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Genetic Algorithm Approach to Entropy Matrix Goal Game via Fuzzy Programming

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ABSTRACT

In this paper the entropy function of the matrix game has been considered as one of the objectives and formulate a new game model, named Entropy Matrix Goal Game Model. Non-linear programming model for each player has been established. The equilibrium condition on matrix game is based on expected pay-off, so this equilibrium condition might be violated by mixed strategies when replications are not allowed. To avoid this inconvenience, we have considered G-goal security strategy. The concept of G-goal Security Strategy which assures the property of security against an opponent's deviation in strategy has been introduced. To solve these models, apply fuzzy programming technique through proposed Genetic Algorithm. A numerical example is included to illustrate the results in this paper.

Keywords: Matrix Game, Goal, G-goal Security Strategy, Entropy, Genetic Algorithm.

1. Introduction

Now a days more attention has been paid to matrix game because this approach represents better real-world application of game theory. In fact, each competitive situation that can be modelled as a scaler zero-sum game. In this situation, once the same strategy has to be used in different scenarios, conflicting interest appear between different decision markers as well as within each individual. For instance, the production policies of two firms which are competing for a market can be seen as a scalar game.

Every probability distribution has some ``uncertainty" associated with it. The concept of ``entropy" is introduced to provide a quantitative measure of

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uncertainty. Entropy models are emerging as valuable tools in the study of various social and engineering problems. The maximum entropy principle initiated by Jaynes'[16] is a powerful optimization technique of determining the distribution of random system in the case of partial or incomplete information or data available in the system. The principle has now been broadened and extended and has found wide applications in different fields of science and technology.

In matrix game we see that family of probability distributions of strategies of every players are consistent with given information, we choose the distribution whose uncertainty or entropy is maximum. Each player is interested in making moves which will be as surprising and as uncertain to the other player as possible. For this reason, the players are involved in maximizing their entropies. Consequently, in the mathematical models of matrix game with certain goals, incorporate an entropy function as one of their objectives. This model is known as multicriteria entropy matrix goal game model.

In conventional mathematical programming, the coefficients or parameters of the bimatrix game model is assumed to be deterministic and fixed. But, there are many situations where they may not be exactly known i.e., they may have some uncertainty in nature. Thus the decision-making methods under uncertainty are needed. From this point of view, the fuzzy programming has been incorporated in decision-making method. In fuzzy programming problems, the coefficients, constraints and the goals are viewed as fuzzy numbers or fuzzy sets. In decision-making process, the fuzzy set theory was initiated by Bellman and Zadeh [26]. After that, Tanaka [25] applied the concepts of fuzzy sets to decision making problems by considering the as fuzzy goals. Later on, Zimmermann [27] showed that the classical algorithms could be used to solve multi-objective fuzzy linear programming problems.

In this paper, some references are presented including their work.

Ghose and Prasad [13], have been proposed as a solution concept based on Pareto-optimal security strategies for these games. They also introduced the concept based on the similarity with security levels determined by the saddle points in scalar matrix games. This concept is independent of the notion of equilibrium so that the opponent is only taken into account to establish the security levels for one's own payoffs. When it is used to select strategies, the concept of security levels has important property that the payoff obtained by these strategies cannot be diminished by the opponent's deviation in strategy. Roy [22], has presented the study of two different solution procedures for the two-persons bimatrix game. The first solution procedure is applied to the game on getting the probability to achieve some specified goals along the player's strategy. The second specified goals along with the player's strategy by defining the fuzzy membership function defined on the pay-off matrix of the bimatrix game. Das and Roy [5], have proposed a new solution concept by considering the entropy function to the objectives of the player. These models are known as entropy optimization models on two persons zero sum game. Solution concept is based on the Kuhn Tucker conditions, Maximum Entropy Principle [16], and Minimum Cross-Entropy Principle [18]. Without considering the pay-offs of the players, we have shown that the optimal strategy and the value of the game for each player are equivalent to the results of classical game.

In this paper we propose the models not only considering the pay-offs but their goals and entropies of the players. Clearly, the bi-criteria are appeared in these model. G -goal Security Strategy for these bi-criteria goal game is defined. Several solution techniques are proposed to solve them.

2. Mathematical Model

In a matrix game, a payoff matrix of the players PI and PII is defined as follows:

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \tag{1}$$

The players are represented by PI (the maximizer, who chooses rows) and PII (the minimizer, who chooses columns). As usual, the mixed strategy for players PI and PII are

$$Y = \{ y \in R^m ; \sum_{i=1}^{m} y_i = 1; y_i \geq 0,\ i = 1,2,\dots,m \} \tag{2}$$

$$Z = \{ z \in R^n ; \sum_{j=1}^{n} z_j = 1; z_j \geq 0,\ \ j = 1,2,\dots,n \} \tag{3}$$

We remark that the pure strategies for both players are the extreme points of Y and Z.

