2025

M.Sc. 3rd Semester Examination

APPLIED MATHEMATICS

Paper : MTM - 306A/B/C

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Paper : MTM - 306A

(Operational Research Modelling - I)

1. Answer any ***four*** questions : $2\times4=8$

   (a) What do you mean by a steady state condition in a queueing system? Explain.

   (b) What is meant by stochastic simulation?

   (c) Write short notes on :

   (i) Dangling

   (ii) Redundancy

   (iii) Merge event

   (d) Define the terms : 'Lead time' and 'Planning horizon'.

P.T.O.

(e) What do you mean by the term 'present worth factor'?

(f) Draw a flow of goods of any real-life supply chain.

2. Answer any *four* questions : 4×4=16

(a) In the $(M/M/1:\infty/FCFS/\infty)$ queue model, find the probability of queue size greater than or equal to a given number, say N, and the expected line length.

(b) Write a note on the major advantages and disadvantages of simulation.

(c) State Fulkerson's rules for numbering events in a network.

(d) Derive the conditions that determine the optimal time period of replacing an item whose maintenance cost is increased with time but money value is unchanged.

(e) A baking company sells cake by its weight in kg. It makes a profit of Rs. 3 on every kg sold on the day it is baked. It disposes of all cakes not sold on the date they are baked, at a loss of Rs. 1.50 per kg. If the demand is known to be rectangular distribution between 2000 and 5000 kg, determine the optimal daily amount baked.

(f) Find the optimum order quantity for a single product of which the price breaks are as follows :

Range of quantity	Unit price (Rs)
$0 < Q < 50$	20.00
$50 \le Q < 120$	16.00
$120 \le Q$	14.5

The monthly demand for the product is 450 units. The carrying cost is 20% of the unit price of the product and cost of ordering is Rs. 30.50 per month.

3. Answer any *two* questions : 8×2=16

(a) For the queuing model $(M/M/1):(N/FCFS/\infty)$, derive and solve the steady state differential difference equation.

(b) Explain the generation of random numbers using the power residue method. Illustrate the method with a complete numerical example.

(c) Derive the conditions of finding optimal order quantity of probabilistic inventory model of instantaneous demand without set-up cost.

(d) The following data describe three inventory items. Determine the economic order quantity for each of the three items to be accommodated with in total available storage area of 700 sq. ft.

P.T.O.

Paper : MTM - 306B

(Dynamical Oceanology : Advanced Wave Hydrodynamics)

1. Answer any ***four*** questions : 2×4=8

(a) Define the term 'standing wave' with an example.

(b) Write down the basic difference between tidal waves and surface waves.

(c) State the criteria for shallow water, intermediate and deep-water waves and draw their graphical representations.

(d) Write down a short note on 'inertial waves'.

(e) Define the term 'group velocity'.

(f) Define the term "Rossby radius" and write down its value for shallow ocean whose depth is 100 m.

2. Answer any ***four*** questions : 4×4=16

(a) Derive depth-average continuity equations for shallow water theory.

(b) Prove that the path of the particle describes a straight line with simple harmonic motion of stationary wave when argument is constant and amplitude is decreased as move downwards from the surface of a channel of finite depth.

P.T.O.

Item	Set-up Cost (Rs.)	Demand (units per year)	Cost per unit (Rs.)	Storage area required per unit (sq. ft.)
1	100	2000	10	0.5
2	200	5000	20	0.6
3	75	10000	5	0.3

Where inventory carrying charge of 20% of average inventory valuation per year and shortages are not allowed.

[Internal Assessment - 10 marks]

(c) Define capillary waves and prove that the relation for surface tension (T) of capillary waves is $\frac{\partial^2\phi}{\partial t^2} - g\frac{\partial\psi}{\partial x} + \frac{T}{\rho}\frac{\partial^3\psi}{\partial x^3} = 0$, where symbols have their usual meaning.

(d) Prove that the bottom stream-lines are constant and upper surface is stream-lines free for steady motion of progressive wave.

(e) Determine the longitudinal velocity expression for linear waves in the absence of rotation.

(f) Derive the speed of propagation $C^2 = \left(\frac{g}{m} + \frac{Tm}{\rho}\right)$ due to effect of capillary on surface waves on a channel of finite depth for deep water where symbols have their usual meaning.

3. Answer any *two* questions : 8×2=16

(a) (i) Write down the pressure condition for the wave propagation at the free surface. 3

(ii) Prove that the total energy of progressive wave is $\frac{1}{2}\rho g a^2 \lambda$ where a, λ is the wave amplitude and wave length respectively. 5

(b) (i) Prove that for waves on deep water the group velocity is half of the wave velocity while for shallow water group velocity is equal to the wave velocity. 4

(ii) Show that a submarine whose depth is half a wave length would hardly notice the motion of a submarine due to surface waves. 4

(c) Determine the velocity of propagation for steady motion of progressive waves of an interface. Hence, show that, if velocity of the wind is just great enough to prevent the propagation of wave of length against it, the velocity of propagation of waves of wind is $2C\left(\frac{\sigma}{1+\sigma}\right)^{1/2}$ where σ is the specific gravity of the air and C is the wave velocity when no wind is present. 5+3

(d) Derive Klein-Gordon equation for long surface wave and hence, prove that geostrophic velocity in transverse direction is given by $\bar{v} = \frac{gh}{2C_0}\exp(-|x|/a)$, where symbols have their usual meaning. 4+4

[Internal Assessment - 10 marks]

P.T.O.

