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Multi-objective Stochastic Transportation Problem Involving Log-normal

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ABSTRACT

This paper presents an application to the multi-objective stochastic transportation problem in fuzzy environment. In this paper, we focus on our attention to multi-objective stochastic transportation problem involving an inequality type of constraints in which all parameters (supply and demand) are log-normal random variable and the objectives are non-commensurable and conflicting in nature. At first we convert the proposed multi-objective linear stochastic transportation problem into an equivalent deterministic problem under chance constrained programming technique framework. Then fuzzy programming technique is applied to solve this problem and we obtained the compromise solution. Lastly a numerical example is provided for the sake of illustrate the methodology.

Keywords: Multi-objective Programming, Stochastic Programming, Log-normal Random Variable, Transportation Problem, Fuzzy Programming.

1. Introduction

In the typical problem, a product is transported from m sources to n destinations and their supply $(a_1, a_2, a_3, \cdots, a_m)$ and demand $(b_1, b_2, b_3, \cdots, b_n)$ are respectively. The coefficients C_{ij}^{k} of the objective functions could represent the transportation cost, delivery time, number of goods transported, unfulfilled supply and demand, and others, are provided with transporting a unit of product from sources i to destination j. The mathematical model of the multi-objective

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transportation problem is presented as follows :

$$\min : z_k = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^{k} x_{ij}, \quad k = 1,2,\cdots,K \tag{1}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, \qquad i = 1,2,\cdots,m \tag{2}$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \qquad j = 1,2,\cdots,n \tag{3}$$

$$x_{ij} \geq 0, \ \forall\, i \ and \ j. \tag{4}$$

Here z_k represents the minimum values of k-th objective function and it is assumed that $a_i > 0,\ b_j > 0,$ and $C_{ij}^{\ k} > 0$ and $\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$ (for unbalanced transportation problem).

Dantzig [3] was first to formulate the mathematical model of probabilistic programming in which he has suggested a two stage programming technique for solving a stochastic programming problem by converting the said problem into deterministic programming. Charnes et al. [2] first introduced the chance constrained programming model known as probabilistic programming with suggestion of three models with different types of objective functions, such as the E-model, V-model and P-model. The E-model minimizes the expected value of objective functions, the V-model minimizes the generalized mean square of the objective functions and the P-model maximizes the probability of aspiration levels of the objective functions. Goicoechea et al. [5] described the deterministic equivalents for some probabilistic programming involving normal and other distributions. Sahoo et al. [9] discussed the probabilistic linear programming problem with random variables and they [10] are developed the computation of probabilistic linear programming problem involving normal and log-normal to some random variables with a joint constraints and obtaining its solution by fuzzy programming technique. Kambo [7] discussed the chance constrained and two stage programming methods for solving a stochastic linear programming problem. Bit et al. [1] in 1992 have been presented the multi-objective transportation problem using the fuzzy programming technique on probabilistic constraints.

The kinds of vagueness can be treated as basic approaches of fuzzy programming called flexible programming which is the most successful application of the fuzzy set theory initiated by Zadeh [12]. The fuzzy programming technique to multi-objective linear programming problems was first introduced by Zimmermann [13]. Diaz [4] represented an alternative procedure to generate all non-dominated solution to the multi-objective transportation problem. This approach depends upon the best compromise solution among the set of coefficient solutions. Liu et al. [8] developed on chance constrained programming involving fuzzy parameters. In particular, Hulsurkar et al. [6] applied fuzzy programming to multi-objective stochastic programming problems. We assumed that $C_{ij}^{k}\ (k=1,2,\cdots,K)$ are deterministic constants and $a_i\ (i=1,2,\cdots,m)$ and $b_j\ (j=1,2,\cdots,n)$ may be random variables in multi-objective stochastic transportation problem.

Most of the researchers have been followed by the conventional fuzzy approach for the solution to multi-objective stochastic transportation with two constraints having only one deterministic and other probabilistic. But they have not reviewed for studied the multi-objective stochastic transportation problem with two probabilistic constraints involving log-normal random variables.

