Journal of Physical Sciences, Vol. 13, 2009, 13-19
ISSN: 0972-8791: www.vidyasagar.ac.in/journal

Jordan Left Derivations of Two Torsion Free ΓM – Modules

A.C.Paul and Amitabh Kumer Halder

Department of Mathematics
University of Rajshahi
Rajshahi University – 6205
Rajshahi, Bangladesh
acpaul_math@yahoo.com

Received November 4, 2009; accepted November 11, 2009

ABSTRACT

Let M be a Γ-ring and X be a 2-torsionfree left ΓM-module. The purpose of this paper is to investigate Jordan left derivations on M considering $a\alpha b\beta c = a\beta b\alpha c$, for every $a,b,c \in M$ and $\alpha,\beta \in \Gamma$. We show that the existence of a nonzero Jordan left derivation of M into X implies M is commutative. We also show that if X = M is a semiprime Γ-ring, then the derivation is a mapping from M into its centre. Finally we show that if M is a prime Γ- ring, then every Jordan left derivation d: M→ M is a left derivation.

Mathematics Subject Classification: Primary 16Y30, 16W25; Secondary 16U80.

Keywords: *n*-torsionfree, Jordan left derivations, left ΓM-modules, commutativity, prime Γ-rings, semiprime Γ-rings.

1. Introduction

Let M and Γ be additive abelian groups. M is said to be a Γ-ring if there exists a mapping $M \times \Gamma \times M \to M$ (sending (x, α, y) into $x\alpha y$) such that

(a) $(x + y)\alpha z = x\alpha z + y\alpha z$,
$x(\alpha + \beta)y = x\alpha y + x\beta y$,
$x\alpha(y + z) = x\alpha y + x\alpha z$,
(b) $(x\alpha y)\beta z = x\alpha(y\beta z)$,
for all $x,y,z \in M$ and $\alpha,\beta \in \Gamma$.

A Γ-ring M is commutative if $a\alpha b = b\alpha a$ for all $a,b \in M$ and $\alpha \in \Gamma$. A subset A of a Γ-ring M is a left(right) ideal of M if A is an additive subgroup of M and $M\Gamma A = \{m\alpha a: m \in M, \alpha \in \Gamma, a \in A\}$ $(A\Gamma M)$ is contained in A. The centre of M, written as Z(M), is the set of those elements in M that commute with every element in M i.e.,$Z(M) = \{m \in M: m\alpha x = x\alpha m$, for all $x \in M$ and $\alpha \in \Gamma\}$. M is prime if $a\Gamma M\Gamma b = 0$ with $a,b \in M$, then $a = 0$ or $b = 0$. M is semiprime if $a\Gamma M\Gamma a = 0$ with $a \in M$, then $a = 0$.

Let M be a Γ-ring and X be an additive abelian group. X is a left ΓM-module if there exists a mapping MXΓXX → X (sending (m, α,x) into mαx) such that

(a) $(m_1 + m_2)\alpha x = m_1\alpha x + m_2\alpha x$,

(b) $m\alpha(x_1 + x_2) = m\alpha x_1 + m\alpha x_2$,

(c) $(m_1\alpha m_2)\beta x = m_1\alpha(m_2\beta x)$,

for all $m, m_1, m_2 \in M$, $x, x_1, x_2 \in X$ and $\alpha, \beta \in \Gamma$.

X is n-torsionfree if nx = 0, for x∈M implies x = 0,where n is an integer. An additive mapping d: M → X is a derivation if d(aαb) = aαd(b) + d(a) αb, a left derivation if d(aαb) = aαd(b) +bαd(a), a Jordan derivation if d(aαa) = aαd(a) +d(a) α a and a Jordan left derivation if d(aαa) = 2aαd(a), for all a,b∈M and α∈Γ.

Y.Ceven [4] studied on Jordan left derivations on completely prime Γ-rings. He obtained that the existence of a nonzero Jordan left derivation on a completely prime Γ-ring makes Γ-ring commutative with an assumption. He also showed that a Jordan left derivation on a completely prime Γ-ring is a left derivation with the same assumption. In this paper, an example of a Jordan left derivation is given for Γ-rings. Mustafa Asci and Sahin Ceran [6] investigated a nonzero left derivation d on a prime Γ-ring M for which M is commutative with the conditions d(U)⊆U and d^2-(U)⊆Z, where U is an ideal of M and Z is the centre of M. They also showed that M is commutative if d_1 and d_2 are nonzero left and right derivations on M and $d_2(U)\subseteq U$ and $d_1d_2(U)\subseteq Z$.

