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A Comparative Study of Wavelet Transform and Fourier Transform

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ABSTRACT

Wavelet transform is increasing being used in analysis and detection of signals. In this article we discuss the concept of wavelets, different forms of wavelets, and their Fourier transforms are shown. We have also discussed the comparative advantages of wavelet transforms over Fourier transforms in analyzing signals. In this study we try to present how discrete functions are represented in wavelet form, especially in the Haar wavelet representation.

Keyword: Wavelet, Wavelet transform, Haar Function, Haar Basis Function, Fourier transform, Time-frequency signal.

1. Introduction

Wavelet transform of a function is the improved version of Fourier transform. Wavelet analysis is an exciting new method for solving difficult problems in mathematics, physics and engineering, data compression, signal processing, image processing, pattern recognition, computer graphics and other medical image technology. The first known connection to modern wavelets dates back to Jean Baptiste Joseph Fourier in the nineteenth century. The next known link to wavelets came from Alfred Haar in the year 1909. After Haar's contribution to wavelets there was a gap of time in research in this field until Paul Levy's work and slight advances were made in the field of wavelets from the 1930's to the 1970's. The next major advancement came from Jean Morlet around the year 1975. Morlet had made quite an impact on the history of wavelets; however, he was not satisfied with his efforts. In 1981 Morlet in collaboration with Alex Grossman worked on an idea that Morlet discovered while experimenting on a basic calculator.

The Fourier transform is a powerful tool for processing signals that are composed of some combination of sine and cosine signals. Mallat (1999), Wells (1993) and Strang (1989) have shown that wavelets also allow filters to be constructed for stationary and non-stationary signals.

However, wavelets have been applied in many other areas including non-linear regression and compression. An offshoot of wavelet compression allows the amount of determinism in a time series to be estimated by Walnut (2001), Wojtaszczyk (1997). Charles (1991), Christensen (2004), Daubechies (1992), Addition, Paul S. (2002), Debnath (2002). Meyer (1993) extensively worked on wavelets. In this paper we have discussed about different wavelets, their properties and advantages. But for wavelet analysis, we can use approximating functions that are contained neatly in finite domains. Wavelets are well suited for approximating data with sharp discontinuities. We have also tried to represent discrete functions into Haar wavelets.

2. Materials and Methods

2.1 Wavelets: Wavelets are functions that are confined in finite domains and are used to represent data or a function. In an analogous way to Fourier analysis which analyzes the frequency content in a function using sines and cosines, wavelet analysis analyzes the scale of a function's content with special basis functions called wavelets. For details we refer to Debnath, L. (2002). Equivalent mathematical conditions for wavelet are:

$$(i)\ \int_{-\infty}^{\infty} |\psi(x)|^2\, dx < \infty, \quad (ii)\ \int_{-\infty}^{\infty} |\psi(x)|dx = 0, \quad (iii)\ \int_{-\infty}^{\infty} \frac{|\hat{\psi}(\omega)|^2}{|\omega|} d\omega < \infty$$

where $\hat{\psi}(\omega)$ is the Fourier Transform of $\psi(x)$, (iii) is called the admissibility condition.

2.2 Discrete Wavelet Transform: The foundations of the DWT go back to 1976 when Croiser, Esteban and Galand devised a technique to decompose discrete time signals. Our discrete wavelets are not time-discrete, only the translation and the scale step are discrete. For details we refer to Addition, P. S. (2002). It turns out that it is better to discretize it in a different way, first we fix two positive constants a_0 *and* b_0 and define

$$\psi_{j,k}(x) = a_0^{-j/2}\psi(a_0^{-j}x - kb_0) \qquad \text{where both } j \text{ and } k \in Z \tag{1}$$

the discrete wavelet transform of a given function $f(x)$ is defined by

$$W_\psi f(j,k) = \langle f, \psi_{j,k} \rangle = \int_{-\infty}^{\infty} f(x)\overline{\psi}_{j,k}(x)\,dx = a_0^{-\frac{j}{2}} \int_{-\infty}^{\infty} f(x)\overline{\psi}(a_0^{-j}x - kb_0)\,dx \tag{2}$$

where both f and ψ are continuous, $\overline{\psi}_{j,k}(x)$ is the complex conjugate of $\psi_{j,k}(x)$. For computational efficiency, $a_0 = 2$ and $b_0 = 1$ are commonly used so that results lead to a
binary dilation of 2^j and a dyadic translation of $k2^j$.

