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ROLE OF RETURN MAXIMISATION, RISK REDUCTION AND SHARE RETURN COVARIANCES IN MARKOWITZ PORTFOLIO EFFICIENCY

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Abstract

One of the main challenges in portfolio management under Markowitz model is to determine the proportion of funds to be invested in each company's share to optimize Sharpe ratio. It is all the more challenging when the portfolio size increases in terms of number of shares. Usually to tackle these problems the investors and fund managers will apply return or risk or by any other logical method. In this article we have used four techniques to optimize the Sharpe ratio. We select the shares based on their return variance and order them in ascending and descending orders. These variances are taken as the basis for allocating funds among the shares first in sequential order, then in ascending order, then in descending order and finally in random weights order. When funds are allocated in descending order of their variance, the portfolios produced maximum return with reasonable variance and maximized the Sharpe ratio. The random allocation of funds as suggested by Markowitz to get efficient frontier for identifying optimum weights is time consuming and requires a lot of number crunching. Our descending order allocation is quick and effective in maximizing return for small and medium sized portfolios. In larger portfolios the random allocation produces higher return with higher Sharpe ratios than the variance allocated portfolios. This is because, out of 1000 iterations, the maximum Sharpe ratios were extracted and presented for comparison. Our findings will benefit the small and medium size portfolios which will result in considerable savings in time and number crunching.

Key words: *Allocation; Diversification; Markowitz; Portfolio; Risk; Sharpe Ratio; Variance*

Introduction

Portfolio diversification is the most discussed topic in investment. The main objectives are to maximize the return and minimize the risk, which ultimately translate into Sharpe ratio (SR) maximization, as it quantifies return per unit of risk. Under Markowitz model the variance and covariance combination with proportion of funds invested (weights) determine the risk of the

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portfolio. Whether it is a small portfolio or larger portfolio the return is always a linear combination of returns and weights, but risk is different. The risk is not only the linear combination of standard deviations (SDs) and weights, but also the covariance matters. The linear combination of SDs, weights and covariances determine the risk of the portfolio. Choosing the correct shares, determining the weights and satisfying the constraints imposed by the regulators are the three main issues in constructing portfolios. The parameters applied to choose the right shares vary from investor to investor and it ranges from earnings per share to lesser variance in return. The constraints imposed by the regulators are also could be satisfied but determining the proportion of funds to be invested in each share is the challenging task in any portfolio construction. Markowitz theorem proposes millions of combinations of shares with different random weights to decide the optimum portfolio which is tedious and time consuming. The numerical computations are not only tedious but also cumbersome as the size increases the covariances increase combinations of two shares. For a five assets portfolio there are 10 covariances and for a ten shares portfolio 45 covariances exist and these are to be included in determining the risk of the portfolios.

Research Problem

The prime objective of portfolio formation is to reduce the risk without reducing the return, because return and risk move in tandem. When higher return is expected this lands in higher risk in a single share, but in portfolios as per Markowitz the covariance between the shares also influence the risk but not the return. This is the diversification effect which is to be minimized by combining not only right shares but also by right proportion. Determining the right proportion is the challenging task. Markowitz recommends trial and error method to decide the optimum proportion by repeating millions of times with different weights. This takes time and large number crunching.

As per financial behavior theory there are three types of investors, one group risk averters, another risk seekers and the third group is risk neutral. The risk averters will choose the securities which exhibit lesser variance in the past returns and the risk seekers will do diametrically opposite. The risk averters' main objective is the safety of funds invested and they are satisfied with a low return. But risk seekers choose the higher variance securities in their portfolios because their objective is to maximize the return by taking calculated risk and not blind risk; else the growth in portfolio value is low. The risk of a portfolio is also affected by the number of securities included in the portfolio and their proportion. Over diversification occurs when more shares are included in the portfolio and vice versa. Determining the number of shares to be included to avoid over or under diversification is another challenging decision faced by a portfolio manager.

Theoretically, the portfolio diversification strategy can increase the return and reduce the risk of investment and ends up with higher SR. To test empirically the diversification effect in terms

of variance and number of shares in portfolio, this paper takes various combination of random weights first, inverse variance weights in the ascending order secondly and finally same inverse variance weights in the descending order. This is to identify the optimum weights quickly to maximize the SR. The mathematical algorithm is given in the methodology and MATLAB coding in the appendix.

Significance of Research

Though numerous studies tested the effectiveness of Markowitz in portfolio building and management still the mistry of optimum weight selection evades. This portfolio management knowledge is crucial not only for diversification but also for portfolio management as the mutual fund sector is growing year by year collecting millions of Ringgits and investing them in stock market promising fair return for the unit holders. The government of Malaysia encourages unit trusts as they provide impetus for active market and the liquidity. The findings will be beneficial for investors, analysts, and mutual funds companies in justifying their investment portfolio size. From the perspective of academicians, it adds value to the growing knowledge of investment portfolio selection, diversification and optimum number of financial assets to include.

Literature review

Harry Markowitz initiated a seminal work of portfolio optimization and then published in 1952. Markowitz (1952) modeled the concept of portfolio diversification to reduce the portfolio risk while maximize its portfolio return. The prominent part of his work is that the combination of mean and risk of the portfolio. In his paper, he pointed that big portfolio size cannot maximize the portfolio return and minimize the portfolio risk. In order to reach the utility, there are two fold. First, is to select shares from various industries. This can induce to lower portfolio variance. The second order is to have low covariance among them by choosing shares with difference economic activities. Chan, Karceski, and Lakonishok (1999) suggested optimizing portfolio including covariance. In addition to these is low portfolio correlation most preferably or even negative relationship.

Hue and Kwan (1987) employed six industry indices of the Stock Exchange of Singapore from July 1975 to June 1985 and ran a factor analysis on them. They suggested that choosing low covariance in stock selection can benefit from diversification across industry. They concluded that invest in three sectors rather than in all the sectors for deriving the benefits of diversification.

There are also studies that investigate the source of portfolio return. As initiated by Markowitz, the portfolio diversification discussed using only stock selection. Effective fund allocation among stock is equally important as stock diversification. Professor David Swenson, Chief Investment Officer of Yale University agreed with Ibbotson (2010) findings that 90% of the variability of returns in institutional portfolios is attributable to asset class allocation as compared to stock selection and market timing.

Markowitz model has been introduced under the assumption of equal weight. Despite that, many do not agree with it. In a simulation study, Ng and Goh (2011) examines the performance of portfolios of stocks listed in the Malaysian bourse. They analyzed the effects of different portfolio sizes and fund allocation methods on return per unit of risk, or risk reward. They concluded that equal weighted mean variance method had outperformed by conditional optimal and minimized variance allocation methods. One of the study by Zhang (2011) found that naive portfolio selection was out performed by Sharpe Ratio optimize sophisticated portfolio rule.