In [8], Sapanci and Nakajima defined a derivation and a Jordan derivation on Γ--rings and showed that a Jordan derivation on a certain type of completely prime Γ-rings is a derivation. They also gave examples of a derivation and a Jordan derivation of Γ-rings.

Bresar and Vukman [2] proved that every Jordan derivation on a prime ring is a derivation. Furthermore, in [3], Bresar and Vukman investigated the existence of a nonzero Jordan left derivation of R into X which makes R commutative, where R is a ring and X is a 2-torsionfree and 3-torsionfree left R-module.

In [5], Jun and Kim proved their results without the property 3-torsionfree. In this paper, we modify the results of Jun and Kim [5] and a part of M.Bresar and J.Vukman [3] in Γ-rings with Jordan left derivations. We prove that the existence of a nonzero Jordan left derivation of M into X implies M is commutative. We also show that the semiprimeness of the Γ-ring X = M makes the mapping d: M → Z(M) a derivation and d: M → M is a left derivation if X = M is prime and d is a Jordan left derivation.

Throughout this paper, the condition aαbβc = aβbαc, for all a,b,c∈M and α,β∈Γ will represent by (*).

2. Jordan Left Derivations

For proving our main results, we have needed some important results which we have proved here as lemmas. So we start as follows.

Lemma 2.1 Let M be a Γ-ring satisfying (*) and X a 2-torsionfree left ΓM-module. Let d: M → X be a Jordan left derivation. Then

(a) d(aαb + bαa) = 2aαd(b) + 2bαd(a),

(b) $d(a\alpha b\beta a) = a\beta a\alpha d(b) + 3a\alpha b\beta d(a) - b\alpha a\beta d(a)$,
(c) $d(a\alpha b\beta c + c\alpha b\beta a) = (a\beta c + c\beta a)\,\alpha d(b) + 3a\alpha b\beta d(c) + 3c\alpha b\beta d(a) - b\alpha c\beta d(a) - b\alpha a\beta d(c)$,
(d) $(a\alpha b - b\alpha a)\beta a\alpha d(a) = a\alpha\,(a\alpha b - b\alpha a)\beta d(a)$,
(e) $(a\alpha b - b\alpha a)\beta(d(a\alpha b) - a\alpha d(b) - b\alpha d(a)) = 0$,
for all $a,b,c\in M$ and $\alpha,\beta\in\Gamma$.
The proof of this lemma is given in Y.Ceven [4].

Lemma 2.2 Let M be a Γ-ring satisfying (*) and let X be a 2-torsionfree ΓM-module. Then there exists a Jordan left derivation $d: M \to X$ such that
(a) $d(a\alpha a\beta b) = a\alpha a\beta d(b) + (a\beta b + b\beta a)\,\alpha d(a) + a\alpha d(a\beta b - b\beta a)$,
(b) $d(b\alpha a\beta a) = a\alpha a\beta d(b) + (3b\beta a - a\beta b)\,\alpha d(a) - a\alpha d(a\beta b - b\beta a)$,
(c) $(a\alpha b - b\alpha a)\beta d(a\alpha b - b\alpha a) = 0$,
(d) $(a\alpha a\beta b - 2a\alpha b\beta a + b\alpha a\beta a)\,\alpha d(b) = 0$,
for all $a,b,c\in M$ and $\alpha,\beta\in\Gamma$.

