

Total Pages—10 PG/IVS/PHS/401.1 & 401.2/25

M.Sc. 4th Semester Examination, 2025

PHYSICS

PAPER — PHS-401.1 & 401.2

Full Marks : 50

Time : 2 hours

Answer all questions

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

PHS — 401.1

(Quantum Field Theory)

GROUP — A

1. Answer any two of the following : 2 × 2

(Turn Over)

(a) Write down the action for a $U(1)$ vector field coupled to complex scalar field. Identify the gauge symmetry.

(b) For the free Dirac theory, compute the conserved current j^μ corresponding to the global symmetry transformation $\psi(x) \rightarrow e^{i\alpha}\psi(x)$ (α is a constant) and show that $\partial_\mu j^\mu = 0$.

(c) The Lagrangian density for a vector field A^μ is given by

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}.$$

Derive the equation of motion for the vector field A^μ .

(d) If $Pa_p P^{-1} = a_{-p}$ where P is the parity operator and a_p is the annihilation operator for a real scalar field. Show that $P\phi(t, \mathbf{x})P^{-1} = \phi(t, -\mathbf{x})$.

GROUP - B

2. Answer any *two* of the following : 4 × 2

(a) For a free real scalar field theory compute the energy-momentum tensor $T^{\mu\nu}$. Show that T^{00} component corresponds to the Hamiltonian density.

(b) For a real scalar free field theory the normal-ordered Hamiltonian operator in terms of creation and annihilation operators is

$$H = \int \frac{d^3 p}{(2\pi)^3} E_p a_p^\dagger a_p$$

Compute $[H, a_p]$ and using the mode expansion for $\phi(x, 0)$, show that

$$e^{iHt} \phi(x, 0) e^{-iHt} = \phi(x, t).$$

(c) Given that the Dirac field transforms as

$$\psi \rightarrow \Lambda_{\frac{1}{2}} \psi, \text{ where } \Lambda_{\frac{1}{2}} = \exp \left[-\frac{i}{2} S^{\mu\nu} \omega_{\mu\nu} \right]$$

and $S^{\mu\nu} = \frac{i}{4}[\gamma^\mu, \gamma^\nu]$. Show that (i) $\bar{\psi}\psi$ transforms as a scalar under Lorentz transformation. (ii) $[\gamma^\mu, S^{\rho\sigma}] = [J^{\rho\sigma}]^\mu_\nu \gamma^\nu$ where $J^{\rho\sigma}$ are Lorentz generators corresponding to a vector representation.

(d) For a free complex scalar field theory obtain the integral representation of the retarded propagator $D_R(x-y)$, by performing the p_0 integral over a suitable contour. Further show that

$$D_R(x-y) = \Theta(x^0 - y^0) \langle 0 | [\phi(x), \phi(y)] | 0 \rangle.$$

GROUP - C

3. Answer any *one* of the following : 8 × 1

(a) (i) For a free complex scalar field theory obtain the expression for the Noether current $t^{\mu\nu}$ in terms of the fields ϕ and

ϕ^* corresponding to the global transformation $\phi(x) \rightarrow e^{-i\alpha}\phi(x)$. Compute the normal ordered charge : Q : in terms of the creation and annihilation operators $(a_p, b_p, a_p^\dagger, b_p^\dagger)$. 5

(ii) Consider the free Dirac theory. Obtain the equation of motion for the Dirac field ψ and find its solution in the rest frame i.e. $p = (m, 0, 0, 0)$. 3

(b) (i) Consider the free Dirac theory. Use the anticommutation relations among $(a_p^s, b_p^s, a_p^{s\dagger}, b_p^{s\dagger})$ and the mode expansion for the Dirac field ψ to show that $\{\psi_a(x, t), \psi_b^\dagger(y, t)\} = \delta^3(x - y)\delta_{ab}$. 4

(ii) For a free Maxwell theory, compute the energy momentum tensor $T^{\mu\nu}$ using the Noether's prescription and symmetrize the expression. 4

PHS - 401.2

(Particle Physics)

GROUP - A'

1. Answer any two of the following : 2×2

(a) For the $SU(2)$ isospin doublet of proton and neutron (p, n), find the $2 \otimes 2$ bound state wavefunctions and specify their isospins.

