# Non-Relativistic Formalism of Resonant Tunneling in the Semiconductor Superlattices

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## ABSTRACT

Theoretical investigations of non-relativistic mechanism is explored for the determination of transmission coefficient across a multibarrier GaAs-Al$_y$Ga$_{1-y}$As heterostructure for the entire energy range of $\varepsilon \langle V_0$, $\varepsilon = V_0$ and $\varepsilon \rangle V_0$. The resonant tunneling states are also determined. The effect of number of barriers and number of cells in the well and barrier regions on the resonant energies are studied in detail. We have computed resonant tunneling lifetime, group-velocity and traversal time of electrons tunneled through the multibarrier system. The results obtained reveal that the electrons in the higher bands corresponding to resonant energies could tunnel out faster than those electrons in the lower band. Furthermore, the presence of an additional resonant peak in resonant energy spectrum indicated the presence of a new surface state in the multibarrier system.

***Keywords:*** *Multibarrier resonant tunneling, Transmission Coefficient, Tunneling Lifetime, Group velocity, Traversal Time.*

## 1. Introduction

The resonant tunneling (RT) of an electron wave through multiple potential barriers is a basic phenomenon in quantum mechanics. Since the pioneering work of Tsu and Esaki [1] on resonant tunneling phenomena through double barrier structures the topic has generated considerable theoretical and experimental interest. The motivation may be attributed to the potential and extensive applications of the resonant tunneling phenomenon in high-speed electronic and optoelectronic devices which include lasers, modulators, photodetectors, signal processing devices, etc. [1]. The interest in this field has been catapulted to a new height with the advent of epitaxial growth techniques, particularly molecular-beam epitaxy (MBE) and metalorganic chemical vapour deposition (MOCVD), through which fabrication of perfect superlattices and multi-quantum well structures became a reality. The tunneling through a multibarrier system (MBS) provides a deeper understanding of the transport phenomena through semiconductor superlattices. An understanding of the time dependent aspects of tunneling is clearly required for the construction of a kinetic theory for such systems. The simple question of tunneling time seems a natural one from which to start.

In a multibarrier structure, the transmission coefficient is the relative probability of an incident electron crossing the multiple barriers. Resonant tunneling in the MBS corresponds to unit transmission coefficient across the structure. One of the most striking features of the multibarrier systems is the occurrence of quasi-level resonant tunneling energy states. Incident electrons on the MBS with energies equal to any one of these quasi-level resonant energy states, suffers resonant tunneling. Resonant tunneling is a consequence of the phase coherence of the electron waves in the quantum wells of the MBS. These quasi-level resonant energy states group themselves into tunneling energy bands separated by forbidden gaps. Each allowed energy band comprises (N-1) number of resonant energy states; N being the number of barriers in the MBS.

The resonant quasi-level lifetime, which is referred here after as resonant lifetime (RTL), is one of the important issues concerning the development of the novel electronic devices based on RT. A striking feature of RTL in the multibarrier systems with more than two barriers is the occurrence of special minima in the resonant lifetime for quasi level resonant energies in the middle of the allowed tunneling energy bands of the MBS. Further, the group velocity of the electrons corresponding to the resonant energy states obtained from the ε-k relation in the resonant tunneling energy bands can be used to calculate the traversal time of the electron across the multibarrier structure for the corresponding incident energy. The traversal time defined in the present context is the time with which the maximum of a wave packet crosses the MBS. As such to optimize the performance of these quantum tunneling devices, an accurate knowledge of their quasi-levels and the corresponding lifetimes and traversal times are necessary. These times are particularly important for estimating the frequency limit and the operation speed of these high speed devices.

With a few exceptions [2-4] investigations of RTL have been dedicated mostly to the double barrier systems [5-8]. The reported works in MBS have only discussed the RT for electrons with incident energies ε in the range $\varepsilon \langle V_0$, $V_0$ being the height of the potential barrier. A study of RT with incident energies $\varepsilon \geq V_0$ is expected to bring the features of the RT energy bands more clearly. Further the resonant tunneling energies, RTL and the transition time depend on the parameters such as the height of the potential barrier controlled through the mole fraction of barrier material vis-à-vis the well material, the thickness of the barrier and well layers and the number of barriers in the structure. We have not come across any such attempt to study theoretically relations of resonant energies and RTL on these factors. However, it was only suggested that δ-doped double barrier structures would be faster [6].