2. Mathematical Model

In this paper, we have consider the mathematical model for multi-objective stochastic transportation problem involving log-normal random variables as follows:

Model - 1:

$$\min : z_k = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^{k} x_{ij}, \quad k=1,2,\cdots,K \tag{5}$$

subject to

$$\Pr\left(\sum_{j=1}^{n} x_{ij} \le a_i\right) \ge 1-\alpha_i, \qquad i=1,2,\cdots,m \tag{6}$$

$$\Pr\left(\sum_{i=1}^{m} x_{ij} \ge b_j\right) \ge 1-\beta_j, \qquad j=1,2,\cdots,n \tag{7}$$

$$x_{ij} \ge 0, \ \forall \ i \ and \ \ j.$$

where $0<\alpha_i<1, \forall\ i$ and $0<\beta_j<1, \forall\ j$.

We assumed that $a_i\ (i=1,2,\cdots,m)$ and $b_j\ (j=1,2,\cdots,n)$ are specified with log-normal random variables.

Now the following cases are to be considered

1. Only $a_i, (i=1,2,\cdots,m)$ are assumed as log-normal random variables.
2. Only $b_j, (j=1,2,\cdots,n)$ are assumed as log-normal random variables.
3. Both $a_i, (i=1,2,\cdots,m)$ and $b_j, (j=1,2,\cdots,n)$ are assumed as log-normal random variables.

2.1 Only $a_i, i = 1,2,\cdots,m$ are assumed as log-normal random variables:

It is assumed that a_i, $i = 1,2,\cdots,m$ are independent log-normal random variables with mean = $\mu_{a_i} = E(\ln a_i)$ and variance = $Var(\ln a_i) = \sigma^2_{a_i}$, which are known to us. We know that

$$mean\ of\ a_i = E(a_i) = \exp\left(\mu_{a_i} + \frac{\sigma^2_{a_i}}{2}\right),\quad i = 1,2,\cdots,m \tag{8}$$

$$variance\ of\ a_i = Var(a_i) = \exp\left(2\mu_{a_i} + \sigma^2_{a_i}\right)\exp(\sigma^2_{a_i} - 1),\quad i = 1,2,\cdots,m \tag{9}$$

The probability density function of i-th random variable a_i, $i = 1,2,\cdots,m$ is

$$f(a_i) = \frac{1}{\sqrt{(2\pi)}\sigma_i a_i}\exp\left[-\frac{1}{2}\left(\frac{\ln a_i - \mu_i}{\sigma_i}\right)^2\right],\quad 0 < a_i < \infty, \sigma_i > 0. \tag{10}$$

As $a_i, (i = 1,2,\cdots,m)$ is a log-normal random variable, so the equation (6) can be represented as follows:

$$\Pr\left(\ln\sum_{j=1}^{n} x_{ij} \le \ln a_i\right) \ge 1 - \alpha_i,\quad i = 1,2,\cdots,m \tag{11}$$

The above constraints can be expressed as:

$$\Pr\left[\frac{\ln(\sum_{j=1}^{n} x_{ij}) - E(\ln a_i)}{\sqrt{Var(\ln a_i)}} \le \frac{\ln a_i - E(\ln a_i)}{\sqrt{Var(\ln a_i)}}\right] \ge 1 - \alpha_i,\quad i = 1,2,\cdots,m \tag{12}$$

On rearranging, we get

$$\Pr\left[\frac{\ln\sum_{j=1}^{n} x_{ij} - E(\ln a_i)}{\sqrt{Var(\ln a_i)}} \ge \frac{\ln a_i - E(\ln a_i)}{\sqrt{Var(\ln a_i)}}\right] \le \alpha_i,\quad i = 1,2,\cdots,m \tag{13}$$

where $\frac{\ln a_i - E(\ln a_i)}{\sqrt{Var(\ln a_i)}}$ is a standard normal random variable with zero mean and unit variance and $\sigma_{a_i} = \sqrt{Var(ln(a_i))}$. Here $\Phi(.)$ represents the cumulative density function of the standard normal random variable and if K_{α_i} denotes the value of the standard normal variable then we have $\alpha_i = \Phi(-K_{\alpha_i})$.