Paper : MTM - 306C

(Computational Fluid Dynamics)

1. Answer any ***four*** questions : $2\times4=8$

(a) How many types of variable arrangement are in the Computational Fluid Dynamics? Discuss them by arranging the x- and y-components of velocities and pressure.

(b) Draw the p-control volume and hence apply finite volume method to two-dimensional continuity equation for incompressible viscous fluid flow.

(c) Derive the expression for u_w (the value of u-velocity at the west face of the control volume) for two points upwind scheme in non-uniform grids, and hence simplify this expression for uniform grid.

(d) A generalization of discretization of one-dimensional unsteady heat conduction equation (with thermal conductivity α) $\frac{\partial T}{\partial t}=\alpha\frac{\partial^2 T}{\partial x^2}$ can be obtained by writing

$$\frac{\Delta T_j^{n+1}}{\Delta t}-\alpha\left[(1-\beta)L_{xx}T_j^n+\beta L_{xx}T_j^{n+1}\right]=0$$

Symbols have their usual meaning. Find the value of β for which the above scheme becomes FTCS, Crank-Nicolson and fully implicit.

(e) Define the Courant-Friedrichs-Lewy (CFL) number 'C' and what is the physical meaning of CFL condition $C \leq 1$?

(f) Draw the schematic of the finite difference solution process.

2. Answer any ***four*** questions : 4×4=16

(a) How many types of time integration methods possible? Write advantages and disadvantages of any two.

(b) Discretise the one-dimensional unsteady heat conduction equation $\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$ using Richardson scheme and write its truncation error. Also draw the schematic diagram to show the points needed for this discretization.

(c) Draw the control volume for u-velocity and place the variables (velocity and pressure) on the respective faces for Quadratic Upwind Interpolation for Convective Kinematics (QUICK).

Then write the expressions for u-velocity at the east and west faces of the said control volume for both negative and positive fluxes. Also with help of appropriate symbol, compose two expressions for negative and positive fluxes into one for both east and west faces separately.

P.T.O.

(d) (i) Draw the (j, k)th control volume for irregular grid. Apply the finite volume method on the following equation for the aforesaid control volume

$$\frac{\partial q}{\partial t}+\frac{\partial F}{\partial x}+\frac{\partial G}{\partial y}=0 \text{ ----(1)}$$

where the symbols have their usual meaning.

(ii) If the global grid is uniform and coincides with lines of constant x and y, then show that the above discretisation coincides with a centered difference representation for the spatial terms of the above original differential equation (i).

3+1

(e) Roughly the truncation error E can be written as $E = A\left(\Delta x^{k}\right)$, with symbols have their own meaning. By plotting on a log-scale, discuss the convergence rates of five-points and three-points symmetry formulas for first order derivative. Also discuss when do you need to apply higher or lower order formulas for derivative on a non-uniform grid.

(f) Discretise the PDE $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ using the Forward Time Central Space (FTCS) and hence discuss the consistency of this discretise equation with that of the governing equation.

3. Answer any *two* questions : 8×2=16

(a) (i) Let the non-dimensional form of x-component Navier-Stokes equations for laminar two-dimensional incompressible fluid flow be

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right).$$

where Re is the Reynolds number. Convert it to its conservation form with the help of continuity equation.

(ii) Draw the control volume for u-velocity.

(iii) Hence apply the finite volume method to the above x-momentum equation for space derivative and three-points backward finite difference formula for time derivative to get the discretise equation in terms of fluxes only.

2+2+4

(b) (i) Apply finite volume method to the one-dimensional steady transport equation.

(ii) Using a linear interpolation for the advection term and second order central difference scheme for the diffusion term to calculate unknowns on each faces, derive the most simplified form of the discretised equation derived in the above part-(a).

P.T.O.

(c) Consider the equation

$$\frac{\partial U}{\partial t}=\frac{\partial^2 U}{\partial x^2},\ 0<x<1,\ t>0$$ where U is known

for $0\leq x\leq 1$ when $t=0$, and at $x=0$ and 1 when $t>0$.

Obtain the discretise form of the above equation using FTCS scheme. Discuss the analytical treat of convergence of the approximate solution u to the exact solution U when spacing tends to zero. 2+6

(d) Derive the homogeneous algebraic equation for the error term η_j^n introduced at grid points (j, n), where $j = 2, 3, \cdots, J-1$ and $n = 0, 1, 2, \cdots$, for the FTCS scheme when applied to $\frac{\partial u}{\partial t}+a\frac{\partial u}{\partial x}=0$, when a is the known velocity and u is passive scalar unknown. ······(1)

Hence discuss the stability of the FTCS scheme using Von Neumann method.

[Internal Assessment - 10 marks]