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Unsteady Hydro-Magnetic Flow of an Oldroyd Fluid through a Porous Channel with Oscillating Walls

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Abstract. The investigation concerns the time-dependent flow of an incompressible unsteady conducting Oldroyd fluid through a porous channel, which is under the influence of transverse magnetic field. It is assumed that the fluid is injected through lower plate with a constant velocity V and it is sucked off with the same velocity through the upper plate. Laplace transform method has been employed for t and the differential equations, which do not involve the retardation time constant, are solved using a perturbation scheme treating visco-elastic parameter to be small. Results are investigated for relaxation parameter, Hartman number and cross-flow Reynolds number on velocity and skin friction of the flow with graphs.

Keywords: Oldroyd Fluid, Hydromagnetic, Cross-flow Reynolds number, Retardation time, Perturbation technique

1. Introduction

Flow under the influence of magnetic field has wide range of theoretical and practical interest to the researchers for last four decades. In the presence of magnetic field fluid particles experience a force is induced by the electric current which modifies the flow field. This Lorenz force is the interaction between the transverse magnetic field and the electrically conducting fluid. In industry, various fluids pass through duct, which is surrounded by electromagnet, are porous. This porosity and duct shape depends on the industrial applications. If the fluid contains particles, which are influenced by the magnetic field, then the Lorenz force appears to prevent the motion of fluid and random motion of fluid particles become obscure. So, the advantage of this force may be applied to suppress the week turbulent motion of the fluid. Flow through small gap as well as the rheometric experiments is examples of application of flow through parallel plates. Most of the industrial fluids are non-Newtonian and some of them are electrically conducting. Viscoelastic fluids, containing the viscous stresses as well as elastic responses in the constitutive relations exhibit different types of flow phenomenon. The Oldroyd two constant models is one of the viscoelastic fluids which exert viscous frictional forces between the two layers together with elongated preventive fibers constituted by molecules. Here the layers of fluid are the assembled of parallel fibers with equivalent properties. Chemicals like, paints, adhesive, coal products etc. are few examples of viscoelastic fluids. Moreover, bio-chemical materials (in liquid states) are also to be included in this class of fluids. Besides, the environmentally polluted material such as industrial wastes in liquid state may also considered to be another kind of non-Newtonian fluid. All the aforesaid non-

Newtonian fluid may not represent the electrical conductivity; however, fluids with constituent ingredients magnetic material may follow the Oldroyd two constant constitutive relations.

As the frictional forces are very high, the flow of a non-Newtonian fluid is usually one-dimensional. In reality, bulk material compelled to pass through the channel is time-dependent because the displacements of material particles are situation dependent. The pressure gradient induced the flow may be the combination of separate function of time and space or both. A source of fluid may also yield the flow arising from either of the plate. The flow analysis of the well-known pressure gradient is important to study the stability of the turbulent motion. The investigation of heat and mass transfer is important in view of the chemical engineering processes. However, suction and injection of fluid in the channel put forward a due advantage to apply the model for research into modern engineering design as well as biological systems.

Sellers and Walker [2] initiated the hydromagnetic laminar flow between parallel planes with non-uniform magnetic field. Mukhopadhya and Chaudhary [3] presented the fluctuating flow of Oldroyd type viscoelastic fluid, which is passing over an infinite flat plate. Recently Ray et al. [4] obtained an exact solution of conducting Oldroyd two constants viscoelastic fluid flow in a horizontal channel in the presence of transverse magnetic field. They assumed that the flow is sinusoidal time dependent. There are also some important investigations on the non-Newtonian fluid flow with or without magnetic field [5, 6, 7, 8]. Asghar et al. [8] put forward an exact solution of unsteady hydromagnetic flows of an Oldroyd -B fluid by employing Fourier transform method. Uddin et. al. [9] put forward some interesting results on hydro-magnetic stagnation point flow with heat transfer and the nonlinear differential equations are solved by Runge-Kutta method. Hayat et. al. [9] investigated the influence of Hall currents and material parameters of the second grade fluid in a rotating frame of reference. They found periodic solution for steady and unsteady flows. Rajagopal and Bhatnagar [10] discussed the flow of Oldroyd -B fluid past an infinite porous plate and the longitudinal and torsional oscillations of an infinite rod of finite radius. Very fundamental ideas of some periodic and non-periodic flows of an Oldroyd-B can be found in Hayat et al. [11]. In the subsequent studies [12] they investigated the same problem by incorporating a rigid body rotation. In a recent study Hayat and Hutter [13] employed the Laplace transform method for the solution for the solution of an initial value problem arising out of a second order rotating fluid through an infinite porous plate.