We analyze the problem under PI point of view.

Let $G \in R$, be a goal specified by PI. In order to determine the strategies based on the probability to achieve the goal G, we formulate a zero-sum game called matrix goal game.

Definition 1. The expected payoff of the matrix goal game, with goal G and matrix $A = (a_{ij}),$ for each strategy pair $y \in Y$ and $z \in Z,$ is

$$v(y,z) = y^t A_G z \tag{4}$$

where

$$A_G = (\delta_{ij}),\ \ i = 1,2,\dots,m,\ \ j = 1,2,\dots,n, \tag{5}$$

$$\delta_{ij} = \begin{cases} 1 & if \quad a_{ij} \geq G \\ 0 & if \quad otherwise \end{cases} \tag{6}$$

and $v(y,z)$ is the probability to get at least G in the game when PI plays strategy $y \in Y$ and PII plays strategy $z \in Z$.

As $v(y,z)$ depends on the strategy that PII plays, we will consider this probability in the worst case; i.e. we assume that PII will choose a strategy $z \in Z$ that gives the minimum value of $v(y,z)$. Then, for each $y \in Y$, PI will get

$$v(y) = \min_{z\in Z} v(y,z) = \min_{z\in Z} y^t A_G z = \min_{1\le j\le n} \sum_{i=1}^{m} y_i \delta_{ij} \tag{7}$$

Definition 2. The G-security level for PI of a matrix game with matrix $A=(a_{ij})$ is the maximum probability that PI can guarantee to himself for obtaining goal G, irrespective the action of PII. It is given by

$$v = \max_{y\in Y} v(y) = \max_{y\in Y}\min_{z\in Z} v(y,z) = \max_{y\in Y}\min_{z\in Z} y^t A_G z \tag{8}$$

Definition 3. A strategy $y \in Y$ is a G-goal security strategy (GGSS) for PI if $v = \min_{z\in Z} y^t A_G z$, where v is the G-goal security level of the matrix G-goal game.
The following result characterized GGSS and gives a procedure to solve matrix goal game.

Theorem 1. The G-goal security strategy and maximum probability to obtain at least goal G are given by the solution of the two-person zero-sum game whose pay off matrix is the matrix A_G.

Proof. For $y \in Y$ and $z \in Z$, the expected pay-off of the zero-sum game with pay-off matrix A_G is

$$v(y,z) = y^t A_G z = \sum_{i=1}^{m}\sum_{j=1}^{n} y_i z_j \delta_{ij} \tag{9}$$

For each $i = 1,2,\ldots,m$, let Z_i be the sum of the z_j's for the columns that have an element equal to 1 in the i^{th} row, i.e.,

$$Z_i = \sum_{j=1}^{n} z_j \delta_{ij} \qquad i = 1,2,\ldots,m \tag{10}$$

The probability of obtaining at least goal G when the players use strategies y and z, respectively, is

$$p = \sum_{i=1}^{m} y_j Z_i = \sum_{i=1}^{m}\sum_{j=1}^{n} y_j z_j \delta_{ij} = v(y,z) \tag{11}$$

Hence the theorem.

Similar definition and theorem can be drawn for PII point of view.
An optimal strategy y and maximum probability v for getting at least goal G of

player PI obtained by the solution of the following linear programming model.

Model 1

$$\max : v$$

subject to

$$\sum_{i=1}^{m} y_i \delta_{ij} \geq v, \ j = 1,2,\ldots,n \tag{12}$$

$$\sum_{i=1}^{m} y_i = 1; \ y_i \geq 0, \ i = 1,2,\ldots,m \tag{13}$$

Similar model for player PII is as follows.

Model 2

$$\min : w$$

subject to

$$\sum_{j=1}^{n} z_j \delta_{ij} \leq w, \ i = 1,2,\ldots,m \tag{14}$$

$$\sum_{j=1}^{n} z_j = 1; \ z_j \geq 0, \ j = 1,2,\ldots,n \tag{15}$$

Again each player is interested in making moves which will be as surprising and as uncertain to the other player as possible. For this reason, the two players are involved in maximizing their entropies. The mathematical form of entropies are as follows:

$$H_1 = -\sum_{i=1}^{m} y_i \, ln(y_i) \tag{16}$$

$$H_2 = -\sum_{j=1}^{n} z_j \, ln(z_j) \tag{17}$$

i.e. they are interested in making their strategies as spread out as possible. However they are primarily interested in maximizing their expected payoffs.