Portfolio Risk- Models assumptions

Assumption	Treynor (1962)	Sharpe (1964)	Lintner (1965)	Mossin (1966)
No taxes	Explicit	Implicit	Explicit	Implicit
No frictions (transaction costs)	Explicit	Implicit	Explicit	Implicit
Agents are price takers who all face identical prices	Explicit	Implicit	Explicit	Implicit
Agents maximize expected utility of future wealth	Explicit	Explicit	Explicit	Explicit
Utility represented as a function of return and risk	Explicit	Explicit	Explicit	Explicit
All agents agree that variance (or standard deviation) is the measure of security risk	Explicit	Explicit	Explicit	Explicit
Agents prefer more return to less and display risk aversion	Explicit	Explicit	Explicit	Explicit
A riskless asset (paying an exogenously determined positive rate of interest) exists, and all agents agree that it is riskless	Explicit	Explicit	Explicit	Explicit
All agents share the same subjective probability distribution of expected future prices	Explicit	Explicit	Explicit	Explicit
Fractional shares may be held	Implicit	Implicit	Explicit	Explicit
Short sales are allowed	Explicitly allowed	Explicitly disallowed	Explicitly allowed	Explicitly allowed
Leverage is allowed	Explicitly allowed	Explicitly disallowed	Explicitly allowed	Implicitly allowed
The number of shares of each security is constant	Implicit	Implicit	Implicit	Implicit
Agents share the same single period time horizon	Explicit	Explicit	Implicit	Implicit

Source:French (2003), https://ciber.fuqua.duke.edu/

Table 2 is a summary of Treynor, Sharpe, Lintner, and Mossin assumptions by French (2003). These equilibrium assumption comparisons exaggerate the work of four scholars. The assumptions contributed by these scholars are segregated as either explicit or implicit. French was dissatisfied that Treynor's works were rich in content but unpublished which gave place for later intellects to have similar assumption expression and widely cited.

Portfolio Diversification and Risk Reduction

There are various studies in determination of a profitable portfolio size. However, through the literature, the number of stocks for a well-diversified portfolio is rather vague. According to the past literature, the number of stocks required to construct a well-diversified portfolio had significantly increased from around ten for retail investor up to few hundred (Statman, 2004) or even thousands for a mutual fund company (Horie, 2010). Gupta *et al.*(2001) reported the average well-diversified portfolio of the Malaysian stocks is found to contain at least 27 randomly chosen securities.

The size of a portfolio has something to do with the effect of diversification. Evans and Archer (1968) study the effect of portfolio size to the portfolio risk. The finding is that as the portfolio diversifies, portfolio risk declines asymptotically. Portfolio with 15 stocks is the limit that risk is stabilized. They concluded that no more than 15 stocks would be need to achieve diversification. Elton and Gruber (1977) proposed that the portfolio variance reduction gradually diminishes as the portfolio diversifies. Statman (1987) argues that investor with leveraging should have 30 stocks while for the investor with lending should have 40 stocks.

Risk Adjusted Return

The weight of each shares can use as a sorting method in allocating the fund proportionately. The weighing of all the individual shares is weighted and adds in order to form a well-diversified portfolio. This portfolio is believed to have only systematic risk whereby the unique risk is diversified away. (Podobnik, Balen, Jagric, & Kolanovic, 2000). The purpose of creating a well-diversified portfolio is to maximize the portfolio return while minimizes the portfolio risk. The portfolio return and portfolio risk is moving in a different direction. In order to track these two directions, the risk-adjusted returns are an ideal tool to solve the problem. Enrico (2005) argued that the risk reward combination in portfolio selection was dominated by stochastic behavior of asset returns. In this article, the risk reward was characterized in the second order portfolio dominance that addressed the problem of stochastic dominance.

The portfolio allocation models specify either mean-variance approach or market index and share index approach of CAPM. Both the model portfolios allocate funds based on certain predetermined principles. In future, whatever is expected may not come true. However, the portfolios are to be evaluated performance wise to identify which model works well. For this performance evaluation, the Sharpe ratio and Treynor ratio are used (Hendrik & Marco, 2006).

Investment portfolio performance is not predetermined and so the investors are curious in looking forward for their portfolio performance. Thus, Hendrik and Wilkens (2006) tested

their investment holding by using Sharpe ratio and Treynor ratio and plug into investor-specific measure.

Sharpe ratio

Sharpe (1966) introduced a measure for the performance of mutual funds by considering both systematic and unsystematic risks (called total market risk) as the measurement of excess return. According to the Sharpe (1994), he proposed the idea of the Sharpe ratio. This ratio is use to do the risk adjustment on portfolio return. It can be using either ex-ante Sharpe or ex-post Sharpe. However, only the ex-post Sharpe ratio is under review. Historical data is required to test this ratio. If a study is carried out for observation on the ex-post data, then ex-post Sharpe ratio is relevant. If a decision is needed for some investment, then ex-ante Sharpe ratio will be suitable for the test.

Porfolio Optimization

Prior to the existence of algorithm program, researchers were unable to perform portfolio optimization analysis. Only up to a level of information, they were able to obtain results. Even though total risk can be brought down by diversification, the ability to form a best portfolio was out of the past researchers capacity except that they use the market information. With the development of scientific computing, researchers are capable to go to second level of decision-making information. In doing so, the investment funds can be allocated more efficiently to achieve the investment objective in a single time horizon (Lin, 2012).

Tremendous research had been done on optimizing. In order to get the maximum likelihood of high performance portfolio, research had done on optimizing portfolio selection, and portfolio allocation. Lin (2012) proposed PONGSA model to optimize the portfolio selection process. Cohen and Natoli (2003) had pursuit of risk and utility in portfolio optimization. They represented the risk as chances of failure and utility as chances of success. However, the optimum portfolio is carried out under parabola form for minimizing the portfolio risk and hyperbola form for portfolio utility. Konno and Kobayashi (1997) constructed a stock-bond portfolio optimization model. Yoshimoto (1996) proposed Portfolio Optimization System with Transaction Costs (POSTRAC) in portfolio optimization, because the portfolio performance is unaffected when transaction cost is considered.

Paudel and Koirala (2006) analyzed 30 stocks from 129 stocks listed on Nepalese stock market during the period 1997 through 2006. They tested the robustness of Markowitz and Sharpe models in Nepalese stock market by optimizing the portfolio at a cutoff rate. They recommended the use of the mean variance model in the Nepal stock market.

Methodology

Principles of portfolio construction

There are two aspects to be considered while constructing portfolios:

(i) the selection of shares or financial assets to be included in a portfolio, and

(ii) the right amount to be invested in each selected share so as to maximise the return and minimise the risk.

Selection of shares

Selection of a share or a financial asset to buy is a complex decision. Several financial variables will be considered like earnings per share, price earnings ratio, the current market price, the activeness of the share in the market, the company's future earning potential, past dividend record and so on. There are no single criteria to select the share. The soft factors such as good team of management, company's popularity, the type of industry in which it is operating etc. also play a very important role in selection of shares.

Deciding the amount to be invested

The amount to be invested in a company's share is based on the amount available for investment and also the number of companies to be included in the portfolio. If it is a few companies share portfolio, larger amount will be invested in a company's share and vice versa. The quantum of money invested in a company's share is known as weight. Weight is nothing but the proportion of money invested in a particular company's share. These weights determine the return and risk of the portfolios significantly. The successful fund manager is one who determines the weight efficiently. In small portfolios the manager may be successful but in case of large portfolios the optimisation techniques are employed by repeatedly assigning weights and choosing the best portfolio.

