse magnetic field of strength B_0 is imposed perpendicular to the plate. Since the plate is infinitely long, all physical quantities, except pressure, will be a function of y only. The flow is generated due to the buoyancy force in the presence of a uniform transverse magnetic field.

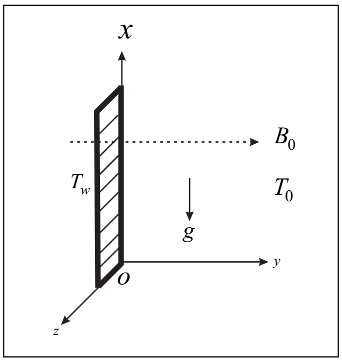

Fig.7: Geometry of the problem

[7] *Published in* **Meccanica** *on line (2012)*

Results: An exact solution of the governing equations describing the flow has been obtained. The velocity field, induced magnetic field and bulk temperature distribution in the boundary layer flow have been discussed. It is found that the primary velocity u_1 increases while the secondary velocity w_1 decreases with an increase in Hall parameter m. It is also found that the induced magnetic field components b_x and b_z decrease near the plate and reversed results occur away from the plate with an increase in Hall parameter m. It is seen that the magnitude of the bulk temperature θ_x in the x-direction decreases with an increase in either Hall parameter m or squared-Hartmann number M^2. On the other hand, the magnitude of the bulk temperature θ_z in the z-direction increases with an increase in Hall parameter m whereas it decreases with an increase in squared-Hartmann number M^2. It is seen that the shear stress components at the plate $\eta = 0$ due to the primary and secondary flows increase with an increase in either Hall parameter m or Grashof number Gr .

CHAPTER 9

Hall effects on unsteady MHD free convective flow past an accelerated moving vertical plate with viscous and Joule dissipations

Consider the unsteady hydrodynamic flow of a viscous incompressible electrically conducting fluid past a uniformly accelerated vertical plate on taking viscous and Joule dissipations into account. Choose a Cartesian co-ordinates system such that the x-axis is taken along the vertical plate in an upward direction, z-axis is perpendicular to the plate and y-axis is taken normal to the zx-plane. At time $t \leq 0$, both the fluid and the plate are at rest with constant temperature T_∞. At time $t > 0$, the plate at $z = 0$ starts to move in its own plane with a uniform velocity ct, $c(> 0)$ being a constant and T_w is the plate temperature. A uniform transverse magnetic field H_0 is applied perpendicular to the plate (see Fig.8). As the plate is infinitely long, the fluid velocity components and temperature distribution are functions of z and t only.

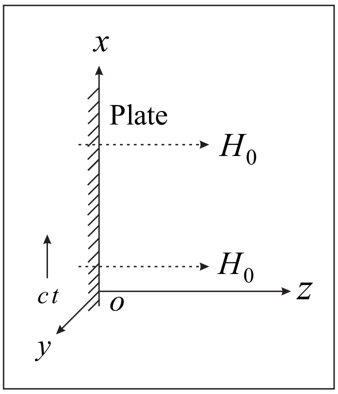

Fig.8: Geometry of the problem

Results: The governing non-linear partial differential equations have been solved numerically using

Crank-Nicolson's type of implicit finite difference method with a tri-diagonal matrix manipulation and an iterative procedure. It is found that the primary velocity u and the magnitude of the secondary velocity v increase with an increase in either Hall parameter m or Eckert number Ec or Grashof number Gr. It is also found that the fluid temperature θ increases with an increase in either Hall parameter m or Eckert number Ec or Grashof number Gr or Prandtl number Pr. Further, it is found that the absolute value of the shear stresses τ_x and τ_y at the plate $\eta = 0$ increase with an increase in either Hall parameter m or Eckert number Ec. The rate of heat transfer $-\left(\frac{\partial \theta}{\partial \eta}\right)_{\eta=0}$ at the plate $\eta = 0$ decreases with an increase in either Eckert number Ec or Prandtl number Pr or time τ.

CHAPTER 10

Future works

For future work, Chapters 2, 3, 4 and 5 can be extended to the problem on taking Hall currents into account. Chapters 6 and 7 can be extended to an unsteady case. Chapter 8 and 9 can be extended to the problem in rotating system. These problems can also be extended in porous medium.

References

[1] Hartmann, J., Kgl. Danske Vidensk. Selsk. Mat. Fys. Medd., 15 (1937) 6.

[2] Alfv́en, H., Nature, 150 (1942) 405.

[3] Bullard, E. C., Mon. Nat. Roy. Astro. Soc., Geophys. Suppl., 5 (1948) 245.

[4] Cowling, T. G., Magnetohydrodynamics, interscience publ., Inc, New York (1957).

[5] Chandrasekhar, S., Radiative transfer, Dover New York, (1960).

[6] Shercliff, J. A., A text book of Magneto hydrodynamics, Pergamon Press, London, (1965).

[7] Ferraro V. C. A. and Plumpton, C., An introduction to magneto fluid mechanics, Clarandon Press, Oxford, (1966).

[8] Cramer, K. P. and Pai, S. L., Magneto-fluid dynamics for engineers and applied physics, McGraw-Hill book Co. New York, (1978).

[9] Davidson, P. A., An Introduction to magnetohydrodynamics, Cambridge Univ. Press, (2001).

Published

1. **Hall effects on MHD Couette flow in rotating system**, *International Journal of Computer Applications (ISSN: 0975 -8887) 35(13) (2011) 22-30.*

2. **Radiation effects on free convection MHD Couette flow started exponentially with variable wall temperature in presence of heat generation**, *Open Journal of Fluid Dynamics (ISSN : 2165-3852) 2 (2012) 14-28.*

3. **Effects of radiation on MHD free convective Couette flow in a rotating system**, *International Journal of Engineering Research and Applications (ISSN: 2248-9622) 2(4) (2012) 2346-2359.*

4. **Transient MHD natural convection between two vertical walls heated/cooled asymmetrically**, *International Journal of Computer Applications (ISSN: 0975 -8887) 52(3) (2012) 27-34.*

5. **MHD natural convection between vertical parallel plates with oscillatory wall temperature**, *Journal of Computer and Mathematical Science (ISSN: 0976-5727) 3(4) (2012) 426-438.*

6. **Hall effect on MHD free convection boundary layer flow past a vertical flat plate**, *Meccanica (2012) online.*

7. **Combined effects of Hall Currents and Rotation on Steady Hydromagnetic Couette flow**, *Research Journal of Applied Sciences, Engineering and Technology (ISSN: 2040-7459) 5(6) (2013) 1864-1875.*

Communicated for publication

1. **Hall effects on unsteady MHD free convective flow past an accelerated moving vertical plate with viscous and Joule dissipations.**