The present paper aims to study (i) the tunneling in the multibarrier systems in a comprehensive manner using the non-relativistic for incident energies for both $\varepsilon \langle V_0$ and $\varepsilon \geq V_0$ (ii) finding an analytical relation between the wave vectors and the resonant energy states in the tunneling energy bands, (iii) examine the dependence of resonant tunneling energies, RTL and for the MBS on various factors like the height of the potential barrier, the thickness of the barrier, and well layers and the number of barriers in the structure. (iv) the determination of group velocity of the electrons in these quasi-resonant states. The group velocities will be used for the determination of the traversal time across the MBS for the electrons in these quasi resonant states.

We consider a MBS constructed by growing two different semiconducting materials in alternate layers, for example, GaAs and $Al_yGa_{1-y}As$, having similar band structure but different energy gaps leading to a potential distribution in the growth direction. The MBS potential is assumed to take the form of alternate rectangular barriers and wells at the conduction and valence band edges along the growth direction and is considered to be superimposed on the intrinsic periodic potential of the host crystal. The effect of periodic crystal potential of the host material is incorporated through the inclusion of band effective mass of the material. The low gap material, GaAs, forms the well while the large gap material $Al_yGa_{1-y}As$ forms the barrier of the superlattice. The barrier height at the conduction band edge is assumed [9] to be 88% of the difference between the band gaps of two materials. The problem of RT is carried out non–relativistically in the single electron approximation with the use of space dependent effective mass. The transmission coefficient across the MBS is obtained through the transfer matrix approach taking recourse to an exact solution of the Schrodinger wave equation. The resonant energies were obtained by numerical computation and also the relation of the resonant tunneling energies in the tunneling band is explored analytically. The resonant lifetime of the electrons for incident energies equal to any of the quasi-level resonant tunneling energy states in the tunneling bands is obtained from the energy uncertainty condition as [5]

$$\tau = \frac{\hbar}{2\Delta E_m} \tag{1.1}$$

where, $\tau$ is the RTL, and $\Delta E_m$ is the width of the resonance peak at half maximum corresponding to the resonance energy $E_m$. $\Delta E_m$ is obtained from the transmission coefficient versus incident energy graph through numerical computation. The group velocity of the electrons in the quasi-level resonant tunneling energy states are obtained with the use of Lagranges interpolation technique and then used for determination of the traversal time.

### 2.1 Transmission coefficient in nonrelativistic treatment

To deal with the problem one need to consider the one-dimensional time independent Schrodinger equation for the electron in the potential V(x) which appears as:

$$\left[-\frac{\hbar^2}{2m^*}\frac{d^2}{dx^2}+V(x)\right]\psi(x)=\varepsilon\,\psi(x) \tag{2.1}$$

where

$$V(x)=\begin{cases}V_0 & for\ nd-b/2\le x\le nd+b/2\\ 0 & otherwise\end{cases}$$

The wave functions of the time independent Schrodinger equation in the nth well region appears as:

$$\psi_n^{(w)}(x)=A_{2n-1}\,e^{ik_1x}+B_{2n-1}\,e^{-ik_1x} \tag{2.2}$$

where $k_1^2=2m_w^*\varepsilon/\hbar^2$

The wave function in the nth barrier region with (nd-b/2) ≤ x ≥ ( nd+b/2) can be obtained as

$$\psi_n^{(b)}(x)=\begin{cases} A_{2n}\,e^{-k_2x}+B_{2n}\,e^{k_2x} & for\,\varepsilon\langle V_0 \\ A_{2n}+B_{2n}\,x & for\,\varepsilon=V_0 \\ A_{2n}\,e^{ik_3x}+B_{2n}\,e^{-ik_3x} & for\,\varepsilon\rangle V_0 \end{cases} \tag{2.3}$$

$$k_2^2=2m_b^*\left(V_0-\varepsilon\right)/\hbar^2 \quad and$$

$$k_3^2=2m_b^*\left(\varepsilon-V_0\right)/\hbar^2$$

Use has been made of the following effective mass dependent boundary conditions at the interfaces of the nth barrier with nth and (n+1)th well region which conserves the probability density and the current density across the junctions.