Random weights

Weighs are determining by the MATLAB software through millions of iterations to get a random proportion of funds as a weight for each share in the portfolio. This study observes the behavior of each share allocation instead of disposing them from the portfolio construction.

Portfolio optimisation

As explained earlier the optimisation is done by trial and error method of assigning weights. This trial and error method may stop once the portfolio objective is attained or when a target number of assignments are made. One assignment is called one portfolio. Out of the number of portfolios one portfolio combination will be selected based on the maximum risk-reward ratio (Sharpe Ratio).

Portfolio return and risk

Portfolio return is the weighted average of returns of shares which are included in the portfolio. The risk is computed in a covariance-weight framework or correlation-weight combination under Markowitz.

Markowitz model Return-Risk – 2 Assets Portfolio

Portfolio Return $$Pr = \sum_{i=1}^{n} R_i * w_i \quad (3.1)$$

Portfolio Risk by correlation $$P_\sigma = \sqrt{\sigma_i^2 w_i^2 + \sigma_j^2 w_j^2 + 2r_{i,j}\sigma_i\sigma_j w_i w_j} \quad (3.2)$$

Portfolio Risk by variance and covariance $$P_\sigma = \sqrt{cov_{i,j} w_i w_j + var * (n-2)} \quad (3.3)$$

where

P_r = Portfolio Return
R_i = Return of each security
W_i = Weight (Investment amount in each share)
P_σ = Portfolio Standard Deviation
$r_{i,j}$ = Correlation coefficient between share returns
n = Number of shares in portfolio
$cov_{i,j}$ = Covariance between share returns
var = Variance of share returns

Markowitz model Return-Risk – n Assets Portfolio (*i, j,n*)

Portfolio Return $$Pr = \sum_{i=1}^{n} R_i * w_i + \ldots\ldots\ldots + R_n * w_n \quad (3.4)$$

Portfolio Risk by correlation $$P_\sigma = \sqrt{\begin{matrix} \sigma_i^2 w_i^2 + \sigma_j^2 w_j^2 + 2r_{i,j}\sigma_i\sigma_j w_i w_j \\ + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ + \sigma_n^2 w_n^2 + \sigma_i^2 w_i^2 + 2r_{n,i}\sigma_n\sigma_i w_n w_i \end{matrix}} \quad (3.5)$$

If the portfolio consists of many assets, the above formula will be expanded with ${}^{n}C_r$ correlations. For instance for a four assets portfolio, there will be six lines, and for five assets 10 lines. This is the reason why Markowitz is not popular as the number of lines increases, it goes out of control. For a 20 assets portfolio there will be $20c_2 = 190$ lines will be there. But a MATLAB program can easily handle large portfolios.

Sharpe ratio

Sharpe ratio is used in analyzing the performance of portfolios. As in the literature review, this ratio intended to measure the excess return from portfolio over the risk to be bear by the investor. This ratio is computed as the portfolio return divided by the portfolio standard deviation:

$$SR = \frac{P_r}{P_{sd}} \quad (3.6)$$

In selection of the best portfolio based on Sharpe ratio, the portfolio with the highest ratio value is to be picked.

Data

To update any theory, meticulous empirical analysis is needed to see whether reality is expressed by the theory. To get robust results sampling and data collection are to be carried out accurately. Keeping in mind the above, we have selected almost 50 shares from the Kuala Lumpur stock exchange. Active shares, top losers and top gainers are the three categories under which the shares are listed in the standard newspapers. We selected almost all shares listed under the above three categories. Their code numbers were noted and then from Yahoo finance, the share prices were retrieved for two years from 17th September 2013 to 16th September 2014. Since the data for some companies' not available for the above two years, they were omitted from sample. Finally, by random sample, 30 companies' data were chosen to construct portfolios.

In Yahoo finance, recent data were given in the top row and the previous data were given at the end. To get the proper time series data, we flipped the rows from top to bottom. Time series data were non-stationary and their mean and standard deviations were not stable. The share prices were converted to geometrical returns as explained in methodology.

To better understand the properties of the share price data of these 30 companies, the basic descriptive statistics were computed for both share prices and their returns. The daily closing prices produced very meagre returns as the price changes were in sens (cent). Hence for return, the median and mode were not computed.

Though the share prices are non-stationary their mean, median and mode show whether there is any extreme data. If they are closer to each other there is no extreme price fall or increase. Thirty companies price data reveals no extreme movement in share prices as they are closer except Coasta (mean 3.30, median 2.93 and mode 2.02), FIMA group (mean 6.83, median 6.27 and mode 5.80) and Nestle (mean 65.84, median 67.02 and mode 68). Nine companies' prices are in double digits and the remaining prices are below RM 10. In terms of geometrical returns none of the averages showed negative returns. Seven companies' average returns exceeded 0.002. Other companies' returns were meagre as they were daily return averages. Since the returns are very small numbers, the median and mode were not computed.

Standard deviation (SD) is another important parameter to understand the behavior of prices and returns. SD shows the variation in share prices, higher SD show more variation than lower SD. Three (Nestle, Panasonic and DKSH) companies' prices show SD of more than 2 which indicates higher variation in their prices. Takaso Bhd (whose prices are consistent) shows a SD of less than 0.1.