Proof. Substituting $b\beta a$ and $a\beta b$ for b in Lemma 2.1(a), we get
(1) $d(a\alpha b\beta a + b\beta a\alpha a) = 2a\alpha d(b\beta a) + 2b\beta a\alpha d(a)$ and
(2) $d(a\alpha a\beta b + a\beta b\alpha a) = 2a\alpha d(a\beta b) + 2a\beta b\alpha d(a)$.
Taking (2) minus(1) and then using (*), we get
(3) $d(a\alpha a\beta b - b\alpha a\beta a) = 2a\alpha d(a\beta b - b\beta a) + 2(a\beta b - b\beta a)\,\alpha d(a)$.
Replacing a by $a\alpha a$ in Lemma 2.1(a) and then by (*), we get
(4) $d(a\alpha a\beta b + b\alpha a\beta a) = 2a\alpha a\beta d(b) + 4b\beta a\alpha d(a)$.
By (3) and (4) with the condition that X is 2-torsionfree, we have (a).
Subtracting (3) from (4) and then applying the same condition, we obtain (b).
By Lemma 2.1(e), we have
(5) $(a\alpha b - b\alpha a)\beta(d(a\alpha b) - b\alpha d(a) - a\alpha d(b)) = 0$.
Using Lemma 2.1(a) in (5), we get
(6) $(a\alpha b - b\alpha a)\beta(d(b\alpha a) - a\alpha d(b) - b\alpha d(a)) = 0$.
Taking (5) minus (6), we obtain (c).
By Lemma 2.1(a), Lemma 2.1(b) and (*), we have
$d((a\alpha b - b\alpha a)\beta(a\alpha b - b\alpha a))$
$= -3(a\alpha a\beta b - 2a\alpha b\beta a + b\alpha a\beta a)\,\alpha d(b) - (b\alpha b\beta a - 2b\alpha a\beta b + a\alpha b\beta b)\,\alpha d(a)$.
On the other hand, using (c), we have $d((a\alpha b - b\alpha a)\beta(a\alpha b - b\alpha a)) = 0$.
Thus we have
(7) $3(a\alpha a\beta b - 2a\alpha b\beta a + b\alpha a\beta a)\,\alpha d(b) + (b\alpha b\beta a - 2b\alpha a\beta b + a\alpha b\beta b)\,\alpha d(a) = 0$.
From Lemma 2.1(d),
(8) $(a\alpha a\beta b - 2a\alpha b\beta a + b\alpha a\beta a)\,\alpha d(a) = 0$.
Replacing a by a + b in (8), we obtain
(9) $(a\alpha a\beta b - 2a\alpha b\beta a + b\alpha a\beta a)\,\alpha d(b) - (b\alpha b\beta a - 2b\alpha a\beta b + a\alpha b\beta b)\,\alpha d(a) = 0$.
Adding (7) and (9) , and then using the condition that X is 2-torsionfree , we get
(10) $(a\alpha a\beta b - 2a\alpha b\beta a + b\alpha a\beta a)\,\alpha d(b) = 0$.
Hence from (9) and (10), we obtain (d).

Theorem 2.3 Let M be a Γ-ring satisfying (*) and let X be a 2-torsionfree ΓM-module. Suppose that $a\alpha M\beta x = 0$ with $a\in M$, $x\in X$ and $\alpha,\beta\in\Gamma$ implies that either a = 0 or x = 0. If there exists a nonzero Jordan left derivation d: M → X then M is commutative.

Proof. By Lemma 2.1(d), we have $(x\alpha x\beta y - 2x\alpha y\beta x + y\alpha x\beta x)\,\alpha d(x) = 0$, for all $x,y\in M$ and $\alpha,\beta\in\Gamma$. Replacing $a\alpha b - b\alpha a$ for x and then using Lemma 2.2(c), we get $(a\alpha b - b\alpha a)\,\alpha\,(a\alpha b - b\alpha a)\,\alpha y\beta d(a\alpha b - b\alpha a) = 0$, for all $a,b,y\in M$ and $\alpha,\beta\in\Gamma$. By assumption, either $(a\alpha b - b\alpha a)\,\alpha\,(a\alpha b - b\alpha a) = 0$ or $d(a\alpha b - b\alpha a) = 0$. Suppose that $(a\alpha b - b\alpha a)\,\alpha\,(a\alpha b - b\alpha a) = 0$, for all $a,b\in M$ and $\alpha\in\Gamma$. Applying Lemma 2.1(a), Lemma 2.1(b), $(a\alpha b - b\alpha a)\,\alpha\,(a\alpha b - b\alpha a) = 0$ and (*), we have

(11) $E = d(((a\alpha b - b\alpha a)(x)\beta((a\alpha b - b\alpha a)\alpha y\beta(a\alpha b - b\alpha a)) + ((a\alpha b - b\alpha a)\alpha y\beta(a\alpha b - b\alpha a))\beta((a\alpha b - b\alpha a)(x))$
$= 6(a\alpha b - b\alpha a)\alpha x\beta(a\alpha b - b\alpha a)\alpha y\beta d(a\alpha b - b\alpha a) + (a\alpha b - b\alpha a)\alpha y\beta\{2(a\alpha b - b\alpha a)\beta d(a\alpha b - b\alpha a)\alpha x\}$.