Hydrodynamic (MHD) flow of second grade visco-elastic fluid past a wedge with porous suction or injection has been studied by Hsiao [14] with the consideration of mixed convective heat transfer. He analyzed governing, momentum and energy equations of the fluid flow by the method of combination of series expansion, the similarity transformation and finite-difference method. With the other significant result he concluded that the buoyancy force can accelerate the fluid motion in the boundary layer and enhance the heat transfer performance.

In the present paper we are solving a generalized time dependent Oldroyd two constant fluid flow governing equation along with the constitutive relations by using the perturbation method. The integral transform method has been employed and the derived ordinary differential equations are solved by perturbation technique [14] when the retardation time constant is the perturbation parameter. We have derived the velocity as well as skin friction for the flow of fluid with the introduction of impulsive fluid velocity [15].

2. Basic Equations

The constitutive equation of an incompressible second order fluid based on Oldroyd model [1] is

$$T_{ij} = -p\delta_{ij} + \tau_{ij} \tag{1}$$

$$(1+\lambda_1 \frac{d}{dt})\tau_{ij} = \mu(1+\lambda_2 \frac{d}{dt})e_{ij} \tag{2}$$

$$e_{ij} = \frac{1}{2}(v_{i,j} + v_{j,i}) \tag{3}$$

where T_{ij}, τ_{ij}, e_{ij}, p, μ, λ_1, and λ_2 are the total stress tensor, deviatoric stress tensor, rate of strain tensor, pressure, coefficient of viscosity, relaxation time and retardation time respectively.

$\frac{d}{dt} = \frac{\partial}{\partial t} + v_i \frac{\partial}{\partial x_i}$ The continuity and momentum equations are

$$v_{i,i} = 0 \tag{4}$$

$$\rho(\frac{\partial v_i}{\partial t} + v_i v_{i,j}) = -p_{,i} + \tau_{ij,j} \tag{5}$$

3. Formulation and Solution

Keeping the view of industrial application in mind we consider the flow of Oldroyd type electrically conducting viscoelastic fluid between two parallel plates; here both the plates are in oscillatory motion with a constant mean velocity U . We assume that on one plate the fluid is being injected with some constant velocity V and the opposite plate is sucked off with the same velocity.

It is assumed that the x-direction is parallel to the plate and y-axis, normal to the plates. The magnetic field is applied along the transverse direction of the flow and perpendicular to the plates. In practice most of the non-Newtonian fluids have a small magnetic Reynolds number and for this reason the induced magnetic field may be neglected. We introduce an impulsive fluid velocity u_1 into the governing equation. In view of the aforesaid considerations, the constitutive and momentum equations of motion of one-dimensional flow field for Oldroyd fluid may be represented as

$$(1+\lambda_1 \frac{\partial}{\partial t} + V\lambda_1 \frac{\partial}{\partial y})\tau_{xy} = \mu(1+\lambda_2 \frac{\partial}{\partial t} + \lambda_2 V \frac{\partial}{\partial y})\frac{\partial u}{\partial y} \tag{6}$$

$$\frac{\partial u}{\partial t} + V\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{1}{\rho}\frac{\partial(\tau_{xy})}{\partial y} - \sigma H_0^2 \eta \frac{u}{\rho} - \frac{\sigma H_0^2 \eta}{\rho} u_1 \tag{7}$$

At first we shall investigate the steady flow field with boundary and initial conditions

$$u = 0 \quad \text{at} \quad y = 0 \quad \text{and} \quad y = h; \quad V = 0, \quad \frac{\partial p}{\partial x} = 0 \tag{8}$$

and later unsteady flow with boundary and initial conditions

$$\begin{aligned} &u = U(t), && \text{at} && y = 0 \\ &u = V(t), && \text{at} && y = h \\ &\frac{\partial p}{\partial x} = p(t), && \text{and} && V \neq 0 \end{aligned} \tag{9}$$

