M.Sc. 2nd Semester Examination, 2025

PHYSICS

(*Quantum Mechanics-II*)

PAPER – PHS-201

Full Marks : 25

Time : 1 hour

Answer all questions

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

GROUP – A

1. Answer any *two* questions : 2 × 2

(*a*) Verify that the dimension of the spaces $\{|j_1, j_2; m_1; m_2\rangle\}$ and $\{|j_1, j_2; j, m\rangle\}$ is equal to $(2j_1 + 1)(2j_2 + 1)$.

(Turn Over)

(*b*) A system consists of two spin-half particles. It is known that one of the particles has spin $S_z = \hbar/2$. If the spin of the combined system is measured, list the possible total spins and S_z values that may be obtained, along with their corresponding probabilities.

(*c*) In the tight binding approximation, the one-dimensional energy spectrum is written as $E(k) = E_0 - 2\Delta \cos ka$ where a is the periodicity of the one-dimensional lattice, k the wave number and Δ a constant. For $k \in [-\pi/a, \pi/a]$, draw the plot $E(k)$ vs k and identify the Brillouin zone. Further expand $E(k)$ for small values of k and find the mass m^* of the free particle.

(*d*) Suppose $|n\rangle$ is an eigenstate of a Hamiltonian H with energy eigenvalue E_n. Now consider the state $|\psi\rangle = \frac{1}{2}(1 \pm \pi)|n\rangle$ where π is the parity operator and $[H, \pi] = 0$. What conclusion can you draw about the state $|n\rangle$ if it is known that the energy spectrum is non-degenerate ?

GROUP – B

2. Answer any *two* of the following : 4×2

(*a*) For the spin-half system prove that the time reversal operator Θ can be written as $\Theta = \eta e^{-i\pi S_y} K$, where η is a phase factor, S_y is the spin-half operator and K a complex conjugation operator. Further show that for a spin-half state

$$|\psi\rangle, \Theta^2|\psi\rangle = -|\psi\rangle.$$

(*b*) A system having the Hamiltonian H_0 is perturbed by H_1 so that $H = H_0 + H_1$ where

$$H_0 = E_0\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \text{ and } H_1 = E_0\begin{pmatrix} 0 & \epsilon \\ \epsilon & 0 \end{pmatrix}$$

with $\epsilon << 1$. Find the first and second order shifts in the energy levels of H_0 using perturbation theory. Compute the eigenvalues of H exactly and compare your results.

(*c*) Estimate the ground-state energy of a one-dimensional simple harmonic oscillator using the trial wavefunction

$$\psi(x) = \sqrt{\alpha} e^{-\alpha|x|}. \left[\text{Use: } \int_0^\infty x^n e^{-ax} = n!/a^{n+1}\right]$$

(*d*) Show that the rotation matrix

$$D_n(\phi) = e^{-i\vec{S}\cdot\vec{n}\phi/\hbar}$$

for spin-half ($j = 1/2$) can be written in terms of Pauli matrices $\{\sigma_i\}$ as

$$D_n(\phi) = 1\cos(\phi/2) - i(\vec{\sigma}\cdot\vec{n})\sin(\phi/2).$$

Use this result to show that

$$|+\rangle_n = \cos(\beta/2)\,|+\rangle_z + e^{i\alpha}\sin(\beta/2)|-\rangle_z$$

where β and α are the polar and azimuthal angles, respectively, of the unit vector $\vec{n}$.

GROUP – C

3. Answer any *one* of the following : 8×1

(*a*) (*i*) A system with an unperturbed Hamiltonian H_0 is subjected to a perturbation V which are given as

$$H_0 = E_0\begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \;;\; V = \lambda E_0\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

A. Find the eigenstates of the unperturbed Hamiltonian H_0 and the exact eigenvalues of the total Hamiltonian $H = H_0 + V$ by directly diagonalizing H. 2

B. Now find the energy eigenvalues of H using non-degenerate/ degenerate perturbation theory upto first order in λ and compare your result with the exact values. 2

(*ii*) Find the matrix representation of the angular momentum operator J_x for $j = 1$. Use this result to compute the energy eigenvalues for the Hamiltonian $H = aJ_z + bJ_x^2$ where a and b are constants. 4

(*b*) (*i*) A particle of mass m is confined within a one-dimensional infinite potential well of width L with the walls at $x = 0$ and $x = L$. A perturbation to the Hamiltonian of the form $V_\lambda = \lambda V_0 \sin(\pi x / L)(\lambda << 1)$ is acted on the system. Compute the energy levels of the n-th excited states to the first order in perturbation theory. 4

(*ii*) Consider the angular momentum addition of two spin-half particles. Using the $J_\pm$ ladder operators and orthogonality relations, rewrite all the states in the $|j_1, j_2; j, m\rangle$ basis in terms of those in $|j_1, j_2; m_1, m_2\rangle$ basis. You must show all steps involved in the computation. 4

[Internal Assessment — 5 Marks]