From (1) we get $\psi_{j,k}(x) = 2^{-\frac{j}{2}}\psi(2^{-j}x - k)$. Now eq. (2) can be written as

$$W_\psi f(j,k) = \langle f, \psi_{j,k} \rangle = 2^{-\frac{j}{2}} \int_{-\infty}^{\infty} f(x)\overline{\psi}(2^{-j}x - k)\,dx \tag{3}$$

where $j \; and \; k$ are integers that scale dilate the mother function $\psi(x)$ to generate wavelets. The scale index j indicates the wavelet's width and the location index k gives its position. The discrete wavelet transform of a given function $f(x)$ can be defined in another way which is given by

$$f(x) = \frac{1}{\sqrt{M}} \sum_{k=0}^{2^{j_0}-1} c_{j_0,k}\varphi_{j_0,k}(x) + \frac{1}{\sqrt{M}} \sum_{j=j_0}^{\infty} \sum_{k=0}^{2^j-1} d_{j,k}\psi_{j,k}(x) \qquad \text{for } j \geq j_0 \tag{4}$$

and $$c_{j_0,k} = W_\varphi(j_0,k) = \frac{1}{\sqrt{M}} \sum_{x=0}^{2^J-1} f(x)\varphi_{j_0,k}(x) \tag{5}$$

$$d_{j,k} = W_\psi(j,k) = \frac{1}{\sqrt{M}} \sum_{x=0}^{2^J-1} f(x)\psi_{j,k}(x) \tag{6}$$

Here $f(x),\ \varphi_{j_0,k}(x) \; and \; \psi_{j,k}(x)$ are functions of the discrete variable $x = 0,1,2,\ldots, M-1$ and we consider $j = 0,1,2,\ldots\ldots\ldots, J-1$ & $M = 2^J$.

2.3 Haar wavelet

A function defined on the real line $\Re$ as

$$\psi(t) = \begin{cases} 1 \ \ for \ t \in \left[0, \frac{1}{2}\right) \\ -1 \ \ for \ t \in \left[\frac{1}{2}, 1\right) \\ 0 \quad otherwise \end{cases}$$

is known as the Haar wavelet.

The Haar wavelet $\psi(t)$ is the simplest example of a wavelet. The Haar function $\psi(t)$ is a wavelet because it satisfies all the conditions of wavelet. Haar wavelet seems non smooth at $t = 0, \frac{1}{2}$ and discontinuous at $t = 1$ and it is very well localized in the time domain.

The Fourier transform of $\psi(t)$ is given by

$$\hat{\psi}(\omega) = i \exp\left(-\frac{i\omega}{2}\right) \cdot \frac{\sin^2\left(\frac{\omega}{4}\right)}{\frac{\omega}{4}}, \quad \mathrm{Re}\{\hat{\psi}(\omega)\} = \sin\left(\frac{\omega}{2}\right) \frac{\sin^2\left(\frac{\omega}{4}\right)}{\frac{\omega}{4}}.$$

The graphs of Haar wavelet $\psi(t)$ and its Fourier transform $\hat{\psi}(\omega)$ are shown in Fig.1(a) and Fig.1(b)

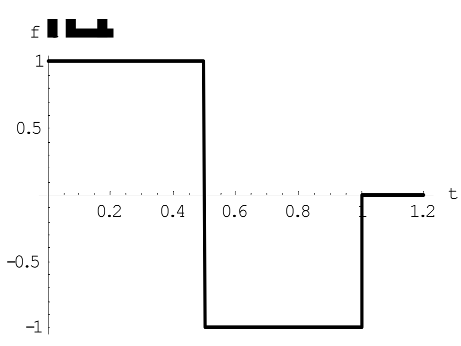

Fig. 1(a). Haar wavelet.