We now introduce the following non-dimensional quantities

$$\bar{u} = \frac{uh}{\upsilon}, \quad \bar{y} = \frac{y}{h}, \quad \bar{t} = \frac{\mu t}{\rho h^2}, \quad \alpha_1 = \frac{\lambda_1 \mu}{\rho h^2}, \quad \alpha_2 = \frac{\lambda_2 \mu}{\rho h^2}, \quad M^2 = \eta^2 H_0^2 \frac{\sigma}{\rho},$$

$$\bar{\tau} = \frac{\tau h^2}{\mu}, \quad \bar{p} = \frac{ph^2}{\upsilon}, \quad R = \frac{Vh}{\upsilon}, \quad R_1 = \frac{hu_1}{\upsilon}$$

$$(1 + \alpha_1 \frac{\partial}{\partial t} + R\alpha_1 \frac{\partial}{\partial y})\tau_{xy} = \upsilon(1 + \alpha_2 \frac{\partial}{\partial t} + \lambda \alpha_2 R \frac{\partial}{\partial y})\frac{\partial u}{\partial y} \tag{10}$$

$$\frac{\partial u}{\partial t} + R\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{\upsilon}\frac{\partial(\tau_{xy})}{\partial y} - M^2 u - M^2 R_1 \tag{11}$$

Now, for steady Newtonian fluid flow the governing equation of motion is

$$\frac{d^2 u_0}{dy^2} - M^2 u_0 = M^2 R_1 \tag{12}$$

Using the non-dimensional form of the boundary condition (8), we obtain a solution as

$$u_0 = R_1 \cosh(My) + \frac{R_1(\cosh M - 1)}{\sinh M}\sinh(My) - R_1 \tag{13}$$

After a little manipulation, we may find the unsteady governing equation as follows

$$\alpha_1 \frac{\partial^2 u}{\partial t^2} + (1 + \alpha_1 M^2)\frac{\partial u}{\partial t} + 2\alpha_1 R \frac{\partial^2 u}{\partial y \partial t} - \alpha_2 \frac{\partial^3 u}{\partial y^2 \partial t} + (R + \alpha_1 R M^2)\frac{\partial u}{\partial y} +$$

$$(\alpha_1 R^2 - 1)\frac{\partial^2 u}{\partial y^2} - \alpha_2 R \frac{\partial^3 u}{\partial y^3} + M^2 u + M^2 R_1 + \frac{\partial p}{\partial x} + \alpha_1 \frac{\partial^2 p}{\partial x \partial t} + \alpha_1 R \frac{\partial^2 p}{\partial x \partial y} = 0 \tag{14}$$

It is encouraging to solve the equations governing the flow of non-Newtonian fluids so as to challenge the mathematical complexity. The analytical solution for the flow of second grade fluid is really difficult to achieve. One prime reason behind this is the order of differential equation is more than the number of available boundary conditions. The difficulty is further accentuated by the fact that a non-Newtonian parameters of the fluid usually occurs in the coefficient of highest derivative. To solve this difficulty we generally seek for a perturbation solution assuming the non-Newtonian fluid parameter to be small [16]. As the governing equations are linear, the Laplace transform technique can be applied for a small time solution. With the compatible data we may enforce the initial condition and the inverse transform could be found easily [17].

Taking Laplace transform w. r. t. t with

$$u(y,0)=u_0(y), \qquad \frac{\partial u(y,0)}{\partial t}=0 \qquad \text{and} \qquad p(0)=0$$

we get

$$-\alpha_2\frac{\partial^3\bar{u}}{\partial y^3}+(\alpha_1R^2-1-\alpha_2 s)\frac{\partial^2\bar{u}}{\partial y^2}+(R+RM^2\alpha_1+2Rs\alpha_1)\frac{\partial\bar{u}}{\partial y}+$$

$$(\alpha_1 s^2+s+\alpha_1M^2s+M^2)\bar{u}$$

$$=\frac{M^2R_1}{s}+(1+s\alpha_1)\bar{p}(s)+(\alpha_1+1+\alpha_1M^2)u_0+2R\alpha_1\frac{\partial u_0}{\partial y} \tag{15}$$