2.1 Entropy Matrix Game Models

With out any loss of generality, let us combine the **Model 1** and the entropy function (16), we formulated a new mathematical model namely Entropy Matrix Goal Game Model which is a multi-objective non-linear programming model. This model is defined for player PI as follows:

Model 3

$$\max : v$$
$$\max : H_1$$

subject to

$$\sum_{i=1}^{m} y_i \delta_{ij} \geq v, \ \ j = 1,2,\ldots,n \tag{18}$$

$$H_1 = -\sum_{i=1}^{m} y_i \ ln(y_i) \tag{19}$$

$$\sum_{i=1}^{m} y_i = 1; \ \ y_i \geq 0, \ \ i = 1,2,\ldots,m \tag{20}$$

Similarly the Entropy Matrix Goal Game Model for player PII is as follows:

Model 4

$$\min : w$$
$$\max : H_2$$

subject to

$$\sum_{j=1}^{n} z_j \delta_{ij} \leq w, \ \ i = 1,2,\ldots,m \tag{21}$$

$$H_2 = -\sum_{j=1}^{n} z_j \ ln(z_j) \tag{22}$$

$$\sum_{j=1}^{n} z_j = 1; \ \ z_j \geq 0, \ \ j = 1,2,\ldots,n \tag{23}$$

3. Solution Procedures

In previous section, we have seen that, **Model 3** and **Model 4** are both multi-objective non-linear programming (MONLP) problem. To get a satisfactory solution of the above models we have introduced the solution techniques which are defined as follows.

3.1. Fuzzy Programming

In fuzzy programming, we first construct the membership function for each objective function in **Model 3**. Let $\mu_{11}(v)$, $\mu_{12}(H_1)$ be the membership functions for both objectives respectively and they are defined as follows:

$$\mu_{11}(v) = \begin{cases} 0 & if \quad v \le v^- \\ \dfrac{v - v^-}{v^+ - v^-} & if \quad v^- \le v \le v^+ \\ 1 & if \quad v \ge v^+ \end{cases} \tag{24}$$

and

$$\mu_{12}(H_1) = \begin{cases} 0 & if \quad H_1 \le H_1^- \\ \dfrac{H_1 - H_1^-}{H_1^+ - H_1^-} & if \quad H_1^- \le H_1 \le H_1^+ \\ 1 & if \quad H_1 \ge H_1^+ \end{cases} \tag{25}$$

where v^+, v^- represents maximum and minimum value of v and H_1^+, H_1^- represents maximum and minimum value of H_1 for player PI.

Similarly we can construct the membership function for each objective function in **Model 4**. Let $\mu_{21}(w)$, $\mu_{22}(H_2)$ be the membership functions for both objectives respectively and they are defined as follows:

$$\mu_{21}(w) = \begin{cases} 1 & if \quad w \le w^- \\ \dfrac{w^+ - w}{w^+ - w^-} & if \quad w^- \le w \le w^+ \\ 0 & if \quad w \ge w^+ \end{cases} \tag{26}$$

and

$$\mu_{22}(H_2) = \begin{cases} 0 & if \quad H_2 \le H_2^- \\ \dfrac{H_2 - H_2^-}{H_2^+ - H_2^-} & if \quad H_2^- \le H_2 \le H_2^+ \\ 1 & if \quad H_2 \ge H_2^+ \end{cases} \tag{27}$$

where w^+, w^- represents maximum and minimum value of w and H_2^+, H_2^- represents maximum and minimum value of H_2 for player PII.

To conversion in a single objective non-linear model from multi-objective non-linear model, we have introduced the concept of fuzzy programming with the help of (32), (33) and the **Model 3**, then we formulated the following single objective non-linear model and this model is denoted by **Model 7**.

Model 7

$$\max : \lambda$$

subject to

$$\lambda \le \frac{v - v^-}{v^+ - v^-} \tag{28}$$

$$\lambda \le \frac{H_1 - H_1^-}{H_1^+ - H_1^-} \tag{29}$$

$$\sum_{i=1}^{m} y_i \delta_{ij} \ge v, \ j = 1,2,\ldots,n \tag{30}$$

$$H_1 = -\sum_{i=1}^{m} y_i \ ln(y_i) \tag{31}$$

$$\sum_{i=1}^{m} y_i = 1; \ y_i \ge 0, \ i = 1,2,\ldots,m \tag{32}$$

and for player II, the similar model may be formulated by the help of **Model 4** and (35) and (36) and this model is denoted by **Model 8**.

