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Appropriate Choice of Lorentzian Approximation of the Fundamental mode from different versions to study Propagation Characteristics of Single Mode Fiber

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ABSTRACT

In this paper, we approximate the fundamental mode of guided propagation in three Lorentzian approximations (LZ0, LZ1, LZ2) sequentially with an expectation of improvements in the performance of propagation characteristics for single mode step index linear optical fiber within scalar variational framework. We show that the improvements in the performance of propagation characterics, normalized propagation constant- ($b(V)$), group delay- ($d(V)$) and especially wave guide dispersion ($g(V)$) version wise in the graphical presentation.

***Keywords**:* Single mode fiber, propagation characteristics, step index fiber, variational analysis, Scalar wave equation.

1. Introduction

We prefer using the single mode conventional and photonic crystal fiber for processing and transmitting information in bit form from user to user. Single mode conventional and photonic crystal fibers are well-known for processing information through the fundamental mode [1] and other applications. In graded index, single mode fiber (SMF), if one accurately knows the fundamental modal field for the corresponding profile, one can obtain the modal spot size and thereby, the various propagation characteristics of the fibers such as splice loss, bending loss [2] etc. The fundamental modal field is obtained from the solution of the differential equation related to the electromagnetic of the optical fiber fitted with the relevant boundary conditions and governed by a relevant refractive index profile [3].

It is well known that analytical solutions of the wave equations exist only in case of step index fibers in terms of Bessel and modified Bessel function [4, 5]. However for graded profiles, one has to take resort to exact numerical solutions. Although the analytical solution and exact numerical solution are accepted as standards for their rigor and accuracy; these solutions are deeply involved. For graded profiles, they do not give closed form expressions in predicting propagation characteristics such as splice loss and bending loss etc. Hence we need approximate methods such as variational [6] and perturbation methods. On the other hand, in analysis of propagation characteristics of associated fiber optics devices like Erbium doped fiber amplifier (EDFA) and Fiber Raman amplifier (FRA), one has to find out the fundamental modal field, ψ in the complex refractive index structure only by deeply involved but simple exact numerical methods [7]. It is known that the perturbation method works excellently when one knows the exact eigen-value and eigen-function of the unperturbed case and the perturbation is very small in comparison to unperturbed profile. Since the exact solution is analytically known for step profile, one can predict the propagation constant and modal field for nearly step profile by perturbative method. But for parabolic and triangular index profiles which are far from step index, one can safely apply variational analysis method. The variational analysis demands a simple and accurate approximation of the fundamental mode in terms of optimization parameters to be found through the optimization of propagation constant derived from the solution of scalar wave equation. Various functions are still being proposed in literature in order to suitably approximate the modal field. It is seen that the simple Gaussian approximation of the fundamental mode is popular but does not give an accurate result. Many approximations modifying the Gaussian function along the radial distance have been proposed but despite producing very accurate results, they require a lot of computational time. This is because, the more one increases the number of optimization parameters, and the more involved is the analysis.

We have already shown three types of Lorentzian functions [8-10] as an approximation of the fundamental modal field which work excellently to predict the propagation constant in low V -region but so it is high time to verify and report the best choice of the Lorentzian version of these through a comparative study of propagation characteristics.

2. **Analysis**

2.1 **Profile of Step index fiber**

The profile of step index fiber is given as,

$$\left.\begin{aligned} n^2(\rho) &= n_1^2(1-\Delta_1 f(\rho)), \quad \rho < 1 \\ &= n_1^2(1-\Delta_1) = n_0^2, \quad \rho \geq 1 \end{aligned}\right\} \qquad (1)$$

where n_1 is the refractive index of the core, grading parameter, $\Delta 1 = (n_1^2 - n_0^2)/n_1^2$, n_0 is the refractive index of cladding; Here, the $\rho = \frac{r}{a}$ where r is radial distance in cylindrical coordinate, a is the core radius. The profile function, $f(\rho) = \rho^q$, q is the profile exponent. When we consider the step index fiber $q \to \infty$ and $\rho^q \to 0$ so $f(\rho) = \rho^q = 0$.

2.2 **Lorentzian Expression of field for three versions:**

The proposed single parameter Lorentzian function for (LZ0) the modal field, ψ is expressed as

$$\psi(\rho) = A(1+\frac{\rho^2}{\omega_0^2})^{-1} \quad \text{for} \quad 0 \le \rho \le \infty \qquad (2)$$

A is a constant and ω_0 is the variational parameter to be optimized w.r.t. V.