In the sequel, we shall treat the retardation time to be small and develop the perturbation solution to the second order in α_2, we get,

$$\bar{u}(y,s)=\bar{u}10(y,s)+\alpha_2\bar{u}11(y,s)+\alpha_2^2\bar{u}12(y,s)+\ldots\ldots\ldots\ldots\ldots\ldots\ldots \tag{16}$$

Substituting (16) into (15) we get

$$(\alpha_1R^2-1)\frac{\partial^2\bar{u}10}{\partial y^2}+(R+\alpha_1RM^2+2\alpha_1Rs)\frac{\partial\bar{u}10}{\partial y}+$$

$$(\alpha_1 s^2+s+\alpha_1M^2s+M^2)\bar{u}10$$

$$=\frac{M^2R_1}{s}+\{1+\alpha_1 s-p(0)\}\bar{p}(s)+(1+\alpha_1+\alpha_1M^2)u_0(y)+2\alpha_1R\frac{\partial u_0}{\partial y} \tag{17}$$

$$(\alpha_1R^2-1)\frac{\partial^2\bar{u}11}{\partial y^2}+(R+\alpha_1RM^2+2\alpha_1Rs)\frac{\partial\bar{u}11}{\partial y}$$

$$+(\alpha_1 s^2+s+\alpha_1M^2s+M^2)\bar{u}11$$

$$=\frac{\partial^3\bar{u}10}{\partial y^3}+s\frac{\partial^2\bar{u}10}{\partial y^2} \tag{18}$$

$$(\alpha_1 R^2 - 1)\frac{\partial^2 \bar{u}_{12}}{\partial y^2} + (R + \alpha_1 RM^2 + 2\alpha_1 Rs)\frac{\partial \bar{u}_{12}}{\partial y}$$

$$+ (\alpha_1 s^2 + s + \alpha_1 M^2 s + M^2)\bar{u}_{12} = \frac{\partial^3 \bar{u}_{11}}{\partial y^3} + s\frac{\partial^2 \bar{u}_{11}}{\partial y^2} \tag{19}$$

Solutions of (17)—(19) are obtained are as follows

$$\bar{u}_{10} = C_1 e^{m_1 y} + C_2 e^{m_2 y} + X_4 \cos(My) + X_5 \sin(My) + X_6 + X_0 \tag{20}$$

$$\bar{u}_{11} = C_3 e^{m_1 y} + C_4 e^{m_2 y} + X_9 \cos(My) + X_{10} \sin(My) \tag{21}$$

$$\bar{u}_{12} = C_5 e^{m_1 y} + C_6 e^{m_2 y} + X_{13} \cos(My) + X_{14} \sin(My). \tag{22}$$

where $X_{1,}$ $X_4,$ $X_5, X_6,$ X_9, X_{10} , X_{13}, X_{14} are various terms appeared in the calculation and are omitted in this paper due to the scarcity of space.

C_i, i=1, 2, …, 6, are to be evaluated by using the transform boundary condition (9). After determining the constants C_i we may apply shifting and convolution theorem of the inverse Laplace transform and obtain the final solution in the following

$$u(y,t) = (1 + \alpha_2 + \alpha_2^2 + \ldots\ldots\ldots\ldots)\sum_{i=1}^{2}\sum_{k=0}^{\infty}\int_0^t V(t-\lambda)e^{n_1(\xi_{ki})(y-1)}\frac{\sinh n_2(\xi_{ki})y}{\frac{d}{ds}[\sinh n_2]_{s=\xi_{ki}}}e^{\xi_{ki}\lambda}d\lambda$$

$$+ (1 + \alpha_2 + \alpha_2^2 + \ldots\ldots\ldots\ldots\ldots)\sum_{i=1}^{2}\sum_{k=0}^{\infty}\int_0^t U(t-\lambda)e^{n_1(\xi_{ki})}\frac{\sinh n_2(\xi_{ki})(1-y)}{\frac{d}{ds}[\sinh n_2]_{s=\xi_{ki}}}e^{\xi_{ki}\lambda}d\lambda$$

