2025

M.Sc. 3rd Semester Examination

APPLIED MATHEMATICS

Paper : MTM - 305 A / B

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Paper : MTM - 305A

(Advanced Optimization)

1. Answer any *four* questions : $2\times4=8$

   (a) What is the basic idea of dynamic programming?

   (b) Define quadratic programming problem. What is the sufficient condition for it?

   (c) What is the necessity of study the "Post Optimality Analysis"?

   (d) What are the basic differences between analytical methods and numerical methods related to optimization?

   (e) Discuss the different types of achievement in goal programming problem.

P.T.O.

(f) Find the conjugate directions of the following real symmetric matrix :

$$\begin{pmatrix} 2 & 4 \\ 4 & 1 \end{pmatrix}$$

2. Answer any ***four*** questions : 4×4=16

(a) Discuss the following effects in post optimality analysis :

(i) Addition of a new variable.

(ii) Deletion of an existing variable.

(b) Minimize $f(x)=\frac{|x-3|}{2}$ in the interval [0, 5] by Golden section method upto five experiments.

(c) A firm produces two products A and B. Each product must be processed through two departments namely 1 and 2. Department 1 has 30 hours of production capacity per day, and department 2 has 60 hours. Each unit of product A requires 2 hours in department 1 and 6 hours in department 2. Each unit of product B requires 3 hours in department 1 and 4 hours in department 2. Management has established the following goals it would like to achieve in determining the daily product mix :

P_1 : The joint total production at least 10 units.

P_2 : Producing at least 7 units of product B.

P_3 : Producing at least 8 units of product A.

Formulate this problem as a goal programming model.

(d) Using steepest descent method minimize $f = x_1^2 + 2gx_1 + x_2^2 + 2fx_2 + c$ starting from any arbitrary point.

(e) Derive the Kuhn-Tucker conditions for quadratic programming problem.

(f) Define unimodal maximization and minimization function. Discuss the elimination of an interval from the interval of uncertainty by unimodal property.

3. Answer any *two* questions : 8×2=16

(a) Solve the following IPP using branch-and-bound method :

Maximize $z = 3x_1 + 4x_2$
subject to the constraints
$4x_1 + 6x_2 \leq 15$
$x_1, x_2 \geq 0$ and integers.

P.T.O.

(b) The optimal result of the LPP

Maximize $z = 2x_1 + 2x_2$
subject to
$5x_1 + 3x_2 \le 8$
$x_1 + 2x_2 \le 4$
and $x_1, x_2 \ge 0$

is given in the following table :

C_B	X_B	b	y_1	y_2	y_3	y_4
2	x_1	4/7	1	0	2/7	−3/7
2	x_2	12/7	0	1	−1/7	5/7
$Z_j - C_j$			0	0	2/7	4/7

Find the optimal results after addition of the following constraints :

(i) $x_1 + x_2 \le 3$

(ii) $x_1 + x_2 \le 2$

(c) Use dynamic programming to show that $p_1 \log p_1 + p_2 \log p_2 + \cdots + p_n \log p_n$ subject to the constraint $p_1 + p_2 + \cdots + p_n = 1$ and $p_i \ge 0$, for all i; is minimum when $p_1 = p_2 = \cdots = p_n = \dfrac{1}{n}$.

(d) Use Beale's method to solve the following quadratic programming problems

Maximize $f = 10x_1 + 25x_2 - 10x_1^2 - x_2^2 - 4x_1x_2$

subject to

$$x_1 + 2x_2 \leq 10$$
$$x_1 + x_2 \leq 9$$
$$x_1,\ x_2 \geq 0$$

[Internal Assessment - 10 marks]

P.T.O.

Paper : MTM - 305B

(Dynamical Meteorology : Thermodynamics in Atmosphere)

1. Answer any ***four*** questions : 2×4=8

   (a) Calculate the workdone when an air parcel expands isothermally at the temperature 27°C from initial pressure 1000 kilo pascal to the final pressure 100 kilo pascal.

   (b) Find the water vapour content of an air column in the atmosphere.

   (c) Differentiate between isothermal process and adiabatic process in the atmosphere.

   (d) Show that the potential temperature is invariant during the adiabatic process.

   (e) Define horizontal and geopotential surfaces in the atmosphere.

   (f) Differentiate between saturated adiabatic process and pseudo-adiabatic process.

2. Answer any ***four*** questions : 4×4=16

   (a) Prove that $pv^{\gamma} = \text{constant}$ when an air parcel follows an adiabatic process in the atmosphere.

   (b) What do you mean by isobaric cooling? Show that the relative increase in dew-point temperature is about 5% of the sum of relative increase in mixing ratio and pressure.

(c) Establish the relation between the specific entropy and the potential temperature. In an adiabatic expansion of an air parcel, the change in internal energy is 50 joule. What is the work done in this process?

(d) Derive the adiabatic lapse rate for moist unsaturated air parcel.

(e) Discuss the variation of pressure with respect to altitude in the atmosphere.

(f) Prove that $\rho = \frac{p}{R_d T}\left\{1-(1-\epsilon)\frac{e}{p}\right\}$ for moist air in the atmosphere.

3. Answer any ***two*** questions : 8×2=16

(a) Explain Parcel Theory. Derive static stability of dry air in the atmosphere.

(b) Define Gibb's free energy. Derive the differential form of Gibb's free energy. State and prove the Clausius-Clapeyron equation in the atmosphere.

(c) (i) Define mixing ratio. Calculate the virtual temperature of the moist air having mixing ratio of 20 gm/kg at 30°C.

(ii) Derive the co-ordinates in Emagram and Skew-T thermodynamic diagrams.

(d) Discuss about the phase change of an ideal gas and derive the relation of dependency of latent heat of evaporation with respect to temperature during the phase change of an air parcel.

[Internal Assessment - 10 marks]