Table1: Descriptive statistic results of 2011 daily share prices

		Mean		Median	Mode	Standard Deviation		Skewness		Kurtosis		Minimum		Maximum	
		SP	Return	SP	SP	SP	Return	SP	Return	SP	Return	SP	Return	SP	Return
1	CIMB	7.50	0.000	7.48	7.62	0.40	0.000	0.77	1.511	0.12	0.328	6.81	0.000	8.65	0.000
2	Takaso	0.25	0.000	0.23	0.22	0.06	0.010	2.45	1.262	7.32	17.292	0.17	-0.046	0.60	0.093
3	Ajiya	2.06	0.002	2.13	2.20	0.26	0.051	0.26	2.575	-0.44	51.484	1.63	-0.354	2.78	0.631
4	Atlan	4.73	0.001	4.70	4.60	0.14	0.014	0.18	0.917	-1.13	6.244	4.45	-0.050	5.05	0.099
5	Coastal	3.30	0.000	2.93	2.02	1.23	0.009	0.34	0.231	-1.49	9.309	1.48	-0.053	5.35	0.052
6	FIMA Group	6.83	0.002	6.27	5.80	1.22	0.028	0.95	-1.922	-0.75	63.966	5.62	-0.350	9.60	0.271
7	Kuluang	3.29	0.001	3.27	3.00	0.27	0.013	0.29	2.359	-1.07	18.010	2.89	-0.045	4.00	0.119
8	Kunusfar	1.71	0.001	1.70	1.60	0.11	0.017	0.06	1.164	-1.19	16.414	1.42	-0.092	1.98	0.128
9	Lay Hong	1.54	0.000	1.42	1.20	0.43	0.019	1.54	0.459	2.00	14.359	1.05	-0.132	3.51	0.140
10	LPI	15.64	0.002	15.60	17.00	1.45	0.027	-0.21	1.636	-1.47	19.497	13.38	-0.117	17.86	0.257
11	Nestle	65.84	0.001	67.02	68.00	2.98	0.007	-1.04	-1.579	-0.24	19.407	58.40	-0.065	69.68	0.029
12	Oriental	8.52	0.000	8.41	7.80	0.87	0.008	0.55	-1.155	-0.81	13.183	6.95	-0.057	10.50	0.043
13	OSK	1.67	0.000	1.62	1.62	0.23	0.010	1.75	-0.795	2.80	8.143	1.40	-0.069	2.55	0.037
14	Panasonic	22.36	0.001	22.00	22.00	1.72	0.014	0.99	1.705	0.23	15.573	19.88	-0.071	27.10	0.131
15	Pet - Gas	21.62	0.000	22.00	23.00	2.05	0.009	-0.25	-0.356	-1.38	14.066	17.10	-0.073	24.88	0.067
16	Scintex	4.88	0.000	5.44	5.79	1.16	0.011	-0.58	-0.465	-1.15	3.191	2.62	-0.059	7.15	0.034
17	Spritzer	1.57	0.002	1.70	0.88	0.42	0.015	-0.25	1.001	-0.97	4.309	0.81	-0.057	2.38	0.089
18	Thaps	6.48	0.002	5.80	5.60	1.53	0.021	1.26	1.151	0.38	5.318	4.72	-0.058	11.44	0.131
19	Technic	3.23	0.001	3.16	3.20	0.28	0.021	1.87	-0.189	4.05	5.728	2.90	-0.099	4.77	0.090
20	Batu kawan	18.96	0.001	19.18	19.60	0.79	0.013	-0.85	0.957	2.83	8.966	13.46	-0.072	20.12	0.085
21	Cayamata	2.89	0.000	2.67	1.65	1.10	0.021	0.38	-5.894	-1.04	220.374	1.50	-0.356	5.39	0.262
22	DKSH	5.59	0.002	6.08	6.40	2.09	0.040	-0.36	-9.414	-1.16	179.428	2.06	-0.683	9.00	0.271
23	Gamuda	4.41	0.002	4.58	4.60	0.45	0.018	-0.82	1.518	-0.78	7.750	3.43	-0.059	5.00	0.132
24	Hong Bank	14.24	0.001	14.20	14.00	0.28	0.012	0.36	0.577	-0.24	7.445	13.44	-0.051	14.98	0.088
25	H.industries	5.41	0.000	5.15	4.40	1.00	0.007	0.73	0.144	-0.51	3.930	4.10	-0.030	8.48	0.033
26	H.leong	15.03	0.001	15.16	15.00	1.26	0.013	0.05	1.431	0.10	9.685	12.16	-0.057	18.30	0.100
27	Laferage	9.67	0.001	9.75	9.70	0.67	0.013	-0.36	-0.076	-0.03	1.283	7.90	-0.047	11.16	0.046
28	MKH BHD	2.93	0.000	2.61	2.60	0.86	0.015	0.60	0.247	-0.83	3.808	1.75	-0.071	4.85	0.072
29	PPBANK	14.21	0.001	14.42	12.20	1.41	0.022	-0.21	-1.867	-0.99	21.800	11.18	-0.211	16.68	0.081
30	TASEK	14.91	0.000	15.10	16.10	1.03	0.013	-1.16	0.247	1.09	1.175	11.10	-0.039	16.50	0.042

Source: Authors' calculations

Returns SD is more meaningful than price SD as prices are non-stationary. Eight companies SD are more than 2% which indicate the higher variation of these returns. Five companies show a return of less than 1% which indicates the closer and narrow price movements. Ajiya Bhd shows the highest SD in returns (5.1%).

SD shows the behavior and pattern of prices and returns but does not show which tail, from average, more prices and returns spread. The skewness indicates the spread in left and right tails. It is the third moment in statistics. Minus skewness indicates that more prices or returns fall in left tail area and known as negative skewness and vice versa. Skewness value of zero indicates symmetric or equal spreads in both tail areas.Any skewness value between -1 to +1 will be considered normal. If it is more than this range then it is considered as negligible and the values are considered as normal. Five companies' prices show a skewness value of more than +1 and their prices are right skewed. Tasek Bhd only shows a negative skewness (-1.16). In terms of return, nine companies' returns show negative skewness and remaining positive skewness. Many companies returns are more skewed and they show values beyond the range stated above. This clearly proves that the returns generated from companies market prices are not normally distributed and have fat tails. Cayamata Bhd (-5.894) and DKSH (-9.414) show very high negative skewness followed by Ajiya with positive skewness (2.575).

Kurtosis indicates the peakedness of the price and return distributions. Leptokurtic with values more than three indicates that the distribution is peaked and opposite is the platykurtic with values less than -3 indicate the flatness of the distribution. Any values between -3 and +3 indicate mesokurtic which normal. The share prices look normal as they indicate lesser kurtosis values. But the returns are with very large values and all have positive values indicating that all the spreads are narrow around the mean indicating high peakedness with fat tails. Cayamata (220.374) and DKSH (179.428) show very huge kurtosis which indicates the closely moving share prices around the mean producing very narrow positive and negative returns not spreading much.

Minimum and maximum gives the range within which the prices and returns move. It is a good indicator for determining the spread. Return range is more important than the price range as prices are non-stationary. Four companies Ajiya (-0.354; 0.631), FIMA group (-0.35; 0.271), Cayamata (-0.356; 0.262) and DKSH (-0.683; 0.271) show the largest variation in terms of minimum and maximum returns, indicating that these prices are highly volatile.

The above characteristics of share price and return behavior are independent which means they are not connected with another share price or return. In portfolios these shares are grouped and bundled together in some proportion which in turn decides their behavior collectively. This collective behavior is different from individual behavior for several reasons, but prominent among them are the covariance and the weight (proportion) of composition of these shares in portfolio. The collective behavior is analysed below under four different features of portfolios.

Portfolios and diversification of risk

Deciding the proportion of funds (weights) to be invested in a share is the most crucial portfolio decision. These weights decide the effectiveness of risk reduction and return maximization and ultimately end in maximising Sharpe ratio. Until today from Markowitz there is no perfect answer for the weights. They are decided by either heuristic method or by judgement or by random allocation with trial and error method. The iteration method is the most popular method but this is the most time consuming and costly method. An attempt is made to allocate the funds quickly and effectively to achieve effectiveness by maximizing the Sharpe ratio.

To test the variance allocation 30 active companies' share prices which are listed in Kuala Lumpur stock exchange were downloaded from Yahoo finance website. These shares are arranged in the descending order and sorted by date to make them in the ascending order. Since the prices are non-stationary they have been converted to returns and their behavior was tested in the same order in which the shares were downloaded.

In the past the funds allocation was mostly based on past return, risk, popularity of the company, share price etc. The size of the portfolios was also decided by judgement: smaller diversification may be with less than 10 shares and larger diversification with more than 10 shares. The maximum number may be up to 30, beyond which it may be over – diversified. Over diversification may not bring effective return as small funds are allocated for each company.

Taking the above points into consideration, in this paper we have decided to allocate more funds to less risky shares compared to more risky shares. This is achieved by finding the variance of returns of each share and inverting them to allocate more funds to less risky shares, to satisfy the risk aversion principle.