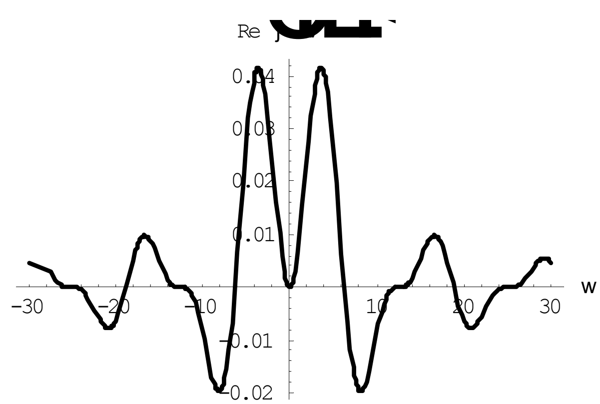

Fig. 1(b). Fourier transform of Haar wavelet.

2.4 Mexican Hat Wavelet

The wavelet which is defined by the second derivative of a Gaussian probability density function

$$\psi(t) = (1-t^2)\exp\left(-\frac{t^2}{2}\right) = -\frac{d^2}{dt^2}\exp\left(-\frac{t^2}{2}\right)$$

is known as Mexican Hat Wavelet.

The Fourier transform of $\psi(t)$ is

$$\hat{\psi}(\omega) = \sqrt{2\pi}\omega^2 \exp\left(-\frac{\omega^2}{2}\right)$$

The graphs of Mexican Hat wavelet $\psi(t)$ and its Fourier transform $\hat{\psi}(\omega)$ are shown in Figs. 2(a) and 2(b). This wavelet has excellent localization in time and frequency domains and clearly satisfies the admissibility condition.

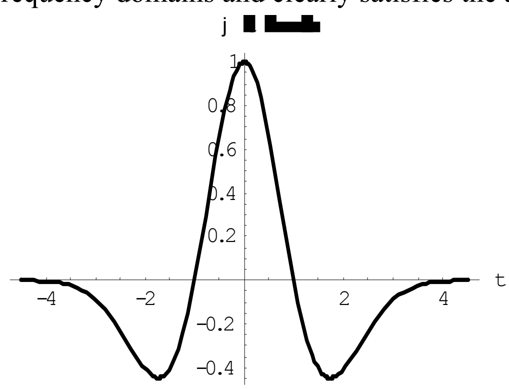

Fig. 2(a). The Mexican Hat wavelet.

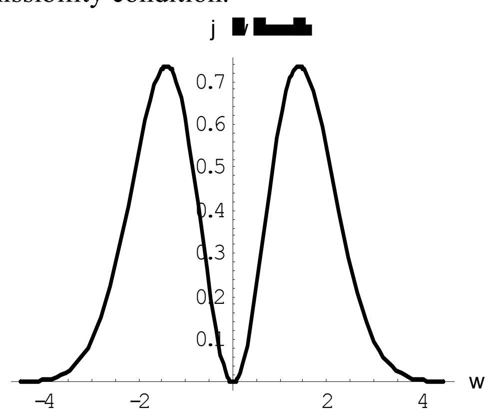

Fig. 2(b). Fourier transform of Mexican Hat wavelet.

