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Vertex Contraction Operations in Advanced Fuzzy Graph Types: Intuitionistic, Pythagorean, Neutrosophic, and Bipolar Extensions

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ABSTRACT

This paper extends the fundamental concept of vertex contraction in fuzzy graphs to advanced fuzzy graph structures, including intuitionistic fuzzy graphs (IFGs), Pythagorean fuzzy graphs (PFGs), neutrosophic graphs (NGs), and bipolar fuzzy graphs (BFGs). Formal definitions of contraction operations are developed to preserve the essential characteristics of each graph type. The effects of these operations on domination parameters are examined, and several theoretical properties are established, including the preservation of hesitancy degrees, the behavior of membership functions, and structural invariants. The proposed framework offers a unified approach to graph simplification and network analysis in uncertain environments characterized by multiple dimensions of uncertainty. Applications to multi-criteria decision-making, social network analysis, and conflict resolution systems are also discussed.

Keywords: Vertex contraction, Intuitionistic fuzzy graphs, Pythagorean fuzzy graphs, Neutrosophic graphs, Bipolar fuzzy graphs, Domination, Hesitancy degree

AMS Mathematics Subject Classification (2010): 05C72, 03E72

Abstract in Bengali

এই গবেষণাপত্রটি ফাজি গ্রাফে শীর্ষবিন্দু সংকোচনের মৌলিক ধারণাটিকে উন্নত ফাজি গ্রাফ কাঠামোতে প্রসারিত করে, যার মধ্যে রয়েছে অন্তর্দৃষ্টিবাদী ফাজি গ্রাফ (IFGs), পাইথাগোরিয়ান ফাজি গ্রাফ (PFGs), নিউট্রোসোফিক গ্রাফ (NGs) এবং বাইপোলার ফাজি গ্রাফ (BFGs)। প্রতিটি গ্রাফের প্রয়োজনীয় বৈশিষ্ট্যগুলি সংরক্ষণের জন্য সংকোচন

ক্রিয়াকলাপের আনুষ্ঠানিক সংজ্ঞা তৈরি করা হয়। আধিপত্য পরামিতিগুলির উপর এই ক্রিয়াকলাপগুলির প্রভাব পরীক্ষা করা হয় এবং বেশ কয়েকটি তাত্ত্বিক বৈশিষ্ট্য প্রতিষ্ঠিত হয়, যার মধ্যে দ্বিধাগ্রস্ততা ডিগ্রি সংরক্ষণ, সদস্যপদ ফাংশনের আচরণ এবং কাঠামোগত পরিবর্তনশীলতা অন্তর্ভুক্ত। প্রস্তাবিত কাঠামোটি অনিশ্চয়তার একাধিক মাত্রা দ্বারা চিহ্নিত অনিশ্চিত পরিবেশে গ্রাফ সরলীকরণ এবং নেটওয়ার্ক বিশ্লেষণের জন্য একটি সমন্বিত পদ্ধতির প্রস্তাব দেয়। বহু-মানদণ্ডের সিদ্ধান্ত গ্রহণ, সামাজিক নেটওয়ার্ক বিশ্লেষণ এবং দ্বন্দ্ব সমাধান ব্যবস্থার প্রয়োগগুলিও আলোচনা করা হয়েছে।

1. Introduction

Fuzzy graph theory, introduced by Rosenfeld in 1975 [7] and rooted in the seminal concept of fuzzy sets proposed by Zadeh [16], has become an essential tool for modeling uncertainty in network structures. Comprehensive treatments of fuzzy graphs and their structural properties can be found in the monograph by Mordeson and Nair [5]. The recent work by Ramya and Lavanya (2023) [6] on edge contraction and neighbourhood contraction operations in classical fuzzy graphs has opened new avenues for graph simplification while preserving important structural properties.

Domination and connectivity have long been central themes in fuzzy graph theory. Early investigations into domination in fuzzy graphs were carried out by Somasundaram and Somasundaram [15], establishing fundamental concepts that continue to influence current research. Later studies extended these ideas to vertex connectivity and real-world applications, such as human trafficking networks [12], highlighting the practical importance of domination-related parameters in uncertain network environments.

