Synopsis of the Ph.D. thesis entitled

SOME STUDIES ON FLUID FLOW IN POROUS MEDIUM

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by

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SYNOPSIS

The thesis has been devoted to study the fluid flow in porous medium. The flow behavior through porous medium, suction or blowing on momentum and the heat transfer characteristics in some of these flows have been studied in detail. An investigation has also made on the flow of a viscous incompressible radiating fluid past a porous plate or disks in a rotating system through porous medium.

Extensive research works have been carried out in recent years to study the fluid flow through porous media. Due to its broad range of applications in science and industry, this field has gained extensive attention lately. The concept of porous media is used in many areas of applied science and engineering such as filtration, acoustics, geomechanics, soil mechanics, rock mechanics, petroleum engineering, bio-remediation, construction engineering, hydrogeology, petroleum geology, geophysics, biology and biophysics, material sciences etc. It is also of great importance in technical fields such as ground water hydrology, mining industry, drainage and irrigation engineering, textile and ceramic engineering, formation and dispersion of fog, distribution of temperature and moisture over agricultural fields and fruit trees, crop damage due to freezing and environmental disorders, recovery or extraction of crude oil, storage of nuclear wastes, packed bed catalytic reactors, atmospheric and oceanic circulations etc. The movement of oil and natural gas through the reservoirs, well drilling and logging are being studied in petroleum engineering. The water in the earth and sand structures, water bearing formations and filter beds for purification of drinking water are the subjects of study in hydrology. In filtration studies, the main concern is to determine how fluid moves through the porous structure leaving behind unwanted material. The flow of blood and other body fluids and electro-osmosis are few examples in medicine and bio-chemical engineering where the role played by porous media is important. The porosity of a porous medium continuously changes and alters the pressure drop characteristics of the medium. The porosity is also important in metallic, plastic, enamel coatings and construction of bridges and high rise buildings. The study of heat transfer processes in porous media is a well developed field of investigation because of its importance in a variety of situations occurring in geothermal systems, microelectronic heat transfer equipment, thermal insulation and thermoacoustic engines which can provide cooling or heating. Porous materials are also used in heat exchangers, building thermal insulators, porous insulators for fire fighting etc. In the nuclear waste disposal industry, to model a suitable canester is very essential for the safety analysis. Axi-symmetric bodies are utilized as canesters. Their disposal to the sea bed or to the earth's crust needs a better understanding of the convection phenomenon in the porous medium. The extraction of petroleum to the last drop from the oil reservoirs in the earth's crust needs a nice knowledge of the convection mechanism and through understanding of the miscible displacement techniques in porous medium. Nield and Bejan [1-3] Kaviany [4], Ingham and Pop [5-7] and Vafai [8] have surveyed topics on fluid flow and heat transfer in porous media.

The thesis has been devoted to the study of some fluid dynamics problems with porous media. The effects of porosity, suction or blowing, radiation, buoyancy force and heat transfer characteristics in some of these flows have been studied analytically and numerically. The thesis consists of **ten** chapters. The **first chapter** includes a brief discussion on certain relevant topics, like basic equations governing the flow of a viscous incompressible fluid through a homogeneous isotropic porous medium and related literature review. The problems related to the present work are studied in **Chapters 2 to 9** and **Chapter 10** deals with the conclusions and suggestions for future works.

Unsteady Flow of Viscous Fluid through a Porous Medium Bounded by a Porous Plate in a Rotating System [1]

Consider the unsteady flow of a viscous incompressible fluid past an infinitely long porous flat plate embedded in a porous medium in a semi-infinite expanse of fluid ($z > 0$). Choose a Cartesian co-ordinates system with x-axis along the plate in the direction of the flow, y-axis perpendicular to it and z-axis normal to the xy-plane. The fluid and the plate are in a state of rigid body rotation with a uniform angular velocity Ω about the z-axis (see Fig. 1). At time $t \leq 0$, both the fluid and the plate are at rest. At time $t > 0$, the plate suddenly starts to move in its own plane with a uniform velocity U in the direction of the flow.

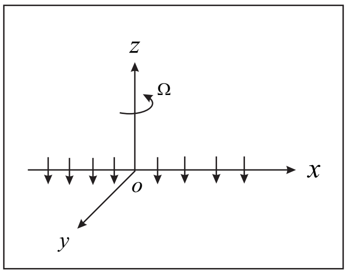

Fig. 1: Geometry of the problem

Conclusion:
The unsteady flow of a viscous incompressible fluid past an infinitely long porous plate embedded in a porous medium in a rotating system has been studied. It is observed that the fluid velocities increase with an increase in porosity parameter. It is interesting to note that for small time, the primary velocity is independent of rotation while the secondary velocity is affected by the rotation. It is also noticed that for small time, there are no inertial oscillations in the flow field but for large time, the fluid flow reaches to the steady state through the inertial oscillations. Furthermore, the inertial oscillations are unaffected by both the suction/blowing and the porosity parameter while they are affected by the rotation parameter only.