To accommodate the size in the portfolio, we have decided to increase the size of the portfolios from three shares to thirty shares. In three share portfolio, larger funds will be invested in a specific company compared to 30 share portfolio. We analyse these portfolios under four headings to assess the effectiveness in terms of Sharpe ratio. First we include the shares in the portfolio in the same order in which we downloaded. Secondly we sort the variances produced by these share returns in the ascending order and include in the portfolios in their order of variance. Thirdly we sort the variance of returns in the descending order and include the shares in this order. Finally we choose the weights subjectively and include the shares in the original order in which we downloaded. We discuss all the four results from three assets portfolio to 30 assets portfolios in terms of Sharpe ratio. To illustrate the results clearly their graphs are given below.

Random inclusion of shares

Twenty eight diversified portfolios were constructed from three shares to thirty shares listed in KLSE. The portfolios' size started from three shares and in every step one share was added

to thirty shares with the same amount to invest. In every step, return, standard deviations and Sharpe ratio were computed. The weights were determined by inversing the variances and normalizing them to sum to one. Table 2 gives the results of sequential selection of shares.

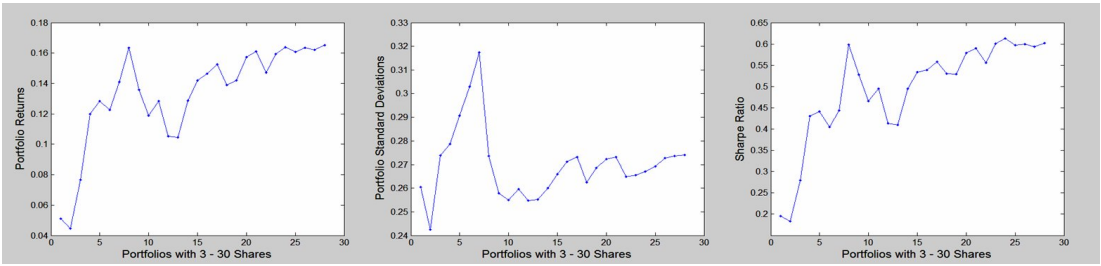
Second column gives the name of companies shares included in portfolios in every step. The first two rows show zeros for all because there is no portfolio with one and two shares. The third column shows the variance of returns of each share which form the basis for weight computation. Takaso returns are highly variable (26.19) followed by Cayamata (15.66). These variance figures are very small as they are daily returns. To understand easily they are scaled by 10,000 and expressed in basis points.

Table 2 : Sequential Share Selection and Sharpe Ratio

Portfolios	Shares included	Variance(10^{-3})	Return	Risk	Sharpe Ratio
	CIMB	1.00	0	0	0
	Takaso	26.19	0	0	0
3 Assets	Ajiya	2.06	0.05	0.26	0.20
4 Assets	Atlan	0.87	0.04	0.24	0.18
5 Assets	Coastal	7.89	0.08	0.27	0.28
6 Assets	FIMA Group	1.65	0.12	0.28	0.43
7 Assets	Kuluang	2.88	0.13	0.29	0.44
8 Assets	Kunusfar	3.66	0.12	0.30	0.41
9 Assets	Lay Hong	7.21	0.14	0.32	0.44
10 Assets	LPI	0.46	0.16	0.27	0.60
11 Assets	Nestle	0.62	0.14	0.26	0.53
12 Assets	Oriental	0.97	0.12	0.25	0.47
13 Assets	OSK	2.06	0.13	0.26	0.49
14 Assets	Panasonic	0.85	0.11	0.25	0.41
15 Assets	Pet - Gas	1.24	0.10	0.26	0.41
16 Assets	Scintex	2.36	0.13	0.26	0.50
17 Assets	Spritzer	4.24	0.14	0.27	0.53
18 Assets	Thaps	4.22	0.15	0.27	0.54
19 Assets	Technic	1.82	0.15	0.27	0.56
20 Assets	Batu kawan	0.56	0.14	0.26	0.53
21 Assets	Cayamata	15.66	0.14	0.27	0.53
22 Assets	DKSH	3.35	0.16	0.27	0.58
23 Assets	Gamuda	1.55	0.16	0.27	0.59
24 Assets	Hong Bank	0.56	0.15	0.26	0.56
25 Assets	H.industries	1.61	0.16	0.27	0.60
26 Assets	H.leong	1.74	0.16	0.27	0.61
27 Assets	Laferage	2.20	0.16	0.27	0.60
28 Assets	MKH BHD	4.68	0.16	0.27	0.60
29 Assets	PPBANK	1.57	0.16	0.27	0.59
30 Assets	TASEK	1.49	0.17	0.27	0.60

Source: Calculation by the authors

The small, medium and large portfolios' return, risk and Sharpe ratios are given in table 2. Up to 10 companies' shares form small portfolios. The returns of these portfolios increase from 5% to 16% with few slight decreases. The risk also increases from 26% to 32% with two decreases slightly. The return is to be maximized and the risk is to be minimized. The Sharpe ratio optimizes both return and risk. This ratio increases from 18% and reaches 60% in the 10 assets portfolio. Four portfolios show Sharpe ratio in the range of 40%. In medium size portfolios the return, risk and Sharpe ratios are almost sable except for small variation. Returns are in the ratio of 14% and 15%, while risks are in the range of 25% to 27%, and the Sharpe ratio is in the range of 41% to 56%. The larger portfolios with 21 shares to 30 shares show a stable return, risk and Sharpe ratio. Returns are in the close range of 14% to 17%, risks are also in close range of 26% and 27% and Sharpe ratios 53% 61%. The results imply that the size of portfolios do not alter the return and risk profile drastically.



1.a Portfolio returns 1.b Portfolio Risks 1.c Portfolio Sharpe ratios

Figure 1: Sequential Share Selection

Figures 1.a, 1.b and 1.c show the same results of Table 2. The visual are clearer in understanding the pattern of the return, risk and Sharpe ratio profiles. Up to 10 shares in a portfolio maximizes all the three parameters. These results imply that inclusion of more than ten shares in a portfolio does not improve the efficiency and effectiveness of portfolios. It seems that there is no incentive and motivation to include more and more shares.