$$+ \sum_{k=0}^{\infty}\sum_{i=1}^{2}.\int_0^t \sum_{j=p}^{4}.\frac{d^n}{ds^n}\left[\frac{Y_1(s)e^{s(t-\lambda)}]}{\prod\limits_{\substack{q=j\\ j<4}}^{3}.(s-r_{1q+1})\prod\limits_{\substack{t=1\\ j>1\\ p<2}}^{j-1}.(s-r_{1t})}\right]_{\substack{s=r_{1j}\\ j=p\\ n=p-1}}$$

$$.\left[\begin{array}{l} e^{n_1(\xi_{ki})y+\xi_{ki}\lambda}\dfrac{\sinh(y-1)n_2(\xi_{ki})}{\frac{d}{ds}[\sinh n_2]_{s=\xi_{ki}}}+e^{n_1(\xi_{ki})(y-1)+\xi_{ki}\lambda}\dfrac{\sinh n_2(\xi_{ki})y}{\frac{d}{ds}\left[\sinh n_2\right]_{s=\xi_{ki}}} \\ \cos M - e^{n_1(\xi_{ki})+\xi_{ki}\lambda}\cos My \end{array}\right]d\lambda +$$

$$\sum_{k=0}^{\infty}\sum_{i=1}^{2}.\int_0^t\sum_{j=p1}^{4}.\frac{d^n}{ds^n}\left[\frac{Y_2(s)e^{s(t-\lambda)}]}{\prod\limits_{\substack{q=j\\ j<4}}^{3}.(s-r_{1q+1})\prod\limits_{\substack{t=1\\ j>1\\ p1<2}}^{j-1}.(s-r_{1t})}\right]_{\substack{s=r_{1j}\\ j=p1\\ n=p1-1}}$$

$$.\left[\begin{array}{l} e^{n_1(\xi_{ki})y+\xi_{ki}\lambda}\dfrac{\sinh(y-1)n_2(\xi_{ki})}{\left[\sinh n_2\right]'_{s=\xi_{ki}}}+e^{n_1(\xi_{ki})(y-1)+\xi_{ki}\lambda}\dfrac{\sinh n_2(\xi_{ki})y}{\left[\sinh n_2\right]'_{s=\xi_{ki}}}\sin M \\ -e^{n_1(\xi_{ki})+\xi_{ki}\lambda}\cos My \end{array}\right]d\lambda$$

$$\sum_{k=0}^{\infty}\sum_{i=1}^{2}.\int_0^t\sum_{j=p2}^{2}\frac{d^n}{ds^n}.\left[\frac{X_3[e^{s(t-\lambda)}]}{\prod\limits_{\substack{q=j\\ j<2}}^{1}.(s-r_{0q+1})\prod\limits_{\substack{t=1\\ j>1\\ p2<1}}^{j-1}(s-r_{0t})}\right]_{\substack{s=r_{0j}\\ j=p2\\ n=p2-1}}\left[\begin{array}{l} e^{n_1(\xi_{ki})y+\xi_{ki}\lambda}\dfrac{\sinh(y-1)n_2(\xi_{ki})}{\frac{d}{ds}[\sinh n_2]_{s=\xi_{ki}}}- \\ e^{n_1(\xi_{ki})y+\xi_{ki}\lambda}\dfrac{\sinh n_2(\xi_{ki})y}{\frac{d}{ds}[\sinh n_2]_{s=\xi_{ki}}}+1 \end{array}\right]d\lambda$$

$$+\sum_{i=1}^{2}.\sum_{k=0}^{\infty}[\int_0^t MR_1 \sum_{j=p3}^{3}.\frac{d^n}{ds^n}\left[\frac{e^{s(t-\lambda)}}{\prod_{\substack{q=j\\ j<3}}^{2}.(s-r_{0q+1})\prod_{\substack{t=1\\ j>1\\ p3<2}}^{j-1}(s-r_{0t})}\right]_{\substack{s=r_{0j}\\ j=p3\\ n=p3-1}}$$