Haar Wavelet Representation of functions:

2.5 Haar Scaling Function: The Haar scaling function can be defined as

$$\varphi(x) = \chi_{[0,1)}(x) = \begin{cases} 1 & \text{if } 0 \le x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

2.6 Haar Wavelet Function: Haar wavelet function $\psi(x)$ in terms of scaling function can be written as

$$\psi(x) = \chi_{\left[0,\frac{1}{2}\right)}(x) - \chi_{\left[\frac{1}{2},1\right)}(x) = \begin{cases} 1, & \text{if } 0 \le x \le \frac{1}{2} \\ -1, & \text{if } \frac{1}{2} \le x < 1 \\ 0, & \text{otherwise.} \end{cases}$$

2.7 Haar Basis Function

The Haar basis function $h_x(z)$ is defined over the continuous and closed interval $z \in [0,1]$ for x=0, 1, 2… M-1, where $M = 2^J$. These functions are contained in the $M \times M$ transformation matrix H. In order to generate H, we define integer K such that $K = 2^j + k - 1$ where $0 \le j \le J-1, k = 0$ or 1 for j=0 and $1 \le k \le 2^j$ for $j \ne 0$.

Then the Haar basis functions are $h_0(z) = h_{00}(z) = \dfrac{1}{\sqrt{M}}, z \in [0,1]$

(7)

$$h_x(z) = h_{jk}(z) = \frac{1}{\sqrt{M}} \begin{cases} 2^{\frac{j}{2}} & ,(k-1)/2^j \le z < (k-.5)/2^j \\ -2^{\frac{j}{2}} & ,(k-.5)/2^j \le z < k/2^j \\ 0 & ,otherwise, z \in [0,1] \end{cases} \tag{8}$$

The ith row of a $M \times M$ Haar transformation matrix contains the elements of

$h_i(z) \quad for \quad z = 0/M, 1/M, 2/M, ..., \dfrac{M-1}{M}$.

Given that

$K = 2^j + k - 1$

(9)

$0 \le j \le J - 1$

(10)

$0 \le j \le J-1, k = 0\, or\, 1\; for\; j = 0\, and\;\; 1 \le k \le 2^j\; for\; j \ne 0$

2.7.1Example: Consider a signal {1, 4,-3, 0, 2,-1, 5, 3} which can be represent by the discrete function $f(0)=1, f(1)=4, f(2)=-3, f(3)=0, f(4)=2, f(5)=-1, f(6)=5, f(7)=3$.

We have $M = 2^J = 8 = 2^3 \therefore J = 3$. From (4.5.4) we get $0 \le j \le 2$ .when j=0, then k=0 or 1 [From (2.7.5)] and $K = 2^0 + 0 - 1 = 0$ or $K = 2^0 + 1 - 1 = 1$ [From equation (2.7.3)] when j=1, then $1 \le k \le 2$ [From eq. (2.7.5)]. Thus for j=1 and k=1 then K=2, & for j=1 and k=2 then K=3. Again for j=2 then $1 \le k \le 4$. So for j=2 and k=1 then $K = 4$ & for j=2 and k=2 then K=5. Also for j=2 and k=3 then $K = 6$ & for j=2 and k=4 then K=7.

K	j	k
0	0	0
1	0	1

2	1	1
3	1	2
4	2	1
5	2	2
6	2	3
7	2	4

Table: the values for K, j and k.

Now we get, $h_0(0) = h_{00}(0) = \frac{1}{\sqrt{M}} = \frac{1}{\sqrt{8}}$.

For $h_1(z) = h_{01}(z) = \frac{1}{\sqrt{8}} \begin{cases} 1 \ , & 0 \le z < 0.5 \\ -1 \ , & 0.5 \le z < 1 \\ 0 \ , & otherwise \ , \ z \in [0,1] \end{cases}$

$h_1(\frac{0}{8}) = h_{01}(0) = \frac{1}{\sqrt{8}}, h_1(\frac{1}{8}) = h_{01}(.125) = \frac{1}{\sqrt{8}}, h_1(\frac{2}{8}) = h_{01}(.25) = \frac{1}{\sqrt{8}},$