Next in order to improve the propagation performance, above version of Lorentzian function (LZ0), we propose and formulate two modified forms as given below and carry out the similar study on propagation characteristics to expect improved result as per expectation in variational formulation. The modal fields for the two improved versions expressed in two sets of equations are respectively proposed as follows:

for the first modified Lorentzian (LZ1) modal field equation

$$\psi(\rho) = \begin{cases} \dfrac{A}{1+\dfrac{\rho^2}{\omega_0^2}} & \text{for} \quad 0 \le \rho \le 1 \\[2ex] \dfrac{Ae^{\frac{1}{\rho_0}}}{1+\dfrac{1}{\omega_0^2}} e^{-\frac{\rho}{\rho_0}} & \text{for} \quad 1 < \rho < \infty \end{cases} \qquad (3)$$

$$\psi(\rho) = \begin{cases} \dfrac{A}{1+\dfrac{\rho^2}{\omega_0^2}} & \text{for} \quad 0 \le \rho \le 1 \\[2ex] \dfrac{Ae^{\frac{1}{\rho_0}}}{1+\dfrac{1}{\omega_0^2}} \dfrac{1}{\sqrt{\rho}} e^{-\frac{\rho}{\rho_0}} & \text{for} \quad 1 < \rho < \infty \end{cases} \qquad (4)$$

In both the cases the field and its derivatives are equated in core-cladding interface for which we get a relation between ρ_0 and variational parameter ω_0. They are $\rho_0 = (\omega_0^2 + 1)/2$ for the case of Lorentzian field (LZ1) as expressed in equation (3) and $\rho_0 = 2(1+\omega_0^2)/(3-\omega_0^2)$ for the case Lorentzian field (LZ2) mentioned in equation (4). The formulations of important propagation characteristics of practical interest like normalized propagation constant , group delay $d(V)$ and wave guide dispersion $g(V)$ based on equation (3) and (4) are presented below.

$$\left.\begin{aligned} b(V) = 1 - \frac{U^2}{V^2};\ d(V) = \frac{d(Vb(V))}{dV} \\ g(V) = V\,\frac{d(d(V)}{dV} \end{aligned}\right\} \quad (5)$$

2.3 **Scalar wave equation**

Now in order to use these functions, we write our scalar wave Maxwell's equation in terms of field ψ(ρ) under weakly guiding approximation [6] for any media as below:

$$\rho^2 \frac{\partial^2 \psi(\rho)}{\partial \rho^2} + \rho \frac{\partial \psi(\rho)}{\partial \rho} + a^2 \rho^2 [k_0^2 n^2(\rho) - \beta^2] \psi(\rho) = 0 \qquad (6)$$

From the above equation and after little computation, we get the following variational expression of normalized propagation constant U which is to be optimized w.r.t single Lorentzian parameter ω_0,

$$U^2 = \frac{V^2 \int_1^\infty \psi^2(\rho)\rho d\rho + \int_0^\infty \left(\frac{d\psi(\rho)}{d\rho}\right)^2 \rho d\rho + V^2 \int_0^1 f(\rho)\psi^2(\rho)\rho d\rho}{\int_0^\infty \psi^2(\rho)\rho d\rho} \qquad (7)$$

where $U^2 = a^2(k_0^2 n_1^2(\rho) - \beta^2)$; U is known as normalized propagation constant and $V^2 = a^2 k_0^2 (n_1^2(\rho) - n_0^2(\rho))$; V is known as normalized frequency.

In the above expression of U^2 we substitute the different ψ for different versions and optimized above expression w.r.t. ω_0 and find relation of Lorentzian parameters and V. This, in turn gives the relation between U and V involving Lorentzian parameter and we find field ψ(ρ), $b(V)$, $d(V)$ and $g(V)$. They give the propagation characteristics of the different versions. The above parameters are expressed in the equation (5) and they are formulated in the different Lorentzian expressions.

3. Results and Discussions

We use the equation (5) to find $b(V)$, normalized propagation constant for three different versions of Lorentzian approximation for different values of V and they are Plotted as shown in the Fig.1. The black curve indicates the analytical one and subsequently we plot the same curve for LZ2, LZ1 and LZ0. The LZ2 curve –green one is closer to analytical and similarly others like LZ1 and LZ0 are coming next. We have taken for step profile in linear regime. So we can claim that results are improving version wise.