$$\psi_n^{(w)}(x)\Big|_{nd-b/2}=\psi_n^{(b)}(x)\Big|_{nd-b/2}$$

$$\frac{1}{m_w^*}\frac{d\psi_n^{(w)}}{dx}\Bigg|_{nd-b/2}=\frac{1}{m_b^*}\frac{d\psi_n^{(b)}}{dx}\Big|_{nd-b/2}$$

$$\psi_n^{(b)}(x)\Big|_{nd+b/2}=\psi_{n+1}^{(w)}(x)\Big|_{nd+b/2}$$

$$\frac{1}{m_b^*}\frac{d\psi_n^{(b)}}{dx}\Bigg|_{nd+b/2}=\frac{1}{m_w^*}\frac{d\psi_{n+1}^{(w)}}{dx}\Big|_{nd+b/2}$$

$M_1$ is the transfer matrix for a single barrier which relates the coefficient matrix $\begin{bmatrix}A_3\\B_3\end{bmatrix}$ of the 3rd well region with that of the first well region $\begin{bmatrix}A_1\\B_1\end{bmatrix}$ after tunneling through the first barrier and the matrix elements of $M_1$ appear as:

$$\left(M_1\right)_{11}=\left(M_1\right)_{22}^*=\begin{cases} \left(\cosh k_2b+\dfrac{k_2^2f^2-k_1^2}{2ik_1k_2f}\sinh k_2b\right)e^{-ik_1b} & for\,\varepsilon\langle V_0 \\ \left(1-\dfrac{k_1b}{2if}\right)e^{-ik_1b} & for\,\varepsilon=V_0 \\ \left(\cos k_3b-\dfrac{k_3^2f^2+k_1^2}{2ik_3k_1f}\sin k_3b\right)e^{-ik_1b} & for\,\varepsilon\rangle V_0 \end{cases} \tag{2.5}$$

$$\left(M_1\right)_{12}=\left(M_1\right)_{21}^*=\begin{cases} \dfrac{k_1^2+k_2^2f^2}{2ik_1k_2f}\sinh k_2b & for\,\varepsilon\langle V_0 \\ \dfrac{k_1b}{2if} & for\,\varepsilon=V_0 \\ \dfrac{k_1^2-k_3^2f^2}{2ik_1k_3f}\sin k_3b & for\,\varepsilon\rangle V_0 \end{cases} \tag{2.6}$$

$$f=m_w^*/m_b^*$$

The 2X2 matrix $M_n$ which relates the coefficient matrix $\begin{bmatrix} A_{2n-1} \\ B_{2n-1} \end{bmatrix}$ of the nth well region with that of the (n+1) th well region $\begin{bmatrix} A_{2n+1} \\ B_{2n+1} \end{bmatrix}$ after tunneling through the nth barrier appears as :

$$M_n = \left(F^*\right)^{n-1} M_1 \, F^{n-1} \tag{2.7}$$

where, $F = \begin{bmatrix} e^{ikc} & 0 \\ 0 & e^{-ikc} \end{bmatrix}$ and n = 1, 2, …., N

Thus the transfer matrix $W_n$ which relates the coefficient matrix of the incoming and outgoing wave in the N barrier system appears as

$$\begin{bmatrix} A_{2N+1} \\ B_{2N+1} \end{bmatrix} = \left[W_N\right] \begin{bmatrix} A_1 \\ B_1 \end{bmatrix} \tag{2.8}$$

$$\text{where, } W_N = M_N M_{N-1} .........M_2 M_1 \tag{2.9}$$

Substituting $M_i$ from eq(2.7) in eq(2.9), the transfer matrix takes the form

$$W_N = \left(F^*\right)^N G^N \tag{2.10}$$

$$\text{where the matrix } G = F\, M_1 \tag{2.11}$$

The matrix $W_N$ is hermitian and its determinant has unit value..
Now $G$ matrix can b*e* diagonalized to the matrix $G_d$ as

$$S^{-1} G\, S = G_d \tag{2.12}$$

$$\text{where, } G_d = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \tag{2.13}$$

and, $S$ is the diagonalizing matrix of the matrix $G$

$\lambda_1$ and $\lambda_2$ are the eigenvalues of the matrix $G$ and satisfy the following relations.