Model 8

$$\max : \delta$$

subject to

$$\delta \le \frac{w^+ - w}{w^+ - w^-} \tag{33}$$

$$\delta \le \frac{H_2 - H_2^-}{H_2^+ - H_2^-} \tag{34}$$

$$\sum_{j=1}^{n} z_j \delta_{ij} \le w, \ i = 1,2,\ldots,m \tag{35}$$

$$H_2 = -\sum_{j=1}^{n} z_j \ ln(z_j) \tag{36}$$

$$\sum_{j=1}^{n} z_j = 1; \ z_j \ge 0, \ j = 1,2,\ldots,n \tag{37}$$

Now to solve the above two models, **Model 7** and **Model 8**, we apply the Genetic Algorithm which is depicted in the next section.

3.2 Genetic Algorithm

Now, we shall develop an algorithm for determining the v^+, v^- and H_1^+, H_1^-. The stepwise procedure of GA is shown as follows:

Step 1 : Initialize the parameters of GA of the proposed Entropy Bimatrix Goal Game model.

Step 2: $t = 0$ (t represents the number of current generation.)

Step 3: Initialize $P(t)$ [$P(t)$ represents the population at the t-th generation].

Step 4: Evaluate $P(t)$
Step 5: Find optimal result from $P(t)$.
Step 6: $t = t+1$.
Step 7: If ($t >$ maximum generation number) go to Step 13.
Step 8: Alter $P(t)$ by mutation.
Step 9: Evaluate $P(t)$.
Step 10: Find optimal result from $P(t)$.
Step 11: Compare optimal results of $P(t)$ and $P(t-1)$ and store better one.
Step 12: Go to Step 6.
Step 13: Print optimal result.
Step 14: Stop.

To implement the above GA for the proposed model, the following basic components are considered:(i) Parameters of GA, (ii) chromosome representation, (iii) initialization, (iv) evaluation function, (v) selection process, (vi) genetic operaters(crossover and mutation). Which are defined as follows.

- **Parameters of GA :** GA depends on different parameters like population size(POPSIZE), probability of crossover(PCROS), probability of mutation(PMUTE) and maximum number of generation (MAXGEN). In our present study, we have taken the value of these parameters as follows:

POPSIZE= 25 PCROS= 0 PMUTE= 0.6 MAXGEN= 80

- **Chromosome representation**

The chromosome is defined as $(y_1^a, y_2^a, y_3^a, \ldots, y_m^a)$ where $y_i^a \in Y, i = 1,2,3,\ldots,m$.

The revised genetic algorithm is illustrated as follows:

- **Initialization**

In this study; y_1^a, y_2^a, $\ldots, y_{m-1}^a$ are randomly given values. Please notice a chromosome must satisfy that $y_1^a + y_2^a + y_3^a + \ldots + y_m^a = 1$. This process is randomly generating each element in $(y_1^a, y_2^a, y_3^a, \ldots, y_m^a)$ and $y_1^a + y_2^a + y_3^a + \ldots + y_m^a = 1$; Moreover the number of chromosome is limited to 25 when each new run begins.

- **Evaluation function**

Once $(y_1^a, y_2^a, y_3^a, \ldots, y_m^a)$ is determined, the corresponding v^a and H_1^a can be computed by (12) and (16).

- **Optimum 1**

For 25 chromosomes we get 25 set of values of v^a and H_1^a. Among these 25 values of v^a we stored maximum value and minimum value in v^{a+} and v^{a-}, respectively. In each iteration, these maximum and minimum values are globally stored in $VMAX1, VMIN1$, respectively. Similarly, among 25 values of H_1^a we stored maximum value in H_1^{a+} and minimum value in H_1^{a-} and they are also globally stored in another locations $HMAX1$ and $HMIN1$ respectively, in each iteration.

- **Selection**

Selection procedure is omitted because here objectives are more than one so we can not choose the weaker chromosome that serve worst value for all objectives.

- **Crossover**

Since it is not easy to design a crossover between chromosomes for satisfying that $y_1^a + y_2^a + y_3^a + \ldots + y_m^a = 1$, therefore no crossover is applied in this study.