On the other hand, by (*) , $(a\alpha b - b\alpha a)\,\alpha\,(a\alpha b - b\alpha a) = 0$ and Lemma 2.2(c), we have

(12) $E = d(((a\alpha b - b\alpha a)\alpha x)\beta((a\alpha b - b\alpha a)\alpha y\beta(a\alpha b - b\alpha a)) + ((a\alpha b - b\alpha a)\alpha y\beta(a\alpha b - b\alpha a))\beta((a\alpha b - b\alpha a)\alpha x))$

$= 3(a\alpha b - b\alpha a)\alpha x\beta(a\alpha b - b\alpha a)\alpha y\beta d(a\alpha b - b\alpha a)$.

Comparing (11) and (12), we get

(13) $3(a\alpha b - b\alpha a)\alpha x\beta(a\alpha b - b\alpha a)\alpha y\beta d(a\alpha b - b\alpha a) + (a\alpha b - b\alpha a)\alpha y\beta\{2(a\alpha b - b\alpha a)\beta d(a\alpha b - b\alpha a)\alpha x\} = 0$, for all $a,b,x,y\in M$ and $\alpha,\beta\in\Gamma$.

And, by (*) and Lemma 2.2(c), we have

(14) $F = d((a\alpha b - b\alpha a)\alpha x\beta(a\alpha b - b\alpha a) + x\beta(a\alpha b - b\alpha a)\,\alpha\,(a\alpha b - b\alpha a))$

$= 3(a\alpha b - b\alpha a)\alpha x\beta d(a\alpha b - b\alpha a)$.

On the other hand, we also have

(15) $F = d((a\alpha b - b\alpha a)\alpha x\beta(a\alpha b - b\alpha a) + x\beta(a\alpha b - b\alpha a)\,\alpha\,(a\alpha b - b\alpha a))$

$= 2(a\alpha b - b\alpha a)\alpha d(x\beta(a\alpha b - b\alpha a))$.

Comparing (14) and (15), we get

(16) $3(a\alpha b - b\alpha a)\alpha x\beta d(a\alpha b - b\alpha a)$

$= 2(a\alpha b - b\alpha a)\alpha d(x\beta(a\alpha b - b\alpha a))$, for all $a,b,x\in M$ and $\alpha,\beta\in\Gamma$.

Using $(a\alpha b - b\alpha a)\,\alpha\,(a\alpha b - b\alpha a) = 0$, we have

(17) $(a\alpha b - b\alpha a)\alpha d(x\beta(a\alpha b - b\alpha a) + (a\alpha b - b\alpha a)\beta x)$

$= 2(a\alpha b - b\alpha a)\alpha x\beta d(a\alpha b - b\alpha a)$, for all $a,b,x\in M$ and $\alpha,\beta\in\Gamma$.

From (16) and (17), we have

(18) $3(a\alpha b - b\alpha a)\alpha\{d(x\beta(a\alpha b - b\alpha a)) + d((a\alpha b - b\alpha a)\beta x)\}$

$= 4(a\alpha b - b\alpha a)\alpha d(x\beta(a\alpha b - b\alpha a))$, for all $a,b,x\in M$ and $\alpha,\beta\in\Gamma$.

Thus

(19) $(a\alpha b - b\alpha a)\,\alpha d(x\beta(a\alpha b - b\alpha a))$

$= 3(a\alpha b - b\alpha a)\,\alpha d((a\alpha b - b\alpha a)\beta x)$, for all $a,b,x \in M$ and $\alpha,\beta \in \Gamma$.

From (19), we get

(20) $(a\alpha b - b\alpha a)\,\alpha d(x\beta(a\alpha b - b\alpha a) + (a\alpha b - b\alpha a)\beta x)$

$= 4(a\alpha b - b\alpha a)\,\alpha d((a\alpha b - b\alpha a)\beta x)$, for all $a,b,x \in M$ and $\alpha,\beta \in \Gamma$.

On the other hand, using $(a\alpha b - b\alpha a)\,\alpha\,(a\alpha b - b\alpha a) = 0$, we have

(21) $(a\alpha b - b\alpha a)\,\alpha d(x\beta(a\alpha b - b\alpha a) + (a\alpha b - b\alpha a)\beta x)$

$= 2(a\alpha b - b\alpha a)\,\alpha x\beta d(a\alpha b - b\alpha a)$, for all $a,b,x \in M$ and $\alpha,\beta \in \Gamma$.