Low to high variance inclusion of shares

All else equal, rational investors will avoid risk unless the risk is compensated with reasonable return. The variance determines the risk and diversification intended to reduce this variation. Keeping this in mind, from the risk averse investors point of view how these small, medium and large portfolios behave is a fitting question to address. Hence we constructed same twenty eight portfolios from three shares to thirty shares in the variance ascending order. We arranged the shares in the order of their variance from smallest to highest. These variances are inverted and normalized to get weights to invest funds. The results are as follows. Up to eight shares portfolio the returns are falling steadily from 10% to 4%. Later the returns take a 'U' turn and increase steadily and reach 17% when all 30 shares are included in the portfolio. The risk steadily increase from 12% and reaches 23% in the large portfolio. As in the return, the

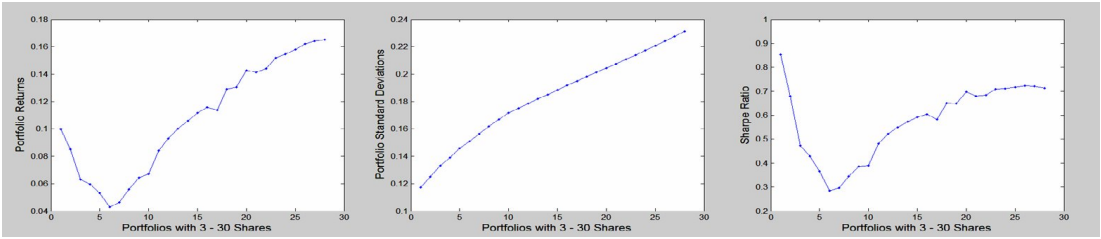
Sharpe ratio also decreases from 85 % to 28 % till the 8 assets portfolio and takes 'U' turn and reaches 71 % in the largest portfolio. Interestingly even this 71% of the largest portfolio is less than the three assets portfolio's ratio which is 85 %.

The patterns of return, risk and Sharpe ratio are clear when the same results are presented in figure form. The highest variance shares produce high returns and also results in very high risk. It is matching with the finance principle that risk and return move in tandem. The Sharpe ratio optimizes both return and risk. The first portfolio produces highest return per unit of risk (0.85%) and later the ratio falls until the eight asset portfolio and later gradually increases to 0.71%. It shows that the variance weighted portfolios are really meaningful in maximizing returns with lesser time and lesser computation, without running millions of iterations to get the optimum weights.

Table 3: Ascending Variance Share Selection and Sharpe Ratio

Portfolios	Shares included	Variance(10^{-3})	Return	Risk	Sharpe Ratio
	LPI	0.46	0	0	0
	Hong Bank	0.56	0	0	0
3 Assets	Batu kawan	0.56	0.10	0.12	0.85
4 Assets	Nestle	0.62	0.08	0.12	0.68
5 Assets	Panasonic	0.85	0.06	0.13	0.47
6 Assets	Atlan	0.87	0.06	0.14	0.43
7 Assets	Oriental	0.97	0.05	0.15	0.37
8 Assets	CIMB	1.00	0.04	0.15	0.28
9 Assets	Pet - Gas	1.24	0.05	0.16	0.30
10 Assets	TASEK	1.49	0.06	0.16	0.35
11 Assets	Gamuda	1.55	0.06	0.17	0.39
12 Assets	PPBANK	1.57	0.07	0.17	0.39
13 Assets	H.Industries	1.61	0.08	0.18	0.48
14 Assets	FIMA Group	1.65	0.09	0.18	0.52
15 Assets	H.Leong	1.74	0.10	0.18	0.55
16 Assets	Technic	1.82	0.11	0.19	0.57
17 Assets	OSK	2.06	0.11	0.19	0.59
18 Assets	Ajiya	2.06	0.12	0.19	0.60
19 Assets	Laferage	2.20	0.11	0.19	0.58
20 Assets	Scintex	2.36	0.13	0.20	0.65
21 Assets	Kuluang	2.88	0.13	0.20	0.65
22 Assets	DKSH	3.35	0.14	0.20	0.70
23 Assets	Kunusfar	3.66	0.14	0.21	0.68
24 Assets	Thaps	4.22	0.14	0.21	0.68
25 Assets	Spritzer	4.24	0.15	0.21	0.71
26 Assets	MKH BHD	4.68	0.15	0.22	0.71
27 Assets	Lay Hong	7.21	0.16	0.22	0.72
28 Assets	Coastal	7.89	0.16	0.22	0.72
29 Assets	Cayamata	15.66	0.16	0.23	0.72
30 Assets	Takaso	26.19	0.17	0.23	0.71

Source: Calculation by the authors



2.a Portfolio returns 2.b Portfolio Risks 2.c Portfolio Sharpe ratios

Figure 2: Ascending Variance Share Selection

High to low variance inclusion of shares

Individual investors risk appetite varies with their risk bearing capacity. Risk seekers want to earn quick money and they choose the risky shares for investment. The higher variance securities are the risky shares. To select the risky shares to be included in the portfolio first we sorted the data in the descending order of variances and then the same procedure followed earlier was adopted to get return, risk and Sharpe ratio. The portfolios' size were increased one share at a time till it reached 30 shares in the highest to lowest variance order. The results are given in table 4 below.

Table 4 : Descending Variance Share Selection and Sharpe Ratio

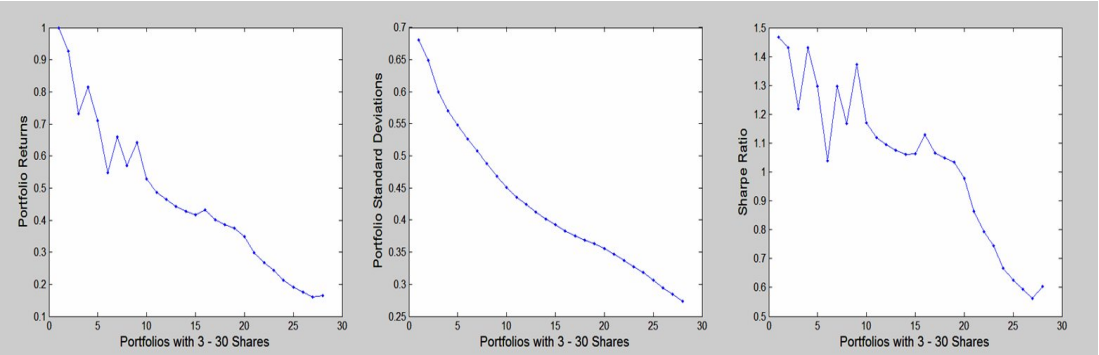
Portfolios	Shares included	Variance(10^{-3})	Return	Risk	Sharpe Ratio
	Takaso	26.19	0	0	0
	Cayamata	15.66	0	0	0
3 Assets	Coastal	7.89	1.00	0.68	1.47
4 Assets	Lay Hong	7.21	0.93	0.65	1.43
5 Assets	MKH BHD	4.68	0.73	0.60	1.22
6 Assets	Spritzer	4.24	0.82	0.57	1.43
7 Assets	Thaps	4.22	0.71	0.55	1.30
8 Assets	Kunusfar	3.66	0.55	0.53	1.04
9 Assets	DKSH	3.35	0.66	0.51	1.30
10 Assets	Kuluang	2.88	0.57	0.49	1.17
11 Assets	Scintex	2.36	0.64	0.47	1.37
12 Assets	Laferage	2.20	0.53	0.45	1.17
13 Assets	Ajiya	2.06	0.49	0.44	1.12
14 Assets	OSK	2.06	0.47	0.42	1.10
15 Assets	Technic	1.82	0.44	0.41	1.08
16 Assets	H.Leong	1.74	0.43	0.40	1.06
17 Assets	FIMA Group	1.65	0.42	0.39	1.06
18 Assets	H.Industries	1.61	0.43	0.38	1.13
19 Assets	PPBANK	1.57	0.40	0.38	1.07
20 Assets	Gamuda	1.55	0.39	0.37	1.05
21 Assets	TASEK	1.49	0.38	0.36	1.03
22 Assets	Pet - Gas	1.24	0.35	0.36	0.98
23 Assets	CIMB	1.00	0.30	0.35	0.86
24 Assets	Oriental	0.97	0.27	0.34	0.79
25 Assets	Atlan	0.87	0.24	0.33	0.74
26 Assets	Panasonic	0.85	0.21	0.32	0.67
27 Assets	Nestle	0.62	0.19	0.31	0.62
28 Assets	Batu kawan	0.56	0.17	0.29	0.59
29 Assets	Hong Bank	0.56	0.16	0.28	0.56
30 Assets	LPI	0.46	0.17	0.27	0.60

Source: Calculation by the authors

The smaller highly risky portfolios produce very good returns starting from RM 1.00 to RM 0.64 in decreasing order when size increases from three shares to 10 shares. The risk also gradually decreases from RM0.68 to RM 0.49. These portfolios produce high Sharpe Ratios ranging from RM 1.47 to RM 1.17. This decline is neither gradual nor steady. These portfolios' ratios go up and down violently.