$$\lambda_{1\,,\,2} = \frac{G_{Tr} \pm \sqrt{G_{Tr}^2 - 4}}{2} \tag{2.14}$$

$G_{Tr}$ is the trace of matrix $G$

$$G_{Tr} = \begin{cases} 2\left[\cos k_1 a \cosh k_2 b + \dfrac{k_2^2 f^2 - k_1^2}{2k_1 k_2 f} \sin k_1 a \sinh k_2 b\right] & for\ \varepsilon \langle V_0 \\ 2\left[\cos k_1 a - \dfrac{k_1 b}{2f} \sin k_1 a\right] & for\ \varepsilon = V_0 \\ 2\left[\cos k_1 a \cos k_3 b - \dfrac{k_3^2 f^2 + k_1^2}{2k_1 k_3 f} \sin k_1 a \sin k_3 b\right] & for\ \varepsilon \rangle V_0 \end{cases} \tag{2.15}$$

$\lambda_1 + \lambda_2 = G_{Tr}$ and $\lambda_1 \lambda_2 = 1$

The above conditions can be summed up as:

$$\begin{aligned} &\lambda_1 = \frac{1}{\lambda_2} = e^{i\theta} \quad \theta = \cos^{-1}\left(G_{tr}/2\right) && for \quad G_{Tr} \langle 2 \\ &\lambda_1 = \lambda_2 = 1 && for \quad G_{Tr} = 2 \\ &\lambda_1 = \frac{1}{\lambda_2} = e^{\theta} \quad \theta = \cosh^{-1}\left(G_{tr}/2\right) && for \quad G_{Tr} \rangle 2 \end{aligned} \tag{2.16}$$

Using these relations the transfer matrix $W_N$ can be written as

$$W_N = \left(F^*\right)^N S\, G_d^N\, S^{-1} \tag{2.17}$$

The Transmission coefficient $T_N$ across N barriers can be obtained as

$$T_N = \frac{\left|A_{2n+1}\right|^2}{\left|A_1\right|^2} \quad . \tag{2.18}$$

In the extreme right [(N=1)th] well, as there is no reflected component, one can set $B_{2N+1} = 0$. Using this fact together with the eqs(2.8)-(2.18) $T_N$ can be obtained as

$$T_n = \frac{1}{\left|\left(W_N\right)_{11}\right|^2} = \frac{1}{1 + \left|\left(W_N\right)_{12}\right|^2} \tag{2.19}$$

where, $\left(W_N\right)_{12} = e^{-ik_1 N c}\, G_{12}\, \dfrac{\lambda_2^N - \lambda_1^N}{\lambda_2 - \lambda_1}$ (2.20)

Substituting Eqs. (2.11) and (2.16) on (2.20), $(W_N)_{12}$ appears as:

$$\left|\left(W_N\right)_{12}\right|^2 = \begin{cases} \left|\left(M_1\right)_{12}\right|^2 \left|\dfrac{\sin n\theta}{\sin\theta}\right|^2 & for \;\; G_{Tr} \langle 2 \\ \left|\left(M_1\right)_{12}\right|^2 N^2 & for \;\; G_{Tr} = 2 \\ \left|\left(M_1\right)_{12}\right|^2 \left|\dfrac{\sinh n\theta}{\sinh\theta}\right|^2 & for \;\; G_{Tr} \rangle 2 \end{cases} \tag{2.21}$$

**2.2 Resonant Tunnelling Energies**

The resonant tunneling across the N-barrier system corresponds to the condition $T_N$ =1. The incident energy of the electron for which the resonant tunneling condition is satisfied is termed as resonant tunneling energy Here we have found the resonant tunneling energies in the multibarrie system from the $T_N$ vs ε curve by a computer program using the search technique . However it would be worth while to point some of the salient features of the resonant tunneling energy states.