Intuitionistic fuzzy graphs (Atanassov, 1986) [3] incorporate both membership and non-membership degrees, allowing for the representation of hesitancy. Pythagorean fuzzy graphs (Yager, 2013) [9] extend this framework by relaxing the constraint to $\mu^2 + \nu^2 \leq 1$, providing greater flexibility in representing uncertainty. Neutrosophic graphs (Smarandache, 2005) [8] introduce truth, indeterminacy, and falsity components, enabling the modeling of incomplete information. Bipolar fuzzy graphs (Lee, 2000) [4] capture dual perspectives through positive and negative membership degrees, essential for modeling systems with opposing forces.

In recent years, fuzzy and advanced fuzzy graph models have been increasingly applied to decision-making, manufacturing systems, and communication networks. Pythagorean fuzzy information has proven particularly effective in multi-criteria decision-making contexts [13]. Connectivity and structural indices have been studied in m-polar and cubic fuzzy networks with applications in industrial manufacturing and trade analysis [10, 11]. More recent works have explored fuzzy connectivity in mobile communication networks [14] and investigated hardness and structural properties of fuzzy contraction operations [1]. Applications to medical and biological networks further demonstrate the growing relevance of fuzzy graph techniques [2].

The objective of this paper is to develop a comprehensive framework for vertex contraction operations across these advanced fuzzy graph types. We establish formal definitions, prove fundamental properties, and investigate the impact of contraction on domination parameters a critical measure of network control and influence.

2. Preliminaries

2.1. Classical fuzzy graphs and contraction

Definition 2.1. A fuzzy graph $G = (V, \sigma, \mu)$ consists of a finite set V of vertices, a fuzzy subset $\sigma: V \rightarrow [0,1]$ representing vertex membership values, and a fuzzy relation $\mu: V \times V \rightarrow [0,1]$ representing edge membership values, such that $\mu(u, v) \leq \min\{\sigma(u), \sigma(v)\}$ for all $u, v \in V$.

Definition 2.2. Let $G = (V, \sigma, \mu)$ be a fuzzy graph and $e = (u, v) \in E$. The edge contraction of e , denoted G/e , merges vertices u and v into a new vertex w , with:

- $V' = (V \setminus \{u, v\}) \cup \{w\}$
- $\sigma'(w) = \max\{\sigma(u), \sigma(v)\}$
- $\mu'(w, x) = \max\{\mu(u, x), \mu(v, x)\}$ for all $x \in V \setminus \{u, v\}$

Definition 2.3. A subset $S \subseteq V$ is a dominating set of G if for every vertex $v \in V \setminus S$, there exists $u \in S$ such that $\mu(u, v)$ is a strong edge. The minimum cardinality of a dominating set is the domination number $\gamma(G)$.

2.2. Advanced fuzzy graph types

Definition 2.4. An intuitionistic fuzzy graph (IFG) $G = (V, A, B)$ consists of a vertex set V , where $A = (\mu_A, \nu_A)$ with $\mu_A: V \rightarrow [0,1]$ (membership) and $\nu_A: V \rightarrow [0,1]$ (non-membership) such that $\mu_A(v) + \nu_A(v) \leq 1$, and $B = (\mu_B, \nu_B)$ for edges such that $\mu_B(u, v) \leq \min\{\mu_A(u), \mu_A(v)\}$ and $\nu_B(u, v) \geq \max\{\nu_A(u), \nu_A(v)\}$. The hesitancy degree is $\pi_A(v) = 1 - \mu_A(v) - \nu_A(v)$.

Definition 2.5. A Pythagorean fuzzy graph (PFG) $G = (V, P, Q)$ extends IFGs by requiring $(\mu_P(v))^2 + (\nu_P(v))^2 \leq 1$ for vertices and $(\mu_Q(u, v))^2 + (\nu_Q(u, v))^2 \leq 1$ for edges.

Definition 2.6. A neutrosophic graph (NG) $G = (V, T, I, F)$ assigns to each vertex v and edge (u, v) three independent values: truth membership $T(v)$, indeterminacy $I(v)$, and falsity $F(v)$, where $T, I, F: V \rightarrow [0,1]$ and $T(v) + I(v) + F(v) \leq 3$.

Definition 2.7. A bipolar fuzzy graph (BFG) $G = (V, \sigma^+, \sigma^-, \mu^+, \mu^-)$ has vertex unctions $\sigma^+: V \rightarrow [0,1]$ (positive membership) and $\sigma^-: V \rightarrow [-1,0]$ (negative membership), with corresponding edge functions μ^+ and μ^- satisfying $\mu^+(u, v) \leq \min\{\sigma^+(u), \sigma^+(v)\}$ and $\mu^-(u, v) \geq \max\{\sigma^-(u), \sigma^-(v)\}$.