The medium portfolios show a steady but a flat decline in returns from 64 % to 39 %. The risk also declines for these medium portfolios which is from 11 shares to 20 shares from 47 % to 37 %. When optimized through Sharpe ratio their decline is not steep but rather gradual from RM 1.37 to RM1.05 except in 18 assets portfolio where there is slight increase in Sharpe ratio.

Largest portfolios include 21 to 30 shares whose variance is low. There are added to the portfolios one each at a time to the existing medium sized portfolios. When these low variance securities are added to the existing portfolio, the returns further decline from 38 % to 17 %. The risk also decreases from 0.37 to 0.27. The Sharpe ratio declines further from RM 1.03 to RM0.60.



3.a Portfolio returns 3.b Portfolio Risks 3.c Portfolio Sharpe ratios

Figure 3: Descending Variance Share Selection

The results produced in table 4 are depicted in pictorial form in 3.a, 3.b and 3.c. The high variance smaller portfolios show unstable return and shape lines with a steady decline standard deviations. The medium portfolios which contain both high variance and medium variance shares show flatter lines but in all three parameters. The lowest variance securities when added to existing portfolios reduce the rate of return, risk and Sharpe ratio at a faster rate than the other two portfolio segments.

These results indicate that when portfolios are built with highest variance weight securities, they produce substantially higher return per unit of risk. When low variance securities are added and if the weight is rearranged with other shares the return and risk decline steadily and also sharply, still producing a good return when compared to other types of portfolios.

Table 5 : Random weight allocation and Sharpe Ratio

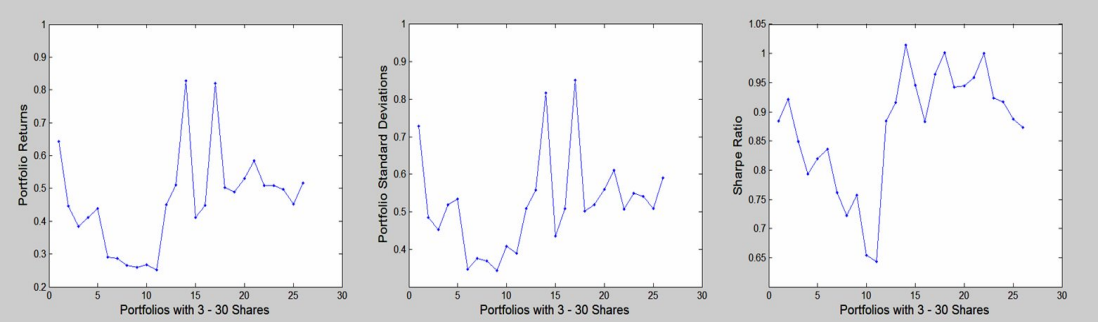
Portfolios	Shares included	Variance(10^{-3})	Return	Risk	Sharpe Ratio
	CIMB	1	0	0	0
	Takaso	26.19	0	0	0
3 Assets	Ajiya	2.06	0.38	0.52	0.73
4 Assets	Atlan	0.87	0.33	0.54	0.60
5 Assets	Coastal	7.89	0.64	0.73	0.88
6 Assets	FIMA Group	1.65	0.45	0.48	0.92
7 Assets	Kuluang	2.88	0.38	0.45	0.85
8 Assets	Kunusfar	3.66	0.41	0.52	0.79
9 Assets	Lay Hong	7.21	0.44	0.54	0.82
10 Assets	LPI	0.46	0.29	0.35	0.84
11 Assets	Nestle	0.62	0.29	0.38	0.76
12 Assets	Oriental	0.97	0.27	0.37	0.72
13 Assets	OSK	2.06	0.26	0.34	0.76
14 Assets	Panasonic	0.85	0.27	0.41	0.65
15 Assets	Pet - Gas	1.24	0.25	0.39	0.64
16 Assets	Scintex	2.36	0.45	0.51	0.88
17 Assets	Spritzer	4.24	0.51	0.56	0.92
18 Assets	Thaps	4.22	0.83	0.82	1.01
19 Assets	Technic	1.82	0.41	0.44	0.95
20 Assets	Batu kawan	0.56	0.45	0.51	0.88
21 Assets	Cayamata	15.66	0.82	0.85	0.96
22 Assets	DKSH	3.35	0.50	0.50	1.00
23 Assets	Gamuda	1.55	0.49	0.52	0.94
24 Assets	Hong Bank	0.56	0.53	0.56	0.94
25 Assets	H.Industries	1.61	0.59	0.61	0.96
26 Assets	H.Leong	1.74	0.51	0.51	1.00
27 Assets	Laferage	2.20	0.51	0.55	0.92
28 Assets	MKH BHD	4.68	0.50	0.54	0.92
29 Assets	PPBANK	1.57	0.45	0.51	0.89
30 Assets	TASEK	1.49	0.52	0.59	0.87

Source: Calculation by the authors

Random weight portfolios

Ignoring the variance as the basis for weight determination, sufficient random numbers were generated to convert them into weights by normalizing them, because the share prices behaves as a random variable which has no property, pattern and they are mostly chaotic. In every iteration, three to thirty shares, portfolios were constructed and their return, risk and Sharpe ratios were recorded. Like this 1000 iterations were performed and the results were saved. These results were sorted in Sharpe ratio in descending order to find the maximum Sharpe ratio portfolio. All portfolios' results in that iteration were selected and reported above in table

5. Only three portfolios show returns of RM 1 and above. The other portfolios are not effective in terms of portfolio Sharpe ratios and returns.



4.a Portfolio returns 4.b Portfolio Risks 4.c Portfolio Sharpe ratios

Figure 4: Sequential share selection, random weight allocation

The returns, risks and Sharpe ratios of the above 28 portfolios are given in the form of figures, which are looking very similar in terms of returns and risks. The Sharpe ratio first falls in 12 portfolios, but later raises steeply to is RM 1 and again in the upper portion they show ups and downs. Random allocation of funds is in way superior to other methods of allocation discussed above.