Laminar Free and Forced Convective Flow between Two Vertical Plates Embedded in Porous Medium [2]

Consider a viscous incompressible fluid flow between two infinitely long vertical parallel plates embedded in a porous medium. The distance between the plates is d. The origin being taken at the left plate of the channel, x-axis vertically upward along the direction of the flow and y-axis perpendicular to it (see Fig. 2). The plate at $y = 0$ has a uniform temperature T_2 while the plate at $y = d$ is subjected to a uniform temperature T_1, where $T_1 > T_2$. Since the channel plates are infinitely long along x-direction, all the physical variables depend on y only. It is assumed that the free and forced convective flow entering the channel is directed vertically upward whereas the pure free convection is motivated by a zero-pressure gradient.

[1] *Published in* **Journal of Porous Media, 2010, 13(7), pp. 645-654**
[2] *Accepted in* **World Journal of Mechanics**

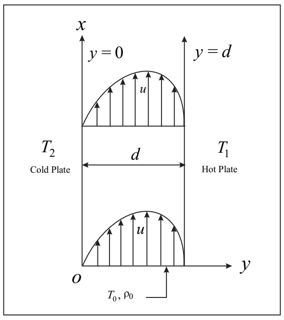

Fig. 2: Geometry of the problem

Conclusion:
The steady laminar free and forced convective flow between two infinitely long vertical parallel plates embedded in a porous medium has been investigated. In the presence of a porous medium, the velocity distribution is symmetric for small values of Grashof number while it is asymmetric for large values of Grashof number. It is also seen that the effect of porosity is prominent near the cold plate while it is nearly unaffected near the hot plate of the channel. It is also observed that for small values of Grashof number, the fluid velocity decreases near the channel plates while it increases at the middle of the channel with an increase in porosity parameter. The shear stress at the cold plate and the absolute value of the shear stress at the hot plate decreases with an increase in porosity parameter. The critical Grashof number for which there is no flow reversal near the cold plate, decreases with an increase in porosity parameter. The bulk temperature increases with an increase in either Grashof number or porosity parameter.

Effects of Radiation on Free Convection Flow in a Vertical Channel Embedded in Porous Medium[3]

Consider a viscous incompressible fluid flow through a porous medium between two vertical walls in the presence of a uniform gravitational field. The distance between the channel walls is d. Employ a Cartesian co-ordinates system with x-axis vertically upward along the direction of the flow and y-axis perpendicular to it. The origin of the reference axes is such that the channel walls are at $y = -\frac{d}{2}$ and $y = \frac{d}{2}$ (see Fig. 3). We do not model the pressure drop across the end caps and consider only the flow far from the end caps.

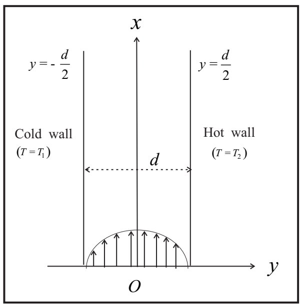

Fig. 3: Geometry of the problem

[3] *Published in* **International Journal of Computer Applications, 2011, 35(6), pp. 38-44**

Conclusion:
Effects of radiation on the free convective flow of a viscous incompressible fluid through a porous medium between two vertical walls have been studied in the presence of a uniform gravitational field. It is observed that both the fluid velocity and the fluid temperature decrease with an increase in radiation parameter. The porosity parameter and buoyancy force (Grashof number) accelerate the fluid velocity while the radiation parameter and temperature parameter have a retarding influence on the fluid velocity. The rate of heat transfer at the cold wall decreases while that at the hot wall increases with an increase in radiation parameter. Also, the critical value of the temperature parameter at the cold wall decreases with an increase in radiation parameter while it increases with an increase in porosity parameter.

Convection of Radiating Gas in a Vertical Channel through Porous Medium[4]

Consider a viscous incompressible fluid flow in a vertical channel embedded in a porous medium. The distance between the channel walls is $2L$. Employ a Cartesian co-ordinates system with z-axis vertically upward along the direction of the flow and y-axis perpendicular to it. The origin of the axes is such that the channel walls are at $y = -L$ and $y = L$ (see Fig. 4). For the laminar flow in the porous medium, the velocity field has only a vertical component and all the physical variables except temperature and pressure are functions of y only.