- **Mutation**

It is applied to single chromosome. It is designed as an order of elements in $(y_1^a, y_2^a, y_3^a, \ldots, y_m^a)$ by randomly determined cut-point. Consider an example: if the original chromosome is $(y_1^a, y_2^a, y_3^a, \ldots, y_m^a)$ and cut-point is randomly determined between the string: y_1^a and y_2^a, $y_3^a, \ldots, y_m^a$, then moreover newly mutated chromosome $(y_1^{'}, y_2^{'}, y_3^{'}, \ldots, y_m^{'})$ is $(y_2^a, y_3^a, \ldots, y_m^a, y_1^a)$.
In each iteration the (POPSIZE * PMUTE) number of chromosome are chosen for mutation.

- **Iteration**

The number of iteration is set to 80 runs, each of which begins with the different random seed.

- **Optimum 2**

After completing all the iterations, we determine v^+ as the maximum among all $VMAX1$ and v^- as the minimum among all $VMIN1$. Also, H_1^+ is the maximum among all $HMAX1$ and H_1^- is the minimum among all $HMIN1$, are determined.

Similar technique apply for player PII.

4. Numerical Example

Let us consider a matrix game as follows:

$$A = \begin{bmatrix} 7 & 9 & 1 & 3 \\ 5 & 2 & 4 & 8 \\ 4 & 6 & 3 & 9 \\ 8 & 2 & 5 & 6 \end{bmatrix} \tag{38}$$

Let $G = 5$, be a goal specified by PI. Then

$$A_G = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \tag{39}$$

The following results are summarized in Table-1 which are computed by Genetic Algorithm.

	maximum value	*minimum value*
v	$v^+ = .929850$	$v^- = .082750$
w	$w^+ = .994700$	$w^- = .501950$
H_1	$H_1^+ = 1.3565053$	$H_1^- = 0.50195$
H_2	$H_2^+ = 1.356053$	$H_2^- = .413787$

Table-1

With the help of above values in Table-1 the mathematical **Model 7** and **Model 8** for the player PI and PII respectively are redefined as follows:

Model 9

$$\max : \lambda$$

subject to

$$\lambda \le \frac{v - .082750}{.929850 - .082750} \tag{40}$$

$$\lambda \le \frac{H_1 - 0.50195}{1.3565053 - 0.50195} \tag{41}$$

$$\sum_{i=1}^{m} \delta_{ij} y_i \ge v, \ \ j = 1,2,\ldots,n \tag{42}$$

$$H_1 = -\sum_{i=1}^{m} y_i \ ln(y_i) \tag{43}$$

$$\sum_{i=1}^{m} y_i = 1;\ y_i \geq 0,\ i = 1,2,\ldots,m \tag{44}$$

and

Model 10

$$\max : \delta$$

subject to

$$\delta \leq \frac{.994700 - w}{.994700 - .501950} \tag{45}$$

$$\delta \leq \frac{H_2 - .413787}{1.3565053 - .413787} \tag{46}$$

$$\sum_{j=1}^{n} \delta_{ij} z_j \leq w,\ i = 1,2,\ldots,m \tag{47}$$

$$H_2 = -\sum_{j=1}^{n} z_j\ ln(z_j) \tag{48}$$

$$\sum_{j=1}^{n} z_j = 1;\ z_j \geq 0,\ j = 1,2,\ldots,n \tag{49}$$

4.1. Results

Since the non-linear **Model 9** and **Model 10** are not easy to solve by any linear programming technique, genetic algorithm may be considered and developed as an efficient approach for **Model 9** and **Model 10**. The aspiration level with two objectives for a given solution, λ^* and δ^* are obtained from above models (**Model 9** and **Model 10**) by the help of Lingo package. The optimal solutions for player PI and player PII are represented in the following Table-2.

aspiration level	*optimal value*	*maximum entropy*	*optimal strategy*
$\lambda^* = 0.4927$	$v^* = 0.5$	$H_1^* = 1.0302$	$y^* = (0.2936, 0.00, 0.2064, 0.5)$
$\delta^* = 0.9227$	$w^* = 0.5400$	$H_2^* = 1.2832$	$w^* = (0.18, 0.18, 0.46, 0.18)$

Table - 2

5. Conclusions

This paper presents the study of two-person matrix(zero-sum) goal game and analyzes the game in entropy environment. Using goal, we consider a solution not only strategy played by the player, but also the probability of getting at least goal value. Therefore, with this approach, each player has information about the probability of achieving the possible outcomes of the entropy matrix game.

A methodology to obtain the GGSS is developed through fuzzy based genetic algorithm and we have shown that all these strategies, together with their associated probabilities, can be obtained as a G-goal efficient solution of the formulated models. Also we conclude that these models are highly significant to the real world practical problem.

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