3. Vertex contraction in intuitionistic fuzzy graphs

3.1. IFG contraction definition

Definition 3.1. Let $G = (V, A, B)$ be an IFG and $e = (u, v)$ be an edge. The intuitionistic fuzzy edge contraction G/e produces $G' = (V', A', B')$ where:

- $V' = (V \setminus \{u, v\}) \cup \{w\}$
- $\mu_{A'}(w) = \max\{\mu_A(u), \mu_A(v)\}$
- $\nu_{A'}(w) = \min\{\nu_A(u), \nu_A(v)\}$
- $\mu_{B'}(w, x) = \max\{\mu_B(u, x), \mu_B(v, x)\}$ for all $x \in V \setminus \{u, v\}$

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- $v_{B'}(w, x) = \min\{v_B(u, x), v_B(v, x)\}$ for all $x \in V \setminus \{u, v\}$

Rationale: The max-min principle for membership and non-membership ensures that the merged vertex w inherits the strongest positive evidence (maximum membership) and the weakest negative evidence (minimum non-membership) from both u and v . This preserves the intuitionistic character while maintaining the constraint $\mu_{A'}(w) + v_{A'}(w) \leq 1$.

3.2. Hesitancy degree analysis

Theorem 3.2. *Let G be an IFG and $G' = G/e$ for edge $e = (u, v)$. The hesitancy degree of the merged vertex w satisfies:*

$$\pi_{A'}(w) \leq \max\{\pi_A(u), \pi_A(v)\} \quad (1)$$

Proof. By definition, $\pi_{A'}(w) = 1 - \mu_{A'}(w) - v_{A'}(w) = 1 - \max\{\mu_A(u), \mu_A(v)\} - \min\{v_A(u), v_A(v)\}$. Without loss of generality, assume $\mu_A(u) \geq \mu_A(v)$ and $v_A(u) \leq v_A(v)$. Then:

$$\begin{aligned} \pi_{A'}(w) &= 1 - \mu_A(u) - v_A(u) \\ &= \pi_A(u) \\ &\leq \max\{\pi_A(u), \pi_A(v)\} \end{aligned}$$

Corollary 3.3. *Edge contraction in IFGs never increases the maximum hesitancy degree of the graph.*

3.3. Domination in contracted IFGs

Definition 3.4. *In an IFG $G = (V, A, B)$, a subset $S \subseteq V$ is an intuitionistic fuzzy dominating set if for every $v \in V \setminus S$, there exists $u \in S$ such that $\mu_B(u, v) = \min\{\mu_A(u), \mu_A(v)\}$ and $v_B(u, v) = \max\{v_A(u), v_A(v)\}$ (i.e., (u, v) is a strong edge in the intuitionistic sense).*

Theorem 3.5. *Let G be an IFG with domination number $\gamma(G) = k$. If $e = (u, v)$ is an edge with $u, v \in S$ for some minimum dominating set S , then $\gamma(G/e) \geq k - 1$.*

Proof. After contracting $e = (u, v)$ to w , the set $S' = (S \setminus \{u, v\}) \cup \{w\}$ forms a dominating set in G/e with $|S'| = k - 1$. By the max-min principle, w dominates all vertices previously dominated by u or v . Therefore, $\gamma(G/e) \leq k - 1$, and since domination number is non-negative, $\gamma(G/e) \geq k - 1$ when the contraction is optimal.

4. Vertex contraction in pythagorean fuzzy graphs

4.1. PFG contraction definition

Definition 4.1. *Let $G = (V, P, Q)$ be a PFG and $e = (u, v)$ be an edge. The Pythagorean fuzzy edge contraction G/e produces $G' = (V', P', Q')$ where the merged vertex w satisfies:*

- $\mu_{P'}(w) = \max\{\mu_P(u), \mu_P(v)\}$
- $v_{P'}(w) = \min\{v_P(u), v_P(v)\}$ if $(\max\{\mu_P(u), \mu_P(v)\})^2 + (\min\{v_P(u), v_P(v)\})^2 \leq 1$
- Otherwise, $v_{P'}(w) = \sqrt{1 - (\max\{\mu_P(u), \mu_P(v)\})^2}$

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Theorem 4.2. *The Pythagorean constraint is preserved under contraction, i.e., $(\mu_{P_r}(w))^2 + (v_{P_r}(w))^2 \leq 1$.*

Proof. By construction, if the direct max-min assignment violates the constraint, we adjust $v_{P_r}(w)$ to the maximum value that satisfies the Pythagorean property. This ensures $(\mu_{P_r}(w))^2 + (v_{P_r}(w))^2 \leq 1$ always holds.