$h_1(\frac{3}{8}) = h_{01}(.375) = \frac{1}{\sqrt{8}},$

$h_1(\frac{4}{8}) = h_{01}(.5) = \frac{1}{\sqrt{8}}, h_1(\frac{5}{8}) = h_{01}(.625) = -\frac{1}{\sqrt{8}}, h_1(\frac{6}{8}) = h_{01}(.75) = -\frac{1}{\sqrt{8}},$

$h_1(\frac{7}{8}) = h_{01}(.875) = -\frac{1}{\sqrt{8}}$

For $h_2(z) = h_{11}(z) = \frac{1}{\sqrt{8}} \begin{cases} \sqrt{2} \ , & 0 \le z < 0.25 \\ -\sqrt{2} \ , & 0.25 \le z < 0.5 \\ 0 \ , & otherwise \ , z \in [0,1] \end{cases}$

$h_2(\frac{0}{8}) = h_{11}(0) = \frac{\sqrt{2}}{\sqrt{8}},$ $h_2(\frac{1}{8}) = h_{11}(.125) = \frac{\sqrt{2}}{\sqrt{8}},$ $h_2(\frac{2}{8}) = h_{11}(.25) = -\frac{\sqrt{2}}{\sqrt{8}}$

$h_2(\frac{3}{8}) = h_{11}(.375) = -\frac{\sqrt{2}}{\sqrt{8}},$

$h_2(\frac{4}{8}) = h_{11}(.5) = 0,$ $h_2(\frac{5}{8}) = h_{11}(.625) = 0,$ $h_2(\frac{6}{8}) = h_{11}(.75) = 0,$

$h_2(\frac{7}{8}) = h_{11}(.875) = 0$

For $h_3(z)=h_{12}(z)=\frac{1}{\sqrt{8}}\begin{cases}\sqrt{2} & , \quad 0.5\le z<0.75\\ -\sqrt{2} & , \quad 0.75\le z<1\\ 0 & , \quad otherwise \quad ,z\in[0,1]\end{cases}$

$h_3(\frac{0}{8})=h_{12}(0)=0$, $h_3(\frac{1}{8})=h_{12}(.125)=0$, $h_3(\frac{2}{8})=h_{12}(.25)=0$,

$h_3(\frac{3}{8})=h_{12}(.375)=0$, $h_3(\frac{4}{8})=h_{12}(.5)=\frac{\sqrt{2}}{\sqrt{8}}$, $h_3(\frac{5}{8})=h_{12}(.625)=\frac{\sqrt{2}}{\sqrt{8}}$,

$h_3(\frac{6}{8})=h_{12}(.75)=-\frac{\sqrt{2}}{\sqrt{8}}$, $h_3(\frac{7}{8})=h_{12}(.875)=-\frac{\sqrt{2}}{\sqrt{8}}$

For $h_4(z)=h_{21}(z)=\frac{1}{\sqrt{8}}\begin{cases}2 & , \quad 0\le z<0.125\\ -2 & , \quad 0.125\le z<.25\\ 0 & , \quad otherwise \quad ,z\in[0,1]\end{cases}$

$h_4(\frac{0}{8})=h_{21}(0)=\frac{2}{\sqrt{8}}$, $h_4(\frac{1}{8})=h_{21}(.125)=-\frac{2}{\sqrt{8}}$, $h_4(\frac{2}{8})=h_{21}(.25)=0$,

$h_4(\frac{3}{8})=h_{21}(.375)=0$, $h_4(\frac{4}{8})=h_{21}(.5)=0$, $h_4(\frac{5}{8})=h_{21}(.625)=0$

$h_4(\frac{6}{8})=h_{21}(.75)=0$, $h_4(\frac{7}{8})=h_{21}(.875)=0$

For $h_5(z)=h_{22}(z)=\frac{1}{\sqrt{8}}\begin{cases}2 & , \quad 0.25\le z<.375\\ -2 & , \quad 0.375\le z<.5\\ 0 & , \quad otherwise, z\in[0,1]\end{cases}$