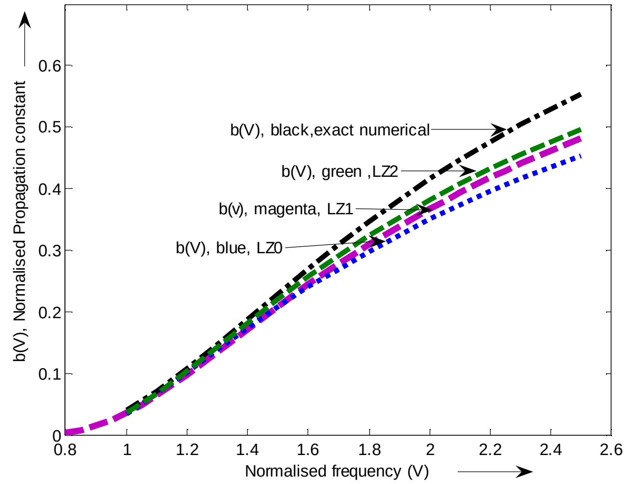

Figure 1: Plot of $b(V)$ vs. V : $b(V)$ of LZ2 is compared with LZ1,LZ0 and exact numerical.

Same equation (5) is used for finding group delay for different versions like LZ2, LZ1 and LZ0.

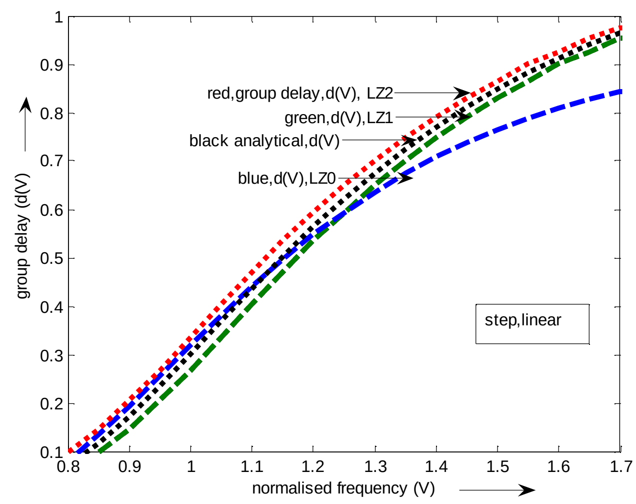

Figure 2: Plot of group delay ($d(V)$ vs normalized frequency (V); Here for step linear profile is presented for LZ2 and LZ2 is compared with Analytical, LZ1 and LZ0.

In the Fig.2 we have shown the plot of group delay $d(V)$ vs. V and we find LZ2 and LZ1 are closer to analytical one compared to LZ0. In the graph, the red one indicates LZ2, blue one indicates LZ0 and green one is LZ1. The group delay for LZ1 and LZ2 are very closer all through for V values from 0.8 to 1.7 but the blue curve start deviating from the analytical for the V values 1.3 onwards.

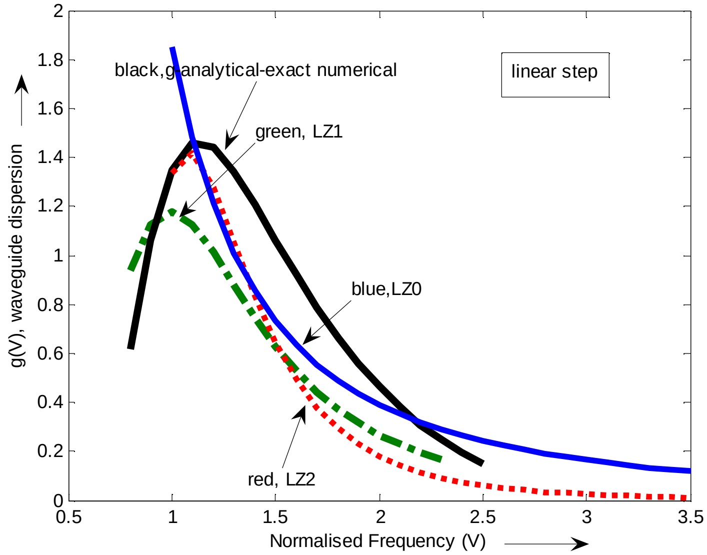

Figure 3: Plot of $g(V)$ vs. V : LZ2 curve is compared with exact numerical, LZ0 and LZ1.

In Fig.3 we have shown the values of group velocity dispersion of three different versions; they are plotted against different values of V . Here we find the trend of all the curves except LZ0 are similar to analytical one; especially the LZ2 is very closer to analytical in low-V region. The curve for dispersion of LZ2 is shown by red spotted one and analytical is plotted by black one and blue curve represent LZ0.

4. **Conclusion**

In this paper we show how the Lorentzian approximations in three different versions produce the important propagation characteristics with different values of normalized frequency. The entire findings are evaluated by adopting the variational analysis principle using scalar wave equation. The characteristics parameters shown are normalized propagation constant, group delay and group velocity dispersion. The most improved version coming out of this intensive study emerges as LZ2 for which the group velocity dispersion curve is following the same trend as well as it is better matching with that of analytical one in low V -region, a region of evanescent coupling

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