4.2. Pythagorean strength index

Definition 4.3. *The Pythagorean strength index of a vertex v is defined as:*

$$PSI(v) = \sqrt{(\mu_P(v))^2 + (1 - v_P(v))^2} \quad (2)$$

measuring the combined strength of positive evidence and lack of negative evidence.

Theorem 4.4. *After contracting $e = (u, v)$ to w , $PSI(w) \geq \max\{PSI(u), PSI(v)\}$.*

This property ensures that contraction preserves or enhances the overall strength of vertices, making it suitable for network simplification while maintaining decision quality in multi-criteria decision-making applications.

4.3. Domination under pythagorean contraction

Definition 4.5. *A Pythagorean fuzzy dominating set requires strong edges where $(\mu_Q(u, v))^2 \geq (\min\{\mu_P(u), \mu_P(v)\})^2$ and $(v_Q(u, v))^2 \leq (\max\{v_P(u), v_P(v)\})^2$.*

The domination behavior in PFGs under contraction follows similar patterns to IFGs, with the additional consideration of the Pythagorean constraint affecting edge strength preservation.

5. Vertex contraction in neutrosophic graphs

5.1. Neutrosophic contraction definition

Definition 5.1. *Let $G = (V, T, I, F)$ be a neutrosophic graph and $e = (u, v)$ be an edge. The neutrosophic edge contraction G/e produces G' where the merged vertex w has:*

- $T'(w) = \max\{T(u), T(v)\}$ (maximum truth)
- $I'(w) = \frac{I(u)+I(v)}{2}$ (average indeterminacy)
- $F'(w) = \min\{F(u), F(v)\}$ (minimum falsity)

Rationale: We adopt an optimistic approach for truth and falsity (maximizing truth, minimizing falsity), while averaging indeterminacy to reflect the combined uncertainty from both vertices.

5.2. Independence of components

Theorem 5.2. *In neutrosophic graphs, the truth, indeterminacy, and falsity components can be contracted independently without violating neutrosophic constraints.*

This independence property makes neutrosophic contraction particularly flexible and allows for component-specific optimization strategies in applications.

5.3. Neutrosophic domination preservation

Definition 5.3. A neutrosophic dominating set requires strong edges where $T_{edge} \geq \min\{T_u, T_v\}$, $I_{edge} \leq \max\{I_u, I_v\}$, and $F_{edge} \leq \max\{F_u, F_v\}$ for dominating vertex u and dominated vertex v .

Theorem 5.4. Let G be a neutrosophic graph with neutrosophic domination number $\gamma_N(G) = k$. After contracting edge $e = (u, v)$ where both u, v belong to a minimum dominating set, $\gamma_N(G/e)$ satisfies $k - 1 \leq \gamma_N(G/e) \leq k$.

6. Vertex contraction in bipolar fuzzy graphs

6.1. BFG contraction definition

Definition 6.1. Let $G = (V, \sigma^+, \sigma^-, \mu^+, \mu^-)$ be a BFG and $e = (u, v)$ be an edge. The bipolar fuzzy edge contraction G/e produces G' where:

- $\sigma^{+'}(w) = \max\{\sigma^+(u), \sigma^+(v)\}$ (maximum positive membership)
- $\sigma^{-'}(w) = \max\{\sigma^-(u), \sigma^-(v)\}$ (maximum negative membership, closest to 0)
- $\mu^{+'}(w, x) = \max\{\mu^+(u, x), \mu^+(v, x)\}$
- $\mu^{-'}(w, x) = \max\{\mu^-(u, x), \mu^-(v, x)\}$

Rationale: The merged vertex inherits the stronger positive properties and the weaker negative properties from both vertices, representing an optimistic merger that emphasizes strengths while downplaying weaknesses.

6.2. Bipolar balance index

Definition 6.2. The bipolar balance index of vertex v is:

$$BBI(v) = \sigma^+(v) + \sigma^-(v) \quad (3)$$

measuring the overall disposition from negative (≤ -1) through neutral (≈ 0) to positive (≤ 1).

Theorem 6.3. After contracting $e = (u, v)$ to w , $BBI(w) \geq \max\{BBI(u), BBI(v)\}$, ensuring the merged vertex maintains or improves the most favorable balance.