$h_5(\frac{0}{8})=h_{22}(0)=0$, $h_5(\frac{1}{8})=h_{22}(.125)=0$, $h_5(\frac{2}{8})=h_{22}(.25)=\frac{2}{\sqrt{8}}$, $h_5(\frac{3}{8})=h_{22}(.375)=-\frac{2}{\sqrt{8}}$,

$h_5(\frac{4}{8})=h_{22}(.5)=0$, $h_5(\frac{5}{8})=h_{22}(.625)=0$, $h_5(\frac{6}{8})=h_{22}(.75)=0$,

$h_5(\frac{7}{8})=h_{22}(.875)=0$

For $h_6(z)=h_{23}(z)=\frac{1}{\sqrt{8}}\begin{cases}2 & , \quad 0.5\le z<.625\\ -2 & , \quad 0.625\le z<.75\\ 0 & , \quad otherwise \quad ,z\in[0,1]\end{cases}$

$h_6(\frac{0}{8}) = h_{23}(0) = 0$, $h_6(\frac{1}{8}) = h_{23}(.125) = 0$, $h_6(\frac{2}{8}) = h_{23}(.25) = 0$,
$h_6(\frac{3}{8}) = h_{23}(.375) = 0$
$h_6(\frac{4}{8}) = h_{23}(.5) = \frac{2}{\sqrt{8}}$, $h_6(\frac{5}{8}) = h_{23}(.625) = -\frac{2}{\sqrt{8}}$, $h_6(\frac{6}{8}) = h_{23}(.75) = 0$,
$h_6(\frac{7}{8}) = h_{23}(.875) = 0$

For $h_7(z) = h_{24}(z) = \frac{1}{\sqrt{8}} \begin{cases} 2 & , \quad 0.75 \le z < 0.875 \\ -2 & , \quad 0.875 \le z < 1 \\ 0 & , \quad otherwise,\ z \in [0,1] \end{cases}$

$h_7(\frac{0}{8}) = h_{24}(0) = 0$, $h_7(\frac{1}{8}) = h_{24}(.125) = 0$, $h_7(\frac{2}{8}) = h_{24}(.25) = 0$,
$h_7(\frac{3}{8}) = h_{24}(.375) = 0$, $h_7(\frac{4}{8}) = h_{24}(.5) = 0$, $h_7(\frac{5}{8}) = h_{24}(.625) = 0$,
$h_7(\frac{6}{8}) = h_{24}(.75) = \frac{2}{\sqrt{8}}$, $h_7(\frac{7}{8}) = h_{24}(.875) = -\frac{2}{\sqrt{8}}$

Now, we construct the 8×8 transformation matrix, $H_{8,}$ is

$$H_8 = \frac{1}{\sqrt{8}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ \sqrt{2} & \sqrt{2} & -\sqrt{2} & -\sqrt{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{2} & \sqrt{2} & -\sqrt{2} & -\sqrt{2} \\ 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix} \quad (12)$$

We have the discrete wavelet transform of a given function $f(x)$ is given by

$$f(x) = \frac{1}{\sqrt{M}} \sum_{k=0}^{2^{j_0}-1} c_{j_0,k} \varphi_{j_0,k}(x) + \frac{1}{\sqrt{M}} \sum_{j=j_0}^{\infty} \sum_{k=0}^{2^{j}-1} d_{j,k} \psi_{j,k}(x) \quad (13)$$

$$c_{j_0,k} = W_\varphi(j_0,k) = \frac{1}{\sqrt{M}} \sum_{x=0}^{2^J-1} f(x) \varphi_{j_0,k}(x)$$

(14)

$$d_{j,k} = W_\psi(j,k) = \frac{1}{\sqrt{M}} \sum_{x=0}^{2^J-1} f(x) \psi_{j,k}(x) \quad (15)$$

Here $f(x), \varphi_{j_0,k}(x)\, and\, \psi_{j,k}(x)$ are functions of the discrete variable $x = 0,1,2,.....,M-1$. We let $j_0 = 0\ and\ M = 2^J$ which are performed over $x = 0,1,2,.....,M-1,\ j = 0,1,2,.....,J-1$.