Then the constraint (11) can be stated as:

$$\Phi\left(\frac{\ln\sum_{j=1}^{n}x_{ij} - \mu_{a_i}}{\sqrt{Var(\ln a_i)}}\right) \le \Phi(-K_{\alpha_i}),\ \ i = 1,2,\cdots,m \tag{14}$$

This inequality will be satisfied only if

$$\frac{\ln\sum_{j=1}^{n}x_{ij} - \mu_{a_i}}{\sqrt{Var(\ln a_i)}} \le -K_{\alpha_i},\ \ i = 1,2,\cdots,m \tag{15}$$

$$\Rightarrow \ln\sum_{j=1}^{n}x_{ij} \le \mu_{a_i} - K_{\alpha_i}\sqrt{Var(\ln a_i)},\ \ i = 1,2,\cdots,m \tag{16}$$

Finally, the probabilistic constraint (6) can be transformed into deterministic constraints as follows:

$$\sum_{j=1}^{n}x_{ij} \le \exp[\mu_{a_i} - K_{\alpha_i}\sigma_{a_i}],\ \ i = 1,2,\cdots,m \tag{17}$$

Therefore, for the case 1 : we have obtained the multi-objective deterministic transportation problem denoted by Model 2 instead of multi-objective probabilistic transportation problem
(Model 1) as follows:

Model -2:

$$\min : z_k = \sum_{i=1}^{m}\sum_{j=1}^{n}C_{ij}^{k}x_{ij},\quad k = 1,2,\cdots,K` \tag{18}$$

subject to

$$\sum_{j=1}^{n}x_{ij} \le \exp[\mu_{a_i} - K_{\alpha_i}\sigma_{a_i}],\ i = 1,2,\cdots,m \tag{19}$$

$$\sum_{i=1}^{m}x_{ij} \ge b_j,\ \ j = 1,2,\cdots,n \tag{20}$$

$$x_{ij} \ge 0,\ \forall\ i\ \ and\ \ j.$$

2.2 Only $b_j, (j = 1,2,\cdots,n)$ are assumed as log-normal random variables:

Assume that $b_j, j = 1,2,\cdots,n$ is independent log-normal random variable with mean = $E(\ln b_j) = \mu_{b_j}$, variance = $Var(\ln b_j) = \sigma^2_{b_j}$, which are known to us. We know that

$$\textit{mean of } b_j = E(b_j) = \exp\left(\mu_{b_j} + \frac{\sigma^2_{b_j}}{2}\right), \quad j = 1,2,\cdots,n \tag{21}$$

$$\textit{variance of } b_j = Var(b_j) = \exp\left(2\mu_{b_j} + \sigma^2_{b_j}\right)\exp(\sigma^2_{b_j} - 1), \quad j = 1,2,\cdots,n \tag{22}$$

As $b_j, (j = 1,2,\cdots,n)$ is a log-normal random variable, so the equation (7) can be represented as follows:

$$\Pr[\ln \sum_{i=1}^{m} x_{ij} \geq \ln b_j] \geq 1 - \beta_j, \; j = 1,2,\cdots,n \tag{23}$$

The above constraints can be expressed as:

$$\Pr\left[\frac{\ln(\sum_{i=1}^{m} x_{ij}) - \mu_{b_j}}{\sqrt{Var(\ln b_j)}} \geq \frac{\ln b_j - \mu_{b_j}}{\sqrt{Var(\ln b_j)}}\right] \geq 1 - \beta_j, \quad j = 1,2,\cdots,n \tag{24}$$

But, $\dfrac{\ln b_j - E(\ln b_j)}{\sqrt{Var(\ln b_j)}}$ is a standard normal random variable with zero mean and unit variance. Here $\Phi(.)$ represents the cumulative density function of the standard normal random variable and if K_{β_j} denotes the value of the standard normal variable then we have $\Phi\left(K_{\beta_j}\right) = 1 - \beta_j$

Therefore, the constraint (23) can be stated as:

$$\Phi\left(\frac{\ln \sum_{i=1}^{m} x_{ij} - \mu_{b_j}}{\sqrt{Var(\ln b_j)}}\right) \geq \Phi(K_{\beta_j}), \quad j = 1,2,\cdots,n \tag{25}$$

The inequality will be satisfied only if

$$\frac{\ln \sum_{i=1}^{m} x_{ij} - \mu_{b_j}}{\sqrt{Var(\ln b_j)}} \geq K_{\beta_j}, \quad j = 1,2,\cdots,n \tag{26}$$

where $\sigma_{b_j} = \sqrt{Var(\ln b_j)}$

$$\Rightarrow \ln \sum_{i=1}^{m} x_{ij} \geq \mu_{b_j} + K_{\beta_j}\sigma_{b_j}, \quad j = 1,2,\cdots,n \qquad (27)$$