$$-\int_0^{t-\lambda} p\{t-\lambda-q)\sum_{i=1}^{2}.\left[\begin{array}{l}\frac{d^n}{ds^n}\sum_{j=p4}^{2}.[\frac{\{1-p(0)\}e^{sq}}{\prod_{\substack{q=j\\ j<2\\ p4<2}}^{1}.(s-r_{0q+1})\prod_{\substack{t=1\\ j>1\\ p4<2}}^{j-1}.(s-r_{0t})}]_{\substack{s=r_{0j}\\ j=p4\\ n=p4-1}}\\ +\frac{\alpha_1}{2}(R^2\alpha_1-1)[(\frac{r_{01}e^{r_{01}q}-r_{02}e^{r_{02}q}}{r_{01}-r_{02}})_{r_{01}\neq r_{02}}+e^{qr_{01}}(1+r_{01}q)_{r_{01}=r_{02}}\end{array}\right]dq$$

$$][\left[e^{n_1(\xi_{ki})y+\xi_{ki}\lambda}\frac{\sinh(y-1)n_2(\xi_{ki})}{\frac{d}{ds}[\sinh n_2]_{s=\xi_{ki}}}-e^{n_1(\xi_{ki})y+\xi_{ki}\lambda}\frac{\sinh n_2(\xi_{ki})y}{\frac{d}{ds}[\sinh n_2]_{s=\xi_{ki}}}+1\right]]d\lambda+$$

$$\alpha_2\sum_{i=}^{2}\sum_{k=1}^{\infty}\int_0^t\sum_{j=p}^{4}\frac{\frac{d^n}{ds^n}(M^2(X_1M-X_2)e^{s(t-\lambda)})_{\substack{s=r_{0j}\\ j=p\\ n=p-1}}}{\prod_{\substack{q=j\\ j<4}}^{3}(r_{1j}-r_{1q+1})\prod_{\substack{t=1\\ j>1\\ p<2}}^{j-1}(r_{1j}-r_{1t})}$$

$$\left[\begin{array}{l}e^{n_1(\xi_{ki})y+\xi_{ki}\lambda}\frac{\sinh(y-1)n_2(\xi_{ki})}{\frac{d}{ds}\left[\sinh n_2\right]_{s=\xi_{ki}}}+e^{n_1(\xi_{ki})(y-1)+\xi_{ki}\lambda}\frac{\sinh n_2(\xi_{ki})y}{\frac{d}{ds}\left[\sinh n_2\right]_{s=\xi_{ki}}}\cos M\\ -e^{n_1(\xi_{ki})+\xi_{ki}\lambda}\cos My\end{array}\right]d\lambda$$

$$+\alpha_2 \sum_{i=1}^{2}\sum_{k=1}^{\infty}\int_0^t \sum_{j=p5}^{4}\left[\frac{d^n}{ds^n}\frac{(-M^2(X_1M+X_2)e^{s(t-\lambda)})}{\prod\limits_{\substack{q=j\\ j<4}}^{3}(s-r_{1q+1})\prod\limits_{\substack{t=1\\ j>1\\ p5<2}}^{j-1}(s-r_{1t})}\right]_{\substack{s=r_{1j}\\ j=p5\\ n=p5-1}}$$

$$\left[e^{n_1(\xi_{ki})(y-1)+\xi_{ki}\lambda}\frac{\sinh n_2(\xi_{ki})y}{\frac{d}{ds}\left[\sinh n_2\right]_{s=\xi_{ki}}}\sin M - e^{n_1(\xi_{ki})+\xi_{ki}\lambda}\sin My\right]d\lambda$$

$$+\alpha_2^2\cdot\sum_{k=0}^{\infty}\sum_{i=1}^{2}\cdot\int_0^t\sum_{j=p6}^{4}\cdot\frac{d^{2n}}{ds^{2n}}\left[\frac{Y_3(s)e^{s(t-\lambda)}]}{\prod\limits_{\substack{q=j\\ j<4}}^{3}.(s-r_{1q+1})^2\prod\limits_{\substack{t=1\\ j>1\\ p6<2}}^{j-1}.(s-r_{1t})^2}\right]_{\substack{s=r_{1j}\\ j=p6\\ n=p6-1}}$$