The given discrete functions are $f(0)=1,\ \ f(1)=4,\ \ f(2)=-3,\ \ f(3)=0,$ $f(4)=2,\ \ f(5)=-1,\ \ f(6)=5,\ \ f(7)=3$ .Here $M=2^J=8=2^3 \therefore J=3$ and with $j_0 = 0$, the summation are performed over $x = 0,1,2,3,4,5,6,7; j = 0,1,2$. We will use the Haar scaling and wavelet functions and assumes that eight samples of $f(x)$ are distributed over the support of the basis functions. So, from the first row of the matrix H_4.

$$\varphi_{0,0}(0) = \varphi_{0,0}(1) = \varphi_{0,0}(2) = \varphi_{0,0}(3) = \varphi_{0,0}(4) = \varphi_{0,0}(5) = \varphi_{0,0}(6) = \varphi_{0,0}(7) = 1$$

Substituting the eight samples into eq. (15), we find

$$c_{0,0} = W_\varphi(0,0) = \frac{1}{\sqrt{8}}\sum_{x=0}^{7} f(x)\varphi_{0,0}(x) = \frac{11}{\sqrt{8}}.$$

Similarly we get $\psi_{j,k}(x)$ corresponding to rows of 2, 3, 4, 5, 6, 7 and 8 of H_8.

Therefore,

$$\psi_{0,0}(0)=1, \psi_{0,0}(1)=1, \psi_{0,0}(2)=1, \psi_{0,0}(3)=1, \psi_{0,0}(4)=-1, \psi_{0,0}(5)=-1, \psi_{0,0}(6)=-1, \psi_{0,0}(7)=-1$$

$$\psi_{1,0}(0)=\sqrt{2}, \psi_{1,0}(1)=\sqrt{2}, \psi_{1,0}(2)=-\sqrt{2}, \psi_{1,0}(3)=-\sqrt{2}, \psi_{1,0}(4)=0, \psi_{1,0}(5)=0, \psi_{1,0}(6)=0, \psi_{1,0}(7)=0$$

$$\psi_{1,1}(0)=0, \psi_{1,1}(1)=0, \psi_{1,1}(2)=0, \psi_{1,1}(3)=0, \psi_{1,1}(4)=\sqrt{2}, \psi_{1,1}(5)=\sqrt{2}, \psi_{1,1}(6)=-\sqrt{2}, \psi_{1,1}(7)=-\sqrt{2}$$

$$\psi_{2,1}(0)=2, \psi_{2,1}(1)=-2, \psi_{2,1}(2)=0, \psi_{2,1}(3)=0, \psi_{2,1}(4)=0, \psi_{2,1}(5)=0, \psi_{2,1}(6)=0, \psi_{2,1}(7)=0$$

$$\psi_{2,2}(0)=0, \psi_{2,2}(1)=0, \psi_{2,2}(2)=2, \psi_{2,2}(3)=-2, \psi_{2,2}(4)=0, \psi_{2,2}(5)=0, \psi_{2,2}(6)=0, \psi_{2,2}(7)=0$$

$$\psi_{2,3}(0)=0, \psi_{2,3}(1)=0, \psi_{2,3}(2)=0, \psi_{2,3}(3)=0, \psi_{2,3}(4)=2, \psi_{2,3}(5)=-2, \psi_{2,3}(6)=0, \psi_{2,3}(7)=0$$

$$\psi_{2,4}(0)=0, \psi_{2,4}(1)=0, \psi_{2,4}(2)=0, \psi_{2,4}(3)=0, \psi_{2,4}(4)=0, \psi_{2,4}(5)=0, \psi_{2,4}(6)=2, \psi_{2,4}(7)=-2$$