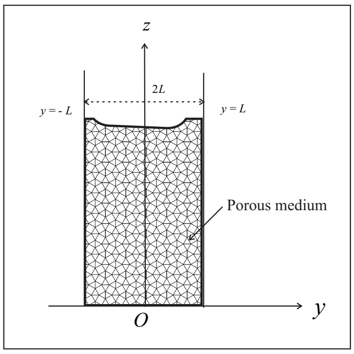

Fig. 4: Geometry of the problem

Conclusion:
The convective flow of a radiating gas between two thermally conducting vertical walls embedded in a porous medium has been studied. It is observed that the fluid velocity increases while the fluid temperature decreases with an increase in the radiation parameter. It is also observed that both the fluid velocity and the fluid temperature increase with an increase in the porosity parameter. It is found that the fluid velocity decreases while the fluid temperature increases with an increase in the thermal conductance parameter. Further, it is found that radiation causes to decrease the rate of heat transfer to the fluid, thereby reducing the natural convection effect. The rate of flow increases with an increase in porosity parameter.

[4] *Published in* **World Journal of Mechanics, 2011, 1, pp. 275-282**

Radiation Effect on Natural Convection near a Vertical Plate Embedded in Porous Medium with Ramped Plate Temperature[5]

Consider the unsteady free convective flow of an optically thin viscous incompressible fluid past a moving infinitely long vertical flat plate coinciding with plane $y = 0$, where the flow is confined to $y > 0$ in a porous medium. Choose a Cartesian co-ordinates system with x-axis vertically upward along the plate and y-axis perpendicular to it (see Fig. 5). At time $t \leq 0$, the plate and the surrounding fluid are at the same constant temperature T_∞. At time $t > 0$, the plate starts to move with a constant velocity U_0 vertically upwards direction and the temperature of the plate is raised or lowered to $T_\infty + (T_w - T_\infty)\frac{t}{t_0}$ when $0 < t \leq t_0$ and the constant temperature T_w is maintained at time $t > t_0$. Since the plate is infinitely long along x-direction, all the physical variables depend on y and t. The flow is considered optically thin gray gas with natural convection and radiation. The radiative heat flux in the x-direction is considered negligible in comparison to y-direction.

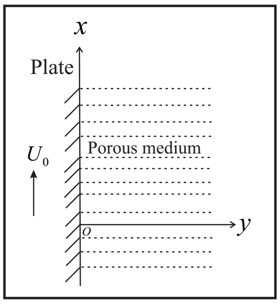

Fig. 5: Geometry of the problem

Conclusion:
An analysis have been made to study the radiation effect on free convective flow past an impulsively started infinitely long vertical plate with the ramped plate temperature in a porous medium. An increase in Grashof number leads to a rise in the fluid velocity due to enhancement in buoyancy force. The fluid velocity is accelerated due to increase in Darcy number. It is found that the fluid temperature decreases as the radiation parameter increases for both the ramped as well as constant plate temperatures. The rate of heat transfer at the plate increases for both the ramped as well as constant plate temperatures with an increase in either radiation parameter or Prandtl number.

Natural Convection Boundary Layer Flow past a Flat Plate of Finite Dimensions [6]

Consider a steady natural convective boundary layer flow past a flat plate of finite dimensions of a viscous incompressible fluid confined to the arbitrary inclination α of the plate embedded in a porous medium. Choose a Cartesian co-ordinates system (x, y, z) in such a way that x and y-axes are in the plane of the plate while the z-axis is normal to the plane of the plate. The fluid and the plate rotate in unison with a uniform angular velocity Ω about the z-axis normal to the plate (see Fig. 6).

[5] *Published in* **Open Journal of Fluid Dynamics, 2011, 1, pp. 1-11**
[6] *Published in* **Journal of Porous Media, 2012, 15(6), pp. 585-593**

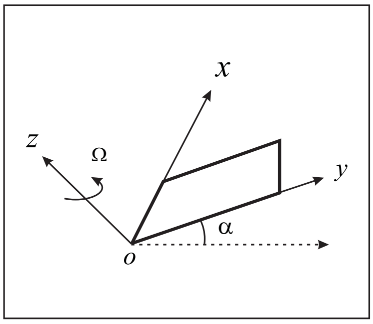

Fig. 6: Geometry of the problem

Conclusion:
The natural convective boundary layer flow past a flat plate of an arbitrary inclination embedded in a porous medium in a rotating environment with finite dimensions has been studied. It is observed that the buoyancy force, angle of attack and Darcy number accelerate both the primary and secondary velocities. On the other hand, the primary velocity decreases while the secondary velocity increases with an increase in rotation parameter. In relevance to a physical situation of interest, it is worth mentioning that the total amount of heat inside the boundary layer region increases with an increase in angle of attack. The bulk temperature θ_x along x-direction decreases while the magnitude of the bulk temperature θ_z along z-direction increases with an increase in Darcy number.

Hydrodynamic Flow Between Two Non-Coincident Rotating Disks Embedded in Porous Medium[7]

Consider the steady flow of a viscous incompressible fluid between two parallel disks embedded in a porous medium rotating with a uniform angular velocity Ω about two non-coincident axes at a distance a apart. Choose a system of cylindrical polar co-ordinates (r, θ, z) with the z-axis normal to the disks as situated symmetrically between two axes of rotation. The axis of rotation of the disk $z = h$ lies to the right and that of the disk $z = 0$ lies to the left of the axis (see Fig. 7).