Finally, the probabilistic constraints (23) can be transformed into deterministic constraints as follows :

$$\sum_{i=1}^{m} x_{ij} \geq \exp[\mu_{b_j} + K_{\beta_j}\sigma_{b_j}], \quad j = 1,2,\cdots,n \qquad (28)$$

Therefore, for the case 2 : we have obtained the multi-objective deterministic transportation problem denoted by Model -3 instead of multi-objective probabilistic transportation problem (Model 1)

Model -3:

$$\min : z_k = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^{k} x_{ij}, \quad k = 1,2,\cdots,K \qquad (29)$$

subject to

$$\sum_{j=1}^{n} x_{ij} \leq a_i, \quad i = 1,2,\cdots,m \qquad (30)$$

$$\sum_{i=1}^{m} x_{ij} \geq \exp[\mu_{b_j} + K_{\beta_j}\sigma_{b_j}], \quad j = 1,2,\cdots,n \qquad (31)$$

$$x_{ij} \geq 0, \ \forall \ i \ \ and \ \ j.$$

2.3 Both $a_i, (i = 1,2,\cdots,m)$ and $b_j, (j = 1,2,\cdots,n)$ are assumed as log-normal random variables:

The mean and variance of a_i and b_j are known and previously defined. In this case, the equivalent deterministic model for the chance constrained programming technique of the multi-objective stochastic transportation problem can be represented as:

Model -4:

$$\min : z_k = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^{k} x_{ij}, \quad k = 1,2,\cdots,K \qquad (32)$$

subject to

$$\sum_{j=1}^{n} x_{ij} \leq \exp[\mu_{a_i} - K_{\alpha_i}\sigma_{a_i}], \quad i = 1,2,\cdots,m \qquad (33)$$

$$\sum_{i=1}^{m} x_{ij} \geq \exp[\mu_{b_j} + K_{\beta_j}\sigma_{b_j}], \quad j = 1,2,\cdots,n \qquad (34)$$

$$x_{ij} \geq 0, \; \forall \; i \; \text{ and } \; j.$$

3. Solution Procedures

Let U_k and L_k are the upper and lower bounds for k-th objective function z_k, where U_k is the highest acceptable level of achievement for the k-th objective and L_k is the aspired level of achievement for the k-th objective. Also, U_k - L_k = d_k is the degradation allowance for the k-th objective.

The steps of fuzzy algorithm with linear membership function for solving the specified problem as follows:

Step 1: Solve the multi-objective stochastic transportation problem as single objective transportation, using one objective at a time and other is ignore.
Step 2: From the results of step - 1, determine the corresponding values for every objective at each solution derived.
Step 3: From the result of step-1 and 2, we construct a pay-off matrix as follows.

$$\begin{bmatrix} z_{11}(X_1) & z_{12}(X_1) & \cdots & z_{1K}(X_1) \\ z_{21}(X_2) & z_{22}(X_2) & \cdots & z_{2K}(X_2) \\ z_{31}(X_3) & z_{32}(X_3) & \cdots & z_{3K}(X_3) \\ \vdots & \vdots & \vdots & \vdots \\ z_{K1}(X_K) & z_{K2}(X_K) & \cdots & z_{KK}(X_K) \end{bmatrix}$$

where $X_1, X_2, X_3, \cdots, X_K$. are the ideal solution with respect to first, second,..., K-th objective functions respectively, and z_{ij} = $z_i(X_i)$, be the i-th row and j-th column elements of the pay-off matrix $(i = 1,2,3,\cdots,K\,;\, j = 1,2,3,\cdots,K)$

Step 4: We define the fuzzy membership function for k-th objective function as follows:

$$\mu_r(z_r) = \begin{cases} 0 & if \quad z_r \geq U_r \\ 1 - \dfrac{z_r - L_r}{U_r - L_r} & if \quad L_r < z_r < U_r \\ 1 & if \quad z_r \leq L_r, r = 1,2,\cdots,K \end{cases} \tag{35}$$

Step 5: We formulated an equivalent deterministic linear programming problem for vector minimum problem (32) using step-4 as follows.