6.3. Applications to conflict resolution

Bipolar fuzzy graphs are particularly suited for modeling systems with opposing forces, such as:

- Social networks with both friendships (positive) and conflicts (negative)
- Business decision-making with pros and cons
- Political alliance and opposition structures

Vertex contraction in BFGs allows for simplifying complex dual-perspective networks while preserving the essential positive-negative dynamics.

7. Comparative analysis and properties

Table 1 summarizes the key properties of contraction operations across different fuzzy graph types.

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7.1. Algorithmic considerations

General Contraction Procedure for Advanced Fuzzy Graphs

G (advanced fuzzy graph), $e = (u, v)$ (edge to contract) G' (contracted graph) Create new vertex w Apply type-specific membership merging rules to w based on u and v each neighbor x of u or v ($x \neq u, v$) Compute edge membership $\mu'(w, x)$ using max operator For multi-component types, compute each component separately Update vertex set: $V' \leftarrow (V \setminus \{u, v\}) \cup \{w\}$ Remove edges incident to u and v ; add edges incident to w Verify type-specific constraints (Pythagorean, neutrosophic, bipolar) G'

Complexity: $O(\deg(u) + \deg(v))$ for a single contraction, where \deg represents vertex degree. Multiple contractions can be optimized using union-find structures.

Table 1: Comparative properties of contraction operations

Property	IFG	PFG	NG	BFG
Membership Merging	Max-min	Max-min with constraint	Max-avg-min	Dual max
Hesitancy/Uncertainty	Non-increasing	Adjusted	Averaged	N/A
Component Independence	Dependent	Dependent	Independent	Semi-independent
Domination Preservation	$\gamma(G/e) \geq \gamma(G) - 1$	Similar	Similar	Similar
Best Application	General uncertainty	MCDM	Incomplete info	Conflicts

8. Applications

8.1. Multi-criteria decision making

Pythagorean fuzzy graphs with contraction operations prove particularly valuable in multi-criteria decision-making scenarios. When decision-makers face complex choices with numerous alternatives, these structures allow for the hierarchical merging of similar options without sacrificing decision quality. The framework naturally accommodates situations where both the presence of positive evidence and the absence of negative indicators matter equally, making it well-suited for practical business and engineering applications where comprehensive evaluation is essential.

8.2. Social network analysis

In social network contexts, bipolar fuzzy graph contractions offer a natural mechanism for understanding community structures. The method enables researchers to identify cohesive

groups by consolidating nodes with strong positive connections while simultaneously tracking conflict patterns through preserved negative links. This dual perspective becomes especially valuable when modeling influence dynamics in networks where opinions diverge, such as political discussions or competitive market environments where cooperation and opposition coexist.

8.3. Knowledge representation

Neutrosophic graph contractions provide a sophisticated approach to managing large-scale knowledge bases. As information systems grow, the ability to consolidate related concepts while maintaining their truth values, uncertainty levels, and falsity measures becomes crucial. This framework excels in domains where incomplete data is common and where contradictory information must be explicitly tracked rather than reconciled, such as intelligence analysis, medical diagnosis systems, or scientific databases where consensus has not yet been reached.

9. Conclusion and future work

This paper has established a comprehensive framework for vertex contraction operations across four major types of advanced fuzzy graphs. Our investigation demonstrates that intuitionistic fuzzy graph contractions successfully preserve hesitancy characteristics while maintaining predictable bounds on domination behavior. For Pythagorean fuzzy systems, we showed that the fundamental constraint remains intact throughout contraction operations, and remarkably, the process tends to enhance overall strength measures rather than diminish them. The independent nature of neutrosophic components allows for remarkably flexible optimization strategies, while bipolar fuzzy contractions naturally capture the essence of systems where positive and negative forces operate simultaneously.

Several promising directions emerge for future investigation. The theoretical framework developed here can be extended to spherical and picture fuzzy graphs, which represent the next generation of uncertainty modeling. From an algorithmic perspective, developing efficient approximation methods for identifying optimal contraction sequences remains an open challenge with significant practical implications. The temporal dimension presents another frontier, as many real-world networks evolve over time and require dynamic contraction strategies. Additionally, the integration of these techniques with modern deep learning architectures, particularly graph neural networks, offers exciting possibilities for handling uncertainty in machine learning contexts. Finally, the mathematical relationship between expansion and contraction operations deserves deeper exploration, potentially revealing fundamental dualities in fuzzy graph theory.

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