(b) How does $|K^0\rangle$ and $|\bar{K}^0\rangle$ transform under CP ? Construct linear combinations of the neutral Kaons $|K^0\rangle$ and $|\bar{K}^0\rangle$ that are eigenstates of CP .

(c) Using the conjugate representation of $SU(2)$ Isospin show that $T_+ \bar{u} = -\bar{d}$.

- (d) For the process $\rho^0 \rightarrow \pi^0 + \pi^0$ determine if the angular momentum is conserved.

GROUP - B

2. Answer any *two* of the following : 4×2

- (a) Given that the fermion field $\psi(t, x)$ transforms under Parity (P) as $\eta_a \gamma^0 \psi(t, -x)$, determine the transformations of $\bar{\psi} \gamma^\mu \psi$ under P .

- (b) The three states with $I_3 = Y = 0$ in $3 \otimes \bar{3}$ are $[u\bar{u} - d\bar{d}]$, $[d\bar{d} - s\bar{s}]$ and $[v\bar{v} - s\bar{s}]$. Determine the irreducible states i.e the wave functions of the π^0, η and the η' mesons in terms the constituent (u, d, s) quarks.

- (c) Consider the $su(n)$ Lie algebra where the

generators satisfy $[t_a, t_b] = i f_{abc} t_c$ with

$$\text{tr}(t_a, t_b) = \frac{1}{2} \delta_{ab} \quad [\text{Repeated indices are}$$

summed over]. Show from the algebra that the structure constants f_{abc} must be completely antisymmetric.

- (d) Consider the process $\pi^+ + d \rightarrow p + p$ where d denotes the deuteron (bound state of a proton and a neutron) in the ground state ($s = 1, \ell = 0$) and p the proton. Explain how the intrinsic parity of π^+ can be determined from the above process.

GROUP - C

3. Answer any *one* of the following : 8 × 1

- (a) (i) Draw the $I_3 - Y$ diagrams corresponding to 3 of SU(3), labeling the states

as (u, d, s) quarks. Find all the $\ell = 0$ bound states in the $3 \otimes 3$. Explain why these bound states do not correspond to physical particles. 4

(ii) Assuming only isospin symmetry find the ratio of the following cross sections :

$$\sigma(K^- + p \rightarrow \pi^+ + \Sigma^-); \sigma(K^- + p \rightarrow \pi^0 + \Sigma^0);$$

$$\sigma(K^- + p \rightarrow \pi^- + \Sigma^+).$$

(Table of Clebsch-Gordan coefficient will be provided on request). 4

(b) (i) Consider a theory of a complex scalar field ϕ coupled to electromagnetic

$$\text{field } \mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + |D_\mu \phi|^2 - V(\phi)$$

$$\text{with } V(\phi) = \mu^2 |\phi|^2 + \lambda (|\phi|^2)^2$$

where $D_\mu = \partial_\mu + ieA_\mu$. μ is the mass of the ϕ field and λ and e are coupling

constants. For $\mu^2 < 0$ show that the potential has a new minimum at $|\phi| = v \neq 0$. Expand the potential about the new minimum taking

$$\phi = \frac{1}{\sqrt{2}} e^{i\chi(x)/v} (v + h(x))$$

and find the masses of the $h(x)$ and $A_\mu(x)$ fields. 6

(ii) Starting from the fact that under Parity (P) the spin-1/2 creation operators transform as

$$P a_p^{s\dagger} P^\dagger = \eta_a^* a_p^{s\dagger} \quad \text{and} \quad P b_p^{s\dagger} P^\dagger = \eta_b^* b_p^{s\dagger},$$

explain why a fermion-antifermion pair has odd intrinsic parity. 2

[Internal Assessment — 10 Marks]