$$\max : \lambda \tag{36}$$

subject to

$$\lambda \le \frac{U_k - z_k}{U_k - L_k}, \quad k = 1,2,\cdots,K \tag{37}$$

and the given constraints (33), (34) and non-negativity conditions as (4) and $\lambda \in [0,1]$. The above relation used to formulate the equivalent deterministic model of the specified problem as follows:

Model -5:

$$\max : \lambda \tag{38}$$

subject to

$$\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^{k} x_{ij} + \lambda(U_k - L_k) \le U_k, \quad k = 1,2,\cdots,K \tag{39}$$

$$\sum_{j=1}^{n} x_{ij} \le \exp[\mu_{a_i} - K_{\alpha_i}\sigma_{a_i}], \; i = 1,2,\cdots,m \tag{40}$$

$$\sum_{i=1}^{m} x_{ij} \ge \exp[\mu_{b_j} + K_{\beta_j}\sigma_{b_j}], \; j = 1,2,\cdots,n \tag{41}$$

$$x_{ij} \ge 0, \; \forall \; i \; and \; j.$$

$$\lambda \ge 0 \tag{42}$$

where

$$\lambda = \min_k \{\mu_k(z_k)\}, \; k = 1,2,\cdots,K$$

4. Case Study

The numerical example is related to a stochastic multi-objective transportation problem in which sources and demands are random variables and follows the log-normal distribution. The decision maker is interested to transport the goods from i - origin to j - destination, so as to satisfied to all requirements as follows:

$$\min : z_1 = x_{11} + 2x_{12} + 7x_{13} + 7x_{14} + x_{21} + 9x_{22} + 3x_{23} + 4x_{24} + 8x_{31} + 9x_{32} + 4x_{33} + 6x_{34} \tag{43}$$

$$\min : z_2 = 4x_{11} + 4x_{12} + 3x_{13} + 5x_{14} + 5x_{21} + 3x_{22} + 9x_{23} + 10x_{24} + 6x_{31} + 2x_{32} + 5x_{33} + x_{34} \tag{44}$$

subject to

$$\Pr\left[\sum_{j=1}^{4} x_{1j} \le a_1\right] \ge 1 - \alpha_1 \tag{45}$$

$$\Pr\left[\sum_{j=1}^{4} x_{2j} \le a_2\right] \ge 1-\alpha_2 \quad (46)$$

$$\Pr\left[\sum_{j=1}^{4} x_{3j} \le a_3\right] \ge 1-\alpha_3 \quad (47)$$

$$\Pr\left[\sum_{i=1}^{3} x_{i1} \ge b_1\right] \ge 1-\beta_1 \quad (48)$$

$$\Pr\left[\sum_{i=1}^{3} x_{i2} \ge b_2\right] \ge 1-\beta_2 \quad (49)$$

$$\Pr\left[\sum_{i=1}^{3} x_{i3} \ge b_3\right] \ge 1-\beta_3 \quad (50)$$

$$\Pr\left[\sum_{i=1}^{3} x_{i4} \ge b_4\right] \ge 1-\beta_4 \quad (51)$$

$$x_{ij} \ge 0,\ i=1,2,3 \ and \ j=1,2,3,4. \quad (52)$$

Now we assume the means and variances of log-normal random variables with the specified probabilistic levels of supplies i.e, a_i for $i=1,2,3$ are represented in the following Table -1.

Mean	Variance	Specified Probability Levels
$E(a_1)=31$	$V(a_1)=6$	α_1 =0.01
$E(a_2)=37$	$V(a_2)=7$	α_2 =0.02
$E(a_3)=40$	$V(a_3)=8$	α_3 =0.03

Table -1

Again, the means and variances of the log-normal random variables with the specified probabilistic levels of demands i.e, b_j for $j=1,2,3,4$ are represented in the following Table -2.

Mean	Variance	Specified Probability Levels
$E(b_1)=10$	$V(b_1)=2$	β_1 =0.04
$E(b_2)=15$	$V(b_2)=3$	β_2 =0.05
$E(b_3)=21$	$V(b_3)=4$	β_3 =0.06

$E(b_4)=26$	$V(b_4)=5$	β_4 =0.07

Table-2

Now using the relation of (8) and (9) the means and standard deviation of the log-normal random variable with specified probabilistic levels instead of supply i.e, a_i for $i=1,2,3$ from the Table-1, are represented in the following Table -3.

