2025

M.Sc. 3rd Semester Examination

APPLIED MATHEMATICS

Paper : MTM - 302

(Transforms and Integral Equations)

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

1. Answer any ***four*** questions : $2\times4=8$

   (a) State Bromwich's integral formula concerning on Laplace transform.

   (b) Find the Fourier transform of $f(x)$ where $f(x) = xe^{-ax^2}$, $a > 0$.

   (c) Define eigen values and eigen vectors in connection with an integral equation.

   (d) Verify the initial value theorem in connection with Laplace transform for the function $2 + \cos(t)$.

   (e) Define the wavelet function and analyze the parameters involving in it.

P.T.O.

(f) Find the solution of the integral equation :

$$y(x)=x+\int_0^1 (xt^2)\,y(t)\,dt.$$

2. Answer any ***four*** questions : 4×4=16

(a) Form an integral equation corresponding to the differential equation $\frac{d^2y}{dx^2}+\lambda xy=1,\ 0\le x\le l$, with the boundary conditions $y(0)=0,\ y(l)=1$.

(b) Prove that the Fourier transform of $\frac{1}{x}$ is $i\sqrt{\frac{\pi}{2}}\,sgn(x)$, where *sgn* is a signum function.

(c) Solve the integral equation $y(x)=1+\lambda\int_0^{2\pi}\sin(x+t)\,y(t)\,dt$ possesses infinite number of solutions.

(d) Evaluate $L\{J_0(t)\}$ with the help of initial value theorem, where $J_0(t)$ is the Bessel's function of order zero.

(e) Find the value of $\sin(t) * t^2$ where $*$ denotes the convolution operator on Laplace transform.

(f) Under certain conditions (to be specified by you), convert the following integral equation $f(x)=\int_0^x k(x,t)\,y(t)\,dt$ into the Volterra integral equation of 2nd kind and then solve it.

3. Answer any *two* questions : 8×2=16

(a) (i) State Parseval's identity on Fourier transform. Using Parseval's relation for Fourier Cosine transform to prove the following :

$$\int_0^{\infty} \frac{\sin \lambda t \sin \mu t}{t^2} \, dt = \frac{\pi}{2} \min(\lambda, \mu). \qquad 1+3$$

(ii) Find the resolvent kernel of the following integral equation and then solve it :

$$\varphi(x) = x^2 + \int_0^x \sin(x-y)\phi(y)\,dy. \qquad 4$$

(b) (i) Solve the following ODE by Laplace transform technique : $y'''(t) + y'(t) = \sin(t)$ with initial conditions, $y(0) = 0$, $y'(0) = -2$, $y''(0) = 0$. 6

(ii) Find the exponential Fourier transform of

$$f(t) = \begin{cases} 1-|t|, & |t|<1 \\ 0, & |t|>1 \end{cases}. \qquad 2$$

(c) (i) Solve the integral equation,

$$ax + bx^2 = \int_0^x \frac{y(t)}{(x-t)^{\frac{1}{2}}} dt. \qquad 4$$

(ii) State and prove the final value theorem concerning on Laplace transform. 4

P.T.O.

(d) Solve the following boundary value problem in the half plane $y > 0$, described by PDE : $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, $-\infty < x < \infty$, $y > 0$, with boundary conditions $u(x, 0) = f(x)$, $-\infty < x < \infty$. u is bounded as $y \to \infty$; u and $\frac{\partial u}{\partial x}$ both vanish as $|x| \to \infty$. 8

[Internal Assessment - 10 marks]