$$\left[\begin{matrix}e^{n_1(\xi_{ki})y+\xi_{ki}\lambda}\dfrac{\sinh(y-1)n_2(\xi_{ki})}{\frac{d}{ds}\left[\sinh n_2\right]_{s=\xi_{ki}}}\cos M + e^{n_1(\xi_{ki})(y-1)+\xi_{ki}\lambda}\\ \dfrac{\sinh n_2(\xi_{ki})y}{\frac{d}{ds}\left[\sinh n_2\right]_{s=\xi_{ki}}}\cos My - 1\end{matrix}\right]d\lambda +$$

$$\alpha_2^2\cdot\sum_{k=0}^{\infty}\sum_{i=1}^{2}.\int_0^t\sum_{j=p6}^{4}\cdot\frac{d^{2n}}{ds^{2n}}\left[\frac{Y_4(s)e^{s(t-\lambda)}]}{\prod\limits_{\substack{q=j\\ j<4}}^{3}.(s-r_{1q+1})^2\prod\limits_{\substack{t=1\\ j>1\\ p6<2}}^{j-1}.(s-r_{1t})^2}\right]_{\substack{s=r_{1j}\\ j=p6\\ n=p6-1}}$$

$$\left[\sin My - e^{n_1(\xi_{ki})(y-1)+\xi_{ki}\lambda} \frac{\sinh n_2(\xi_{ki})y}{\frac{d}{ds}\left[\sinh n_2\right]_{s=\xi_{ki}}} \sin M\right] d\lambda \tag{23}$$

The wall shear stress:

$$(1+\alpha_1(\frac{\partial}{\partial t}+R\frac{\partial}{\partial y})\}\tau_{xy} = \upsilon\{1+\alpha_2(\frac{\partial}{\partial t}+R\frac{\partial}{\partial y})\}\frac{\partial u}{\partial y} \tag{24}$$

Now, it has the Newtonian part $\tau_{xy} = \upsilon\frac{\partial u}{\partial y}$,

And the non-Newtonian part $(\frac{\partial}{\partial t}+R\frac{\partial}{\partial y})(\alpha_1\tau_{xy}) = (\frac{\partial}{\partial t}+R\frac{\partial}{\partial y})(\upsilon\alpha_2\frac{\partial u}{\partial y})$

Here, we have evaluated the approximated value of wall shear stress $\tau = \upsilon(1+\frac{\alpha_2}{\alpha_1})\frac{\partial u}{\partial y}$

4. Results and Discussion

In the present section we shall furnish the results of axial velocity and wall shear stress for different fluid parameters. Since the problem generated here is somewhat different than others, so, no comparison has been made for the verification of the result. In the subsequent calculation we shall consider the following functions at the appropriate places.

$$p(t) = e^{-t}\cos(Ct)/\sqrt{C}, \quad U(t) = U\cos(\omega t), \quad V(t) = U\sin(\omega t)$$

Fig. 1 is the representation of axial velocity distribution for three relaxation constants α_1 (in figure A1=α_1). The other parameter values adopted in this calculation are $R = 0.5$, $M = 9.0$, $\alpha_2 = 0.01$, $R_1 = 1.0$, $\omega = 6.0$, $C = 4.0$, $t = 3.0$, $U = 1.0$. It is indicated that for decreasing values of λ_1, fluid particle assumes its greater speed. In the case, when $\alpha_1 \to 0$, i. e. the relaxation time is small, the energy utilized for the viscoelastic response is small and hence more energy can be stored in the fluid particle. Thus, the fluid particles enriched with energy accelerated the fluid motion. Hence, higher velocity is obvious and expected for small values of λ_1 which is also observed in the present investigation. However, for all values of λ_1, velocity profiles are rectilinear and with the fixed $t = 1.0$, upper wall motion changes with λ_1 which may be the effect of the presence of convective derivative in the constitutive relation. The rectilinearity may be attributed to be the fact of cross-flow Reynolds number associated with the problem.

The influence of magnetic field on the axial velocity depicted graphically in Fig. 2 ($R = 0.5$, $\alpha_2 = 0.01$, $R_1 = 1.0$, $\omega = 6.0$, $C = 4.0$, $t = 3.0$, $U = 1.0$). The impact on the flow behavior of the magnetic field is straightforward and as expected. The increasing values of the Hartmann numbers reduce the flow velocity and increase the backward motion to the respective order of M. At the time of computation, it has been found that M gives only the maximum forward fluid velocity and hence this is the critical value for M in the present model. Moreover, the velocity of the fluid can be determined in $0.005 < M < 20.0$, beyond that the problem as well as the method of solution should be made think over. A careful observation of the graph introduce some important ideas about the velocity distribution; such as, if $M = 4.0$ then the fluid particle velocity increases gradually but for $M = 0.05$ it is quit curvilinear and produce a backflow and the same is followed by increasing the number from $M = 4.0$.