Mean	Standard deviation	Specified Probability Levels
μ_{a_1} =3.430875164	σ_{a_1} =0.078892839	α_1 =0.01
μ_{a_2} =3.608364907	σ_{a_2} =0.07145636	α_2 =0.02
μ_{a_3} =3.687631014	σ_{a_3} =0.049968792	α_3 =0.03

Table-3

Again,using the relation of (21) and (22) the means and standard deviations of the log-normal random variables with specified probabilistic levels instead of demand i.e, b_j for $j=1,2,3,4.$ from the Table -2, are represented in the following Table -4.

Mean	Standard deviation	Specified Probability Levels
μ_{b_1} = 2.292683779	σ_{b_1} = 0.0140721808	β_1 =0.04
μ_{b_2} = 2.701427588	σ_{b_2} = 0.115087908	β_2 =0.05
μ_{b_3} = 3.044093436	σ_{b_3} = 0.09502319	β_3 =0.06
μ_{b_4} = 3.253678247	σ_{b_4} = 0.094003094	β_4 =0.07

Table-4

As discussed in the solution procedures, we have obtained two ideal solutions of the above multi-objective functions stated in equations (43) and (44) with the set of the constraints from (45) to (52), a pay matrtix is formulated and from the pay-off matrix the bounds of the above objective function are obtained i.e, the lower bound is (L_1, L_2) = (265.7626, 256.2620) and for the same problem the upper bound is (U_1, U_2) = (515.195439, 525.282758). Using the membership function of the fuzzy technique, we have derived the following single objective deterministic transportation problem as :

$$\max : \lambda \tag{53}$$

subject to

$$z_1 = x_{11} + 2x_{12} + 7x_{13} + 7x_{14} + x_{21} + 9x_{22} + 3x_{23} + 4x_{24} + 8x_{31} + 9x_{32} + 4x_{33} + 6x_{34}$$

$$z_2 = 4x_{11} + 4x_{12} + 3x_{13} + 5x_{14} + 5x_{21} + 3x_{22} + 9x_{23} + 10x_{24} + 6x_{31} + 2x_{32} + 5x_{33} + x_{34}$$

$$z_1 + 249.432839\lambda \le 515.195439 \tag{54}$$

$$z_2 + 269.030758\lambda \le 525.282758 \tag{55}$$

$$\sum_{j=1}^{4} x_{1j} \le 25.57287834 \tag{56}$$

$$\sum_{j=1}^{4} x_{2j} \le 31.31243425 \tag{57}$$

$$\sum_{j=1}^{4} x_{3j} \le 36.15059581 \tag{58}$$

$$\sum_{i=1}^{3} x_{i1} \ge 10.15548247 \tag{59}$$

$$\sum_{i=1}^{3} x_{i2} \ge 18.12110048 \tag{60}$$

$$\sum_{i=1}^{3} x_{i3} \ge 24.43778598 \tag{61}$$

$$\sum_{i=1}^{3} x_{i4} \ge 29.80520039 \tag{62}$$

$$x_{ij} \ge 0, \lambda \ge 0, \; i = 1,2,3 \; and \; j = 1,2,3,4. \tag{63}$$

The above problem is solved by LINGO package and to obtained the value of aspiration level and compromise solution. They are as follows: λ = 0.7713102, x_{11} = 2.972351, x_{12} = 18.12110, x_{13} = 4.479427, x_{21} = 7.183132, x_{23} = 13.61296. Using the compromise solution we have obtained the optimal objective values and they are z_1 = 322.8053, z_2 = 317.7766.

5. Conclusion

The purpose of this paper is to present a solution procedure for multi-objective stochastic unbalanced transportation problem with log-normal random variables. Initially, the stochastic model of the constraints have been converted into an equivalent deterministic model using chance constrained programming. Then the fuzzy programming is applied to the objective function for solving to the corresponding given specified problems and to obtain a compromise solution from the set of non-dominated solution.

Most of the researchers have dealt with two constraints having one deterministic, another probabilistic constraints. But in our approach, we have presented two different types of probabilistic constraints for practical importance. So our technique is highly fruitful in this sense of real life problems of practical importance. A numerical example is provided to illustrate the methodologies.

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