From (20) and (21) and since X is 2-torsionfree ,we get

(22) $2(a\alpha b - b\alpha a)\,\alpha d((a\alpha b - b\alpha a)\beta x)$

$= (a\alpha b - b\alpha a)\,\alpha x\beta d(a\alpha b - b\alpha a)$,for all $a,b,x \in M$ and $\alpha,\beta \in \Gamma$.

From (13) and (22), we obtain

(23) $3(a\alpha b - b\alpha a)\,\alpha x\beta(a\alpha b - b\alpha a)\,\alpha y\beta d(a\alpha b - b\alpha a) + (a\alpha b - b\alpha a)\,\alpha y\beta(a\alpha b - b\alpha a)\,\alpha x\beta d(a\alpha b - b\alpha a) = 0$, for all $a,b,x,y \in M$ and $\alpha,\beta \in \Gamma$.

Using (*) in (22), and then replacing $y\alpha\,(a\alpha b - b\alpha a)\beta y$ for x, we get $4(a\alpha b - b\alpha a)\,\alpha\,(a\alpha b - b\alpha a)\beta y\alpha d((a\alpha b - b\alpha a)\beta y) = (a\alpha b - b\alpha a)\beta y\alpha\,(a\alpha b - b\alpha a)\beta y\alpha d(a\alpha b - b\alpha a)$, for all $a,b,y \in M$ and $\alpha,\beta \in \Gamma$ .Using (*) and $(a\alpha b - b\alpha a)\,\alpha\,(a\alpha b - b\alpha a) = 0$ in the above relation, we get

(24) $(a\alpha b - b\alpha a)\,\alpha y\beta(a\alpha b - b\alpha a)\,\alpha y\beta d(a\alpha b - b\alpha a)$

$= 0$, for all $a,b,y \in M$ and $\alpha,\beta \in \Gamma$.

Replacing x + y for y in (24), we get

(25) $(a\alpha b - b\alpha a)\,\alpha x\beta(a\alpha b - b\alpha a)\,\alpha y\beta d(a\alpha b - b\alpha a) + (a\alpha b - b\alpha a)\,\alpha y\beta(a\alpha b - b\alpha a)\,\alpha x\beta d(a\alpha b - b\alpha a) = 0$, for all $a,b,x,y \in M$ and $\alpha,\beta \in \Gamma$.

From (23) and (25), and then using that X is 2-torsionfree, we have

(26) $(a\alpha b - b\alpha a)\,\alpha x\beta(a\alpha b - b\alpha a)\,\alpha y\beta d(a\alpha b - b\alpha a)$

$= 0$, for all $a,b,x,y \in M$ and $\alpha,\beta \in \Gamma$.

From (26), it follows that for each $a \in M$ either $a \in Z(M)$ or $d(a\alpha b - b\alpha a) = 0$, for all $a,b \in M$ and $\alpha \in \Gamma$. We consider the case $d(a\alpha b - b\alpha a) = 0$, for all $a,b \in M$ and $\alpha \in \Gamma$. Then by Lemma 2.1(b), Lemma 2.2(b) and (*), we get $2d(b\alpha a\beta a) = 2\{a\alpha a\beta d(b) + a\alpha b\beta d(a) + b\alpha a\beta d(a)\}$.

Using the condition that X is 2-torsionfree , Lemma 2.2(b) and (*) in this relation, we obtain

(27) $(a\alpha b - b\alpha a)\beta d(a) = 0$, for all $a,b \in M$ and $\alpha,\beta \in \Gamma$.

Replacing b(x for b in (27), we have $(a\alpha b\alpha x - b\alpha x\alpha a)\beta d(a) = 0$. This gives $(a\alpha b - b\alpha a)\,\alpha x\beta d(a) + b\alpha\,(a\alpha x - x\alpha a)\beta d(a) = 0$. This implies that $(a\alpha b - b\alpha a)$

$\alpha x\beta d(a) = 0$, for all $a,b,x\in M$ and $\alpha,\beta\in\Gamma$. Therefore, it follows that for each $a\in M$ either $a\in Z(M)$ or $d(a) = 0$. Since d is nonzero, $a\in Z(M)$ .This completes the proof.

Corollary 2.4 Let M be a Γ-ring satisfying (*). Let X = M be a prime Γ-ring. If d: M → M is a Jordan left derivation, then d is a left derivation.