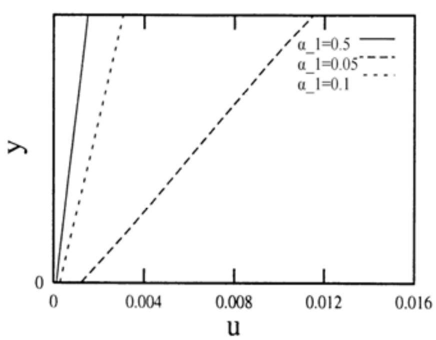

Fig.1 Axial velocity distribution for different

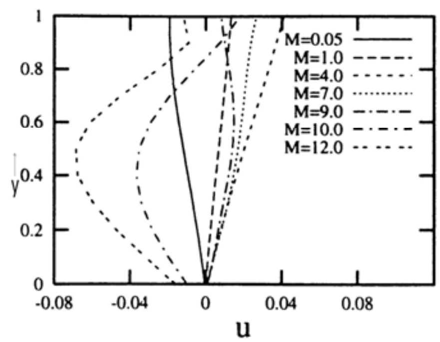

Fig. 2 Axial velocity distribution for various M

In spite of the impulsive term in the governing equation, the computational results (not shown in figure) are not much influenced by impulsive Reynolds number. It can just increase or decrease the magnitude of the axial velocity. The cross-flow Reynolds number is an important parameter subjected by the suction and injection in the present study. In order to validate the model a rigorous computation has been performed and it is observed that the values of R lies between 0.001 and 5.0, when all the other parameter values have a suitable choice. Among many observations we found that the fluid velocity is highly sensible on R as well as α_1. It is to be noted that with this model and method of solution we cannot predict exactly the flow characteristics for different values of α_1 because in the present model it has been considered as a small quantity. At the time instant $t = 0.0$, the shear stress distribution at the wall ($y = 1.0$) in three different suction velocities are given in Fig.3 where. $M = 4.0$, $\alpha_2 = 0.01$, $R_1 = 1.0$, $\omega = 6.0$, $C = 4.0$, $t = 1.0$, $U = 1.0$. In each case distributions are curvilinear. Investigation revealed that fluctuation and magnitude of $\tau_{y=1.0}$ decreases with the increasing values of R. It is noteworthy to see that the phase-angle along with the magnitude of τ differs significantly in the higher values of ω.

An important observation and result of the present investigation has been depicted in Fig. 4 ($M = 4.0$ $R = 0.5$, $\alpha_1 = 1.0$, $\alpha_2 = 0.01$, $R_1 = 1.0$, $M = 4.0$, $U = 1.0$.). This is the representation of wall shear stress ($\tau_{y=0.0}$) at two different instants of time. Together with the idea of Fig. 3, phase-angle of τ increase with the increasing values of ω, so, we may conclude that after certain time τ will linearly increase with ω following the manner of axial velocity.

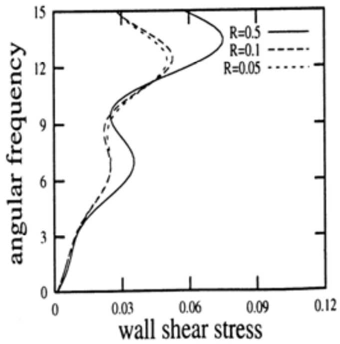

Fig. 3 Shear stress distribution at the wall y=1.0 for various R

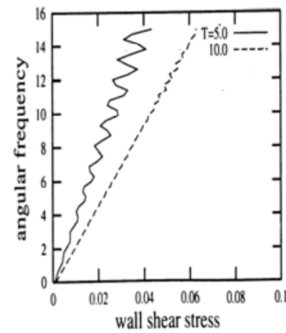

Fig.4 Distribution of wall shear stress at two given time instants(y=0.0)

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