Therefore from (15) we get

$$d_{0,0} = W_\psi(0,0) = \frac{1}{\sqrt{8}}\sum_{x=0}^{7} f(x)\psi_{0,0}(x) = -\frac{7}{\sqrt{8}},$$

$$d_{1,0} = W_\psi(1,0) = \frac{1}{\sqrt{8}}\sum_{x=0}^{7} f(x)\psi_{1,0}(x) = \frac{8\sqrt{2}}{\sqrt{8}},$$

$$d_{1,1} = W_\psi(1,1) = \frac{1}{\sqrt{8}}\sum_{x=0}^{7} f(x)\psi_{1,1}(x) = -\frac{7\sqrt{2}}{\sqrt{8}},$$

$$d_{2,1} = W_\psi(2,1) = \frac{1}{\sqrt{8}}\sum_{x=0}^{7} f(x)\psi_{2,1}(x) = -\frac{6}{\sqrt{8}},$$

$$d_{2,2} = W_\psi(2,2) = \frac{1}{\sqrt{8}} \sum_{x=0}^{7} f(x)\psi_{2,2}(x) = -\frac{6}{\sqrt{8}},$$

$$d_{2,3} = W_\psi(2,3) = \frac{1}{\sqrt{8}} \sum_{x=0}^{7} f(x)\psi_{2,3}(x) = \frac{6}{\sqrt{8}},$$

$$d_{2,4} = W_\psi(2,4) = \frac{1}{\sqrt{8}} \sum_{x=0}^{7} f(x)\psi_{2,4}(x) = \frac{4}{\sqrt{8}}$$

Thus, the discrete wavelet transform of given eight sample functions relative to the Haar scaling wavelets are $\left\{ \frac{11}{\sqrt{8}}, -\frac{7}{\sqrt{8}}, \frac{8\sqrt{2}}{\sqrt{8}}, -\frac{7\sqrt{2}}{\sqrt{8}}, -\frac{6}{\sqrt{8}}, -\frac{6}{\sqrt{8}}, \frac{6}{\sqrt{8}}, \frac{4}{\sqrt{8}} \right\}$.

Using (14) we get
$f(0) = 1$, $f(1) = 4$, $f(2) = -3$, $f(3) = 0$, $f(4) = 2$, $f(5) = -1$, $f(6) = 5$, $f(7) = 3$

Using Mathematica we have drawn the graph of wavelet transform of the given discrete signal

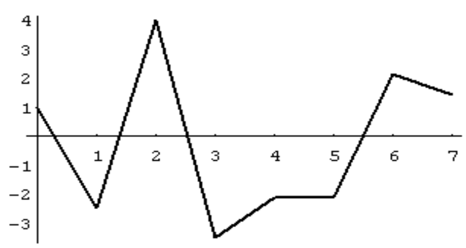

Fig. 3: Graph of Wavelet Transform of the given Function

3. Result and Discussion

From the figures of the wavelet transform we observe that wavelets are well localized in both time and frequency domain whereas the standard curve is only localized in frequency domain. In Fourier analysis signal properties do not change over time and it is called a stationary signal. But most interesting signals contain numerous non- stationary or transitory characteristics like drift, trends, abrupt changes and beginnings and ends of event. These characteristics are often the most important part of the signal. The classical Fourier analysis is not suited for detecting them but the wavelet analysis is suited for detecting them.

4. Conclusion

In our study we discussed about wavelets, wavelet transforms, represent of a function in terms of Haar wavelet. We tried to comparative discussion of

Fourier transform and wavelet transform graphically with mentioning the drawback of Fourier transform. The advantages of wavelet transform are also focused. We have also tried to represent here discrete functions by Haar discrete wavelet. From our above discussion it is clear that wavelet transform is much more efficient than that of Fourier transform. From our above discussion it is clear that the experimental results show that the represent discrete functions in terms of Haar wavelet because wavelets are well-localized in both time and frequency domain, by using wavelet transform we can scale and translate the function and approximate the function by using only a few coefficients.

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