Effectiveness of portfolios under different methods

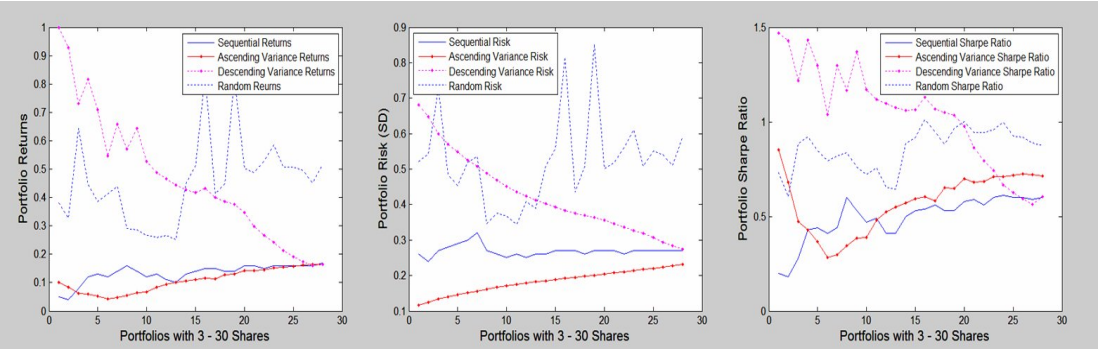
Share return variance is the basis for determining and selection of shares in the first three methods. The fourth method uses the classical random allocation method to assign funds and also inclusion of shares in subsequent portfolios. The return, risk and Sharpe ratios of all portfolios are given in the form of figures for assessing their efficiency below in 5.a, 5.b and 5.c.

The sequential share selection and allocation of funds based on variance of returns produces slightly better returns than the ascending order variance allocation of funds. Both of them show only a marginal difference in portfolios of five to ten shares. The random allocation returns are better and in larger portfolios they are even better than the ascending order variance portfolios. The ascending order variance portfolios show very high returns initially and gradually decline in larger portfolios and touch the descending order portfolio returns. But these returns are not the deciding parameter, because they ignore the risk completely.

The risk pattern of all four methods is akin to returns. Only the degree of variance differs. One can observe that the risk and return behave in the same manner. The ascending order risk gradually increases and the decreasing order risks decrease gradually. The random variance allocation risks increase substantially in large size portfolios.

Figure 5.c shows that all Sharpe ratios computed in all four methods of fund allocation. Sequential allocation Sharpe ratios are in blue colour which increases marginally till 10th portfolio, later in

larger size portfolios the ratio is stable. Ascending variance allocation, the Sharpe ratio declines from up to portfolio eight and later increases but at a decreasing rate. In large sized, portfolios they are stable as in sequential allocation but slightly higher.



5.a Portfolio returns 5.b Portfolio Risks 5.c Portfolio Sharpe ratios

Figure 5: Four share selection methods compared

In descending order allocation, the Sharpe ratios begin with very high values and up to 20 assets portfolios they decline gradually but in the larger sized portfolios it declines sharply. The random allocation shows stable Sharpe ratio in almost all portfolios irrespective of the size. In smaller portfolios the ratio is well below RM 1 but in larger portfolios, it is closer to RM 1. After portfolio 20, the random allocation results are well above all the other methods of allocation.

Conclusion

Selection of share for inclusion in portfolio and determining the right amount to be invested so as to maximize the return and minimize the risk has been tested and retested in several stock markets for the last fifty years. We have chosen thirty shares from Malaysian stock exchange to study portfolio behaviour in terms of Sharpe ratio by allocating the funds available in four different methods. As the risk is measured in terms of variance, we have chosen the variance as the basis for allocation of funds. Without ordering the shares in their variance, we constructed in random order, three-share portfolios to 30-asset portfolios and allocated the funds. The sequential portfolios produce very low Sharpe ratio when compared to all other methods. The descending order allocation produces superior Sharpe ratio in small size portfolios and declines sharply when the size increases. The random weight allocation produces stable Sharpe ratio almost in all sizes. The ascending order variance allocation also produces lower Sharpe ratio when compared to other methods. The results reveal that descending variance allocation produces better Sharpe ratio, if the portfolio size is less than 20, companies' shares and above 20, the random allocation produces better results.

References

Chan L. K., Karceski, J., & Lakonishok, J, (1999). On portfolio optimization: Forecasting covariances and choosing the risk model. *Review of Financial Studies*, 937-974.

Chou, P. H., Li, W. S., & Zhou, G, (2006). Portfolio optimization under asset pricing anomalies. *Japan and the World Economy*, 121-142.

Cohen, M. H., & Natoli, V. D. (2003). Risk and utility in portfolio optimization. *Physica A: Statistical Mechanics and its Applications*, 81-88.

De Giorgi, E. (2005). Reward–Risk Portfolio Selection and Stochastic Dominance. *Journal of Banking & Finance*, 895-926.

Elton, E. J., & Gruber, M. J. (1977). Risk reduction and portfolio size: An analytical solution. *The Journal of Business*, 415-437.

Gupta, G.S., Khoon, C. H. & Shahnon, S. (2001). How many securities make a diversified portfolio in KLSE stocks?. *Asian Academy of Management Journal*, 63-79.

Ibbotson, R. G. (2010). The importance of asset allocation. *Financial Analysts Journal*, 18-20

Konno, H. & Kobayashi, K. (1997). An integrated stock-bond portfolio optimization model. *Journal of Economic Dynamics and Control*, 1427-1444.

Li, B., & Li, J. (2012). Empirical Portfolio Analysis: M-V vs CAPM. *International Proceedings of Economics Development and Research*, 259-266.

Lin, P. C. (2012). Portfolio optimization and risk measurement based on non-dominated sorting genetic algorithm. *Journal of Industrial and Management Optimization*, 549-564

Lintner, J. (1965). Security Prices, Risk, and Maximal Gains From Diversification. *The Journal of Finance,* 587-615.

Markowitz, H. M. (1952). Portfolio Selection. *The Journal of Finance*, 77-91

Ng, K. Y. & Goh, K. L. (2011). Effects of Size and Allocation Method on Stock Portfolio Performance: A Simulation Study. *IACSIT Press, Singapore*, 538-542

Ong, P. W. (1982). Measuring the Expected Return and Risk of Combining Several Shares in an Investment Portfolio. *Securities Industry Review*, 6-16.

Paudel, R. B., & Koirala, S. (2006). Application of Markowitz and Sharpe Models in Nepalese Stock. *Journal of Nepalese Business Studies*, 18-35.

Scholz, H., & Wilkens, M. (2005). Investor-Specific Performance Measurement: A Justification of Sharpe Ratio and Treynor Ratio. *International Journal of Finance*, 17(4), 3671.

Statman, M. (1987). How Many Stocks Make a Diversified Portfolio? *Journal of Financial and Quantitative Analysis*, 60, 44-53.

Statman, M. (2004). The Diversification Puzzle. *Financial Analysts Journal,* 60(4), 44-53.

Xu, Y. (2003). Diversification in the Chinese Stock Market, Working Paper, School of Management, The University of Texas at Dallas.

Yoshimoto, A. (1996). The mean-variance approach to portfolio optimization subject to transaction costs. *Journal of the Operations Research Society of Japan*, 39(1), 99-117.