Total Pages : 4 PG/3rd Sem /MTM/25

2025

M.Sc. 3rd Semester Examination

APPLIED MATHEMATICS

Paper : MTM - 301

(Partial Differential Equations and Generalized Functions)

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

1. Answer any ***four*** questions : 2×4=8

(a) Give an example of a harmonic function in a domain D which has neither a maximum value, nor a minimum value in D.

(b) Let $u(x, y)$ be an integral surface of the equation $a(x, y)u_x + b(x, y)u_y + u = 0$, where $a(x, y)$ and $b(x, y)$ are positive differentiable functions in the entire plane. Define $D = \{(x, y) : |x| < 1, |y| < 1\}$.

Suppose that u attains a local minimum (maximum) at a point $(x_0, y_0) \in D$. Find $u(x_0, y_0)$.

(c) Define Dirac delta function.

P.T.O.

(d) Find the adjoint of the differential operator $L(u) = u_{xx} + u_{xt} + u_t$.

(e) Find the solution of $p^2 y(1+x^2) = qx^2$ where symbols have their usual meaning.

(f) Find the derivative of the Heaviside unit step function.

2. Answer any ***four*** questions : 4×4=16

(a) Solve :

$$\left(x^2D^2 - 2xyDD' + y^2D'^2 - xD + 3yD'\right)u = 8\frac{y}{x}.$$

Symbols have their usual meaning.

(b) Let $u(x, y)$ be a non-constant harmonic function in the disk $x^2 + y^2 < \tau^2$. Define for each $0 < r < \tau$,

$V(r) = \max\limits_{x^2+y^2=r^2} u(x, y)$.

Show that $V(r)$ is a monotonically function in the interval $(0, \tau)$.

(c) Establish the d'Alembert's formula of the Cauchy problem for the homogeneous wave equation.

(d) Solve the problem

$$u_{tt} - u_{xx} = xt, -\infty < x < \infty, t > 0$$
$$u(x, 0) = 0, -\infty < x < \infty$$
$$u_t(x, 0) = e^x, -\infty < x < \infty.$$

(e) Show that the Neumann problem for the Poisson's equation has more than one solution.

(f) Let u be continuous and has the mean value property on a domain $D \subseteq \mathbb{R}^2$. Show that it has continuous derivatives of all orders and all of them have the mean value property on D.

3. Answer any ***two*** questions : 8×2=16

(a) (i) Suppose that $au_{xx} + 2bu_{xy} + cu_{yy} + du_x + eu_y + fu = g$ is hyperbolic in a planar domain D. Assume that a, b, c, d, e, f, g are given functions of x and y. Then show that there exists a coordinate system (ξ, η) in which the above equation has the canonical form $w_{\xi\xi} + l_1[w] = G(\xi, \eta)$, where l_1 is a first-order linear differential operator and G is a function which depends on the above equation. 5

(ii) Solve the following Cauchy problem :

$$u_{tt} - 9u_{xx} = e^x - e^{-x}, \ -\infty < x < \infty, \ t > 0,$$
$$u(x, 0) = x, \ -\infty < x < \infty,$$
$$u_t(x, 0) = \sin x, \ -\infty < x < \infty.$$
3

(b) (i) State and prove strong maximum principle. 5

P.T.O.

(ii) Prove that every non-negative harmonic function in the disk of radius a satisfies
$$\frac{a-r}{a+r}u(0, 0) \le u(r, \theta) \le \frac{a+r}{a-r}u(0, 0).$$ 3

(c) (i) Solve the following problem :

$$u_t = u_{xx} - u,\ 0 < x < 1,\ t > 0$$
$$u(0, t) = u_x(1, t) = 0,\ t \ge 0$$
$$u(x, 0) = x(2-x),\ 0 \le x \le 1.$$ 5

(ii) Establish the Laplace equation in polar coordinates. 3

(d) Consider the equation $u_{xx} - 2u_{xy} + 4e^y = 0,\ y > 0.$

(i) Find the canonical form of the above equation. 4

(ii) Find the general solution $u(x, y)$ of the equation. 2

(iii) Find the solution $u(x, y)$ which satisfies $u(0, y) = f(y)$ and $u_x(0, y) = g(y)$. 2

[Internal Assessment - 10 marks]