Resonant tunneling across the multibarrier system occur for definite values of the incident energy of the incident electron in the entire region of the energy spectrum i.e. for both the regions $\varepsilon < V_0$ and $\varepsilon > V_0$

Eq (2.16) in combination with Eq (2.15) is akin to the energy relation for a lattice of period (*a*+*b*) calculated using effective mass dependent Kronig-Penny model. The

allowed energy bands are restricted to values of $G_{Tr}$ <2 which corresponds to the allowed values of $\cos\theta$ in Eq.(2.16)._Hence the resonant tunneling states group themselves to allowed tunneling energy bands separated by forbidden gaps.
As can be seen from Eq. (2.19) in combination with Eq. (2.21), the resonant state will correspond to $SinN\theta = 0$ where $\theta$ is given in Eq. (16). Thus, there occurs (N-1) values of resonant energies in each band for $\theta = n\pi / N, \quad n = 1, 2, ..., N-1$. These values of $\theta$ correspond to the wave vectors $n\pi/L$ in a superlattice of length $L = Nc$. Thus for the $N$-barrier superlattice with $(N-1)$ wells, each allowed mini energy band will contain $(N-1)$ number of resonance energy states corresponding to the $(N-1)$ values of the wave vector.
During the resonance tunneling, the electron energy resonates at the bound states of quantum well. Hence, the number of allowed bands in these multibarrier systems is found to be equal to the number of bound energy states in the single finite quantum well having the same parameters as that of the MBS. It may be worth noting here that the number of bound states, j, for $\varepsilon<V_0$ in a finite well depends on the width, $a$, and potential height, $V_0$, of the quantum well through the relation :

$$j = Int(\beta) + 1 \tag{2.22}$$

where,
$$\beta = \sqrt{\left(\frac{a}{\pi}\right)^2 \left(k_1^2 + k_2^2\right)_{E=V}} = \sqrt{\frac{8a^2 m_1^* V}{h^2}}$$

and Int ($\beta$)=Integer value of $\beta$
Hence due to phase coherence the number of tunneling bands in multibarrier systems for $\varepsilon<V_0$ will be equal to 'j' which depends on the well width and the height of the potential barrier and is independent of the barrier width.

### 2.3 Traversal Time and Group Velocity

The traversal time of electrons for energies corresponding to resonant tunneling is defined as
$$\tau_R = L / v_g \tag{2.23}$$
where $L$ is the total length traversed by the electron and $v_g$ is the group velocity defiened through the relation $v_g = \frac{1}{\hbar}\frac{dE}{dk}$ . $k$ is the wavevector defined as $\frac{m\pi}{L}$, where $m = 1, 2, \cdots\cdots\cdots (N-1)$. The group velocity $v_g$ is obtained in two steps: (i) First, the points of the E versus k curve are interpolated by Lagrangian interpolation technique, (ii) In the second step, the derivative at the necessary points are computed.
resonant states at any desired value. Although GaAs/$Al_yGa_{1-y}As$ superlattice has been used to test the applicability of the model, the features highlighted in this paper will be true in general for any system including the GaN/$Al_yGa_{1-y}N$ one. We feel that the results of this paper will stimulate experimental studies towards fabrication and analysis of high speed solid state devices based on resonant tunneling in semiconductor superlattices.

### 3.Numerical Analyses

The numerical analyses is basically concerned with (i) the transmission coefficient across multibarrier systems for incident energies $\varepsilon < V_0$ , $\varepsilon = V_0$ and $\varepsilon > V_0$ (ii) determination of resonant tunneling energies for which the transmission coefficient is unity,(iii)the resonant tunneling lifetime and (iv) the traversal time across the barrier. The procedure for computing the transmission coefficient in the non-relativistic treatment is based on numerical computation of Eq.(2.19) . Thereafter, these data were sorted to obtain the resonant tunneling energies in the multibarrier systems through a search program.. The width at half maximum, $\Delta\varepsilon_m$ , around each resonant tunneling energies are obtained by first finding the energies for which $T_c$ is minimum on both the left and the right side of the resonant tunneling peak and then finding the energies $\varepsilon_{L1/2}$ and $\varepsilon_{R1/2}$ on both these sides of the peak where $T_c = (T_{max}+T_{min})/2$; $T_{max}$+being the transmission coefficient at the resonant peak and $T_{min}$ correspond to the minimum transmission coefficient on the corresponding side of the resonant peak. The half width $\Delta\varepsilon_m$ around each peak is obtained from the relation $\Delta\varepsilon_{m=}\ \varepsilon_{R1/2} - \varepsilon_{L1/2.}$ The resonant tunneling life time for each resonant energy is then calculated by using relation (2.24) The traversal time is then calculated for each energy state on the basis of Eq.(2.25) after finding out the group velocity associated with the resonant states in each resonant tunneling band by using Lagranges interpolation technique and then use the principle that traversal time corresponding to a resonant state is equal to the ratio of the length of the multibarrier system and the group velocity associated with the corresponding resonant state. In order to bring forth the variation of the tunneling and traversal times in the multibarrier systems , the problem is studied for various values of barrier height ,barrier width, well width and the number of barriers in the MBS. For the numerical evaluation of $T_c$, $\varepsilon_m$ and $\tau$, we have chosen a realistic system of $GaAs/Al_yGa_{1-y}As$ ($y < 0.45$) superlattice. For this purpose we have considered 2 to 9 barriers. The well is considered to be 1 to10 lattice cells of GaAs . Similarly the barrier is considered to be composed of 1 to10 lattice cells $Al_yGa_{1-y}As$. The effect of crystal potential of the host crystals is considered through the band effective mass The barrier height is considered to be 88% of the difference between the band gaps of well materials (GaAs) and barrier materials ($Al_yGa_{1-y}As$ ). The values of various parameters of the system considered are as follows:

