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Simple approach to find out fiber parameters and characteristics using Marcuse spot sizes applied to Variational Formalism

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ABSTRACT

In this paper, we report for the first time to the best of our knowledge that for a given value of fiber spot size, obtained from Marcuse's relation, one can accurately and easily determine the fiber parameters for a particular normalized frequency using Variational formalism. Then we have shown that with a typical example of fractional power in the core, this technique can also accurately predict propagation characteristics.

1. Introduction

In optical fiber communication, it is very important to calculate precisely the fiber parameters like U, *w* etc. for a particular normalized frequency (V) to study the different propagation characteristics of single mode fiber having step or graded refractive index profile [1-3]. One can obtain such parameter values by solving the well known scalar wave, or semi or full vectorial wave equation for different types of fibers with different refractive index profiles with various numerical or approximation methods. Determination of fiber spot size for particular V value is possible with available Marcuse's formulae, but it is difficult to predict U, W values having only spot size for corresponding V value using such numerical techniques. The use of Variational method can solve this problem. In this approach, the spot size calculated from the available Marcuse's relation is used in the Variational expression of U or *w* or any fiber parameter starting from the well known wave equation to determine all fiber characteristics accurately. In the derivation of the expression of Marcuse's spot size, the overlap integration of the Gaussian fields is maximized [1]. In our investigation, it is seen that our results match fairly with available results obtained from tedious numerical methods.

Our entire analysis is formatted sequentially as follows: the section 2 describes the calculation procedure with Variation approach. We present our results and discussions in the section 3 followed by the conclusion section.

2. Analysis

The refractive index profile for our entire analysis is defined as:

$$n^2(R) = \begin{cases} n_1^2 - (n_1^2 - n_2^2) f(R); R \le 1 \\ n_2^2; R > 1 \end{cases} \qquad (1)$$

where n_1 and n_2 are the refractive indices of the core and cladding regions of the fiber respectively. R is the normalized radial distance. The profile function *f(R)* is defined as:

$$f(R) = \begin{cases} R^q; R \le 1 \\ 1; R > 1 \end{cases} \qquad (2)$$

In the above equation, q is the profile exponent. For step and parabolic profiles the values of q are taken as ∞ and 2 respectively.

The general expression for U^2 is written as:

$$U^2 = \frac{V^2 \int_0^1 \psi^2 R^{q+1} dR + V^2 \int_1^\infty \psi^2 R dR + \int_0^\infty \psi'^2 R dR}{\int_0^\infty \psi^2 R dR} \qquad (3)$$

where ψ is the modal field distribution. For both step and parabolic index fibers the following Gaussian function is chosen:

$$\psi(R) = A\exp(-\frac{R^2}{w_V^2}) \qquad (4)$$

where w_V is the fiber spot size.

The variational expression of U values for step and parabolic index fibers are respectively given as:

$$U_V{}^2 = 1 + 2\ln V = 1 + \frac{2}{w_V{}^2}\ ; \qquad \text{for step profile} \qquad (5)$$

$$U_V{}^2 = \frac{V^2}{2\alpha}\left[1 - e^{-2\alpha}\right] + 2\alpha; \qquad \text{for parabolic profile} \qquad (6)$$

where $\alpha = \frac{1}{w_V{}^2}$

In parabolic index fiber, the fractional power in the core, *f_co* is calculated using w_V from the following relation [2]:

$$f\_co = 1 - e^{-2/w_V^2} \qquad (7)$$

For step index fiber, Marcuse spot size w_M is calculated using well known equation written as [4]:

$$\frac{w_M}{a} \approx 0.65 + \frac{1.619}{V^{3/2}} + \frac{2.879}{V^6} \qquad (8)$$

The V values lies between 0.8 and 2.5.

Again for graded index profile the following equation is utilized [4]:

$$\frac{w_M}{a} \approx \frac{A}{V^{2/(q+2)}} + \frac{B}{V^{3/2}} + \frac{C}{V^6} \qquad (9)$$

where

$$\left.\begin{aligned} A &= \left\{\frac{2}{5}(1+4\left(\frac{2}{q}\right)^{5/6}\right\}^{1/2} \\ B &= e^{0.298/q} - 1 + 1.478\left(1-e^{-0.077q}\right) \\ C &= 3.76 + e^{4.19/q^{0.418}} \end{aligned}\right\} \qquad (10)$$

In this paper, we calculate the values of w_M from equations (8) and (9) for step and graded index fiber. Then we substitute these w_M replacing variational w_V in equation (5) and (6) respectively. These values of U now give new U values, U_M which we present as fiber parameter from Marcuse formulation coupled to Variational method and compare with exact values U_N and Variational values, U_V obtained from Variational optimization. If we optimize equation (6) we get a tedious transcendental equation to solve.

3. Results and discussions

As stated in the earlier section, we present values of U_N, U_M and U_V in Table 1 and Table 2 for step and parabolic index fibers respectively. From both the tables, we see that U_M values closely tally with U_N at par in case of step index fibers and in a better way in case of parabolic index fiber, where $U_N \le U_M < U_V$. Thus, our technique appears to be not only simple but also effective in predicting fiber parameters accurately when spot size is given and transcendental equation is to be solved.

TABLE 1:COMPARISON OF DIFFERENT U VALUES FOR DIFFERENT V VALUES FOR STEP INDEX FIBER

V	U_N	U_M	U_V
1.5	1.316888	1.276084	1.345708
1.9	1.492849	1.460036	1.511194
2.2	1.591060	1.567540	1.605277
2.4	1.645312	1.628303	1.658595
2.8	1.734204	1.730295	1.749068

TABLE 2:COMPARISON OF DIFFERENT U VALUES FOR DIFFERENT V VALUES FOR PARABOLIC INDEX FIBER

V	U_N	U_M	U_V
2.1	1.9077395	1.945949	2.050669
2.5	2.1431626	2.170102	2.220538
3.0	2.3941375	2.411598	2.414642
3.5	2.6124428	2.624616	2.625601

In order to justify the merit of the method further, we compute fractional power in core for parabolic index fiber. Here we see that out curve matches excellently with the exact curve [5] over a large region of *V* values of practical interest, viz. near cut off region. Other propagation characteristics can also be found with our simple variational approach.

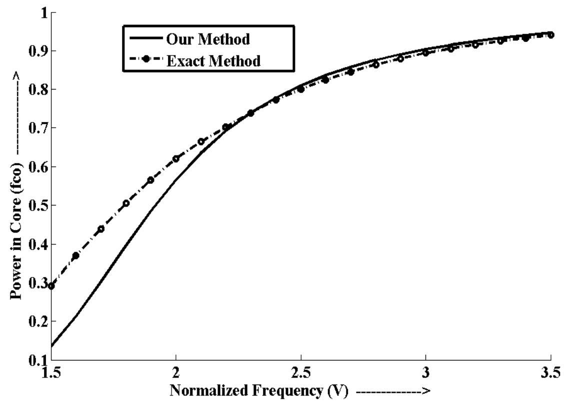

Figure 1: Variation of Power in core ($f_{\_co}$) with Normalized Frequency (V) for Parabolic index Fiber

4. Conclusion

Variational formulation coupled to Marcuse spot size is shown to predict propagation characteristics of single mode fiber in simple and effective way when only spot size is known theoretically or experimentally.

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