Proof. Given that X = M be a prime Γ-ring . By Theorem 2.3, M is commutative. Then $a\alpha b = b\alpha a$, for all $a,b\in M$ and $\alpha \in\Gamma$. Therefore, by Lemma 2.1(a), we have $d(a\alpha b) = a\alpha d(b) + b\alpha d(a)$, for all $a,b\in M$ and $\alpha \in\Gamma$.

Theorem 2.5 Lel M be a Γ-ring satisfying (*) and let X be a left ΓM-module. Let d: M → X be a left derivation.
(a) Suppose that $a\alpha M\beta x = 0$ with $a\in M$, $x\in X$ and $\alpha,\beta\in\Gamma$ implies a = 0 or x = 0. If d (0 then M is commutative.
(b) Suppose that X = M is a semiprime Γ-ring. Then d: M → Z(M) is a derivation.

Proof. Since d: M → X is a left derivation,
(28) $d(a\alpha b) = a\alpha d(b) + b\alpha d(a)$, for all $a,b\in M$ and $\alpha \in\Gamma$.
Replacing b by $b\beta a$ in (28), we have
(29) $d(a\alpha b\beta a) = d(a\alpha (b\beta a)) = a\alpha b\beta d(a) + a\alpha a\beta d(b) + b\beta a\alpha d(a)$ and
(30) $d(a\alpha b\beta a) = d((a\alpha b)\beta a) = a\alpha b\beta d(a) + a\beta a\alpha d(b) + a\beta b\alpha d(a)$.
From (29) and (30), we gwt
(31) $(a\alpha b - b\alpha a)\beta d(a) = 0$, for all $a,b\in M$ and $\alpha,\beta\in\Gamma$.
Writing $c\gamma b$ for b in (31), and then by (*), we get
(32) $(a\alpha c - c\alpha a)\beta b\alpha d(a) = 0$, for all a,b,c(M and (,β(Γ.
By assumption, for each a(M either a(Z(M) or d(a) = 0. But then Z(M) and Ker d = {m(M: d(m) = 0} are additive subgroups of M and M = Z(M) (Ker d. Since Z(M) and Ker d are proper subgroups of M, by Brauer’s trick, either M = Z(M) or M= Ker d. But d (0 , then M = Z(M). This gives (a).
Let X = M be a semiprime Γ-ring. Replacing a by a + m in (32), we get
(33) (a(c – c(a)βb(d(m) + (m(c – c(m)βb(d(a)
= 0, for all a,b,c,m(M and (,β, ((Γ.
For all a,b,c,x,m(M and (,β, (, (,((Γ, we have
((a(c – c(a)βb(d(m)) (x(((a(c – c(a)βb(d(m))
= – ((a(c – c(a)βb(d(m))(x(((m(c – c(m)βb(d(a)) = 0, by (33).
Since M is semiprime, we get from the above relation $(a\alpha c - c\alpha a)\beta b\gamma d(m) = 0$.
In particular, $(a\alpha d(m) - d(m)\alpha a)\beta b\gamma(a\alpha d(m) - d(m)\alpha a)= 0$.This implies that $a\alpha d(m) = d(m)\alpha a$. This shows that $d(m)\in Z(M)$, for every $m\in M$ and we obtain (b).

REFERENCES

1. W.E. Barnes, *On the Γ-rings of Nobusawa,* Pacific J. Math., 18(1966), 411-422.
2. M. Bresar and J. Vukman, *Jordan derivations on prime rings,* Bull. Austral. Math.Soc.,37(1988), 321-322.[3] M. Bresar and J. Vukman, *On the left derivations and related mappings,* Proc. of the AMS., Vol: 110, 1(1990), 7-16.
3. Y. Ceven, *Jordan left derivations on completely prime gamma rings,* C.U. Fen-Edebiyat Fakultesi, Fen Bilimleri Dergisi (2002) Cilt 23 Sayi 2.
4. K.W. Jun and B.D. Kim, *A note on Jordan left derivations,* Bull. Korean Math. Soc., 33(1996), No: 2, 221-228.
5. Mustafa Asci and Sahin Ceran ,*The commutativity in prime gamma rings with left derivation*, International Mathematical Forum,2,2007,no.3,103 – 108.
6. N. Nobusawa, *On a generalization of the ring theory,* Oska J. Math., 1(1964).
7. M. Sapanci and A. Nakajima, *Jordan derivations on completely prime gamma rings,*Math. Japonica, Vol: 46. No: 1, 1997, 47-51.