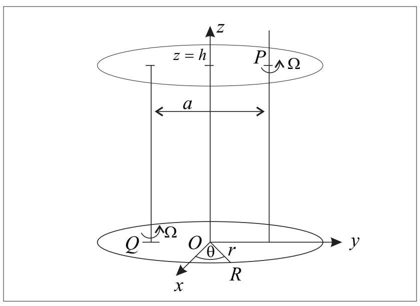

Fig. 7: Geometry of the problem

[7] *Published in* **World Journal of Mechanics, 2011, 1, pp. 50-56**

Conclusion:
The hydrodynamic viscous incompressible fluid flow between two non-coincident rotating disks embedded in a porous medium has been studied. It is observed that both the velocities f and g increase near the lower disk while they decrease near the upper disk with an increase in rotation parameter. On the other hand, the velocity f decreases while the velocity g increases with an increase in porosity parameter near the lower disk and the results are reversed near the upper disk. It is also found that the torque on the disk increases with an increase in rotation parameter while it decreases with an increase in porosity parameter. For large rotation, there exists a thin boundary layer near the disks and the thickness of this boundary layer increases with an increase in porosity parameter.

Effects of Radiation and Viscous Dissipation on Unsteady Free Convective Flow past a Moving Vertical Porous Plate Embedded in a Porous Medium

Consider the unsteady hydrodynamic flow of a viscous incompressible radiative fluid past an infinite moving vertical plate embedded in a porous medium on taking viscous dissipation into account. The x-axis is taken along the plate in the vertically upward direction and y-axis is taken normal to the plate (see Fig. 8). At time $t \leq 0$, both the fluid and plate are at rest with constant temperature T_∞. At time $t > 0$, the plate at $y = 0$ starts to move in its own plane with a velocity u_0 and the plate temperature rise to T_w. It is also assumed that the radiative heat flux in the x- direction is negligible as compared to that in the y-direction. As the plate is infinitely long, the velocity and temperature fields are functions of y and t only.

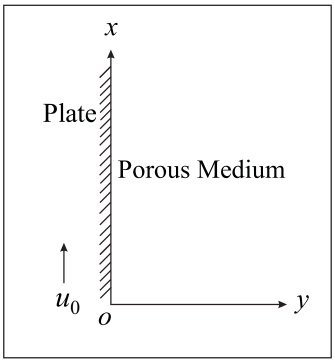

Fig. 8: Geometry of the problem

Conclusion:
The radiation effects on unsteady free convection flow of a viscous incompressible fluid past an infinite moving vertical plate embedded in porous medium have been studied on taking into account the viscous dissipation in the energy equation. It is observed that the radiation has a retarding influence on the fluid velocity. An increase in Darcy number leads to increase the fluid velocity. The fluid velocity and temperature raise due to viscous dissipation. The presence of suction falls the fluid velocity and temperature. Further, it is found that the shear stress at the plate increases in magnitude with an increase in either radiation parameter or time. The rate of heat transfer at the plate increases with an increase in radiation parameter. It is also found that the rate of heat transfer falls with increasing Eckert number.

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[8] Vafai(ed), K. (2005): Handbook of Porous Media (2nd ed), Taylor and Francis, New York.

List of Publication

The following papers have been published/accepted/communicated from this thesis:

1. Unsteady flow of viscous fluid through a porous medium bounded by a porous plate in a rotating system. *Published in* **Journal of Porous Media, 2010, 13(7), pp. 645-653**.
2. Laminar free and forced convective flow between two vertical plates embedded in porous medium. *Accepted in* **World Journal of Mechanics**.
3. Effects of radiation on free convection flow in a vertical channel embedded in porous medium. *Published in* **International Journal of Computer Applications, 2011, 35(6), pp. 38-44**.
4. Convection of radiating gas in a vertical channel through porous medium. *Published in* **World Journal of Mechanics, 2011, 1, pp. 275-282**.
5. Radiation effect on natural convection near a vertical plate embedded in porous medium with ramped plate temperature. *Published in* **Open Journal of Fluid Dynamics, 2011, 1, pp. 1-11**.
6. Natural convection boundary layer flow past a flat plate of finite dimensions. *Published in* **Journal of Porous Media, 2012, 15(6), pp. 585-593**.
7. Hydrodynamic flow between two non-coincident rotating disks embedded in porous medium. *Published in* **World Journal of Mechanics, 2011, 1, pp. 50-56**.
8. Effects of radiation and viscous dissipation on unsteady free convective flow past a moving vertical porous plate embedded in a porous medium. *Communicated*.