a = the well width = $n_w$ x $a_w$, where $n_w$ is the number of cells in the well material in each well slab and $a_w$ is the lattice constant of the well material GaAs.

$a_w = 5.6533$ Å

b= the barrier width = $n_b$ x $a_b$, where $n_b$ is the number of unit cells of the barrier material in each barrier slab and $a_b$ is the lattice constant of the barrier material $Al_{0.3}Ga_{0.7}As$ .

$a_b = 5.6533+7.8\mathrm{x}10^{-4}\ y$

$m_2^*$ = the effective masses of the well (GaAs )and the barrier ($Al_yGa_{1-y}As$) region materials of the superlattice

= $0.065\ m_0$ and $(0.067 + 0.83\ y)\ m_0$ ; $m_0$ is the free electron mass.

$E_{g1}$ and $E_{g2}$ = energy band gap in the well and barrier materials

= 1.428 eV and $(1.424 + 1.247y)$ eV.

$V$ = height of the potential barrier = $0.88\ (E_{g2} - E_{g1})$

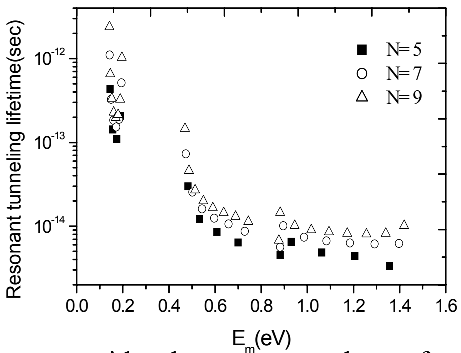
The energy band gap of $Al_yGa_{1-y}As$ becomes indirect when the value of mole fraction ($y$) exceeds 0.45, and hence does not conform to the band diagram (shown in Fig. 1(b)) required to constitute the effective quantum wells for optical devices. Therefore, in the present calculations the values of the mole fraction are restricted to $0 < y \leq 0.45.$ The results have been depicted graphically and discussed. The multi barrier systems are designated in the graphs by the set of numbers (y, $n_b$, $n_w$, N) where N is the number of barriers and y, $n_b$, $n_w$ have the meaning as defined in this section.

To elucidate adequately the motivations behind our study we have indicated tunneling mini energy bands in which the resonant tunneling states are obtained. As regards the numerical treatment of the RTL, the number of discrete energy states that occur in the constituent single quantum well of the superlattice have been mentioned.

## 4 . Results and Discussions

1. The tunneling time and the transmission coefficient of an electron through MBS are obtained by taking into account the space-dependent electron effective mass for the entire range of incident energies **$\varepsilon < V_0$ , $\varepsilon = V_0$ and $\varepsilon > V_0$**
2. The resonant energy states are obtained on the basis of the resonance condition **$T_N$ =1**. The resonant energy states are found to group into allowed energy bands separated by forbidden gaps.
   During the resonance tunneling, the electron energy resonates at the bound states of the single quantum well. The number of allowed mini bands in these MBS are found to be equal to the number of bound energy states in the single finite quantum well having the same parameters as that of the MBS.
3. In a superlattice the increase in periodicity of the lattice leads to splitting of a Brillouin zone to a set of mini energy bands. Consequently, our results depict the resonant tunneling energies and the allowed mini band structure of the superlattice. It is investigated exhaustively that the number of tunneling bands depends on the well width and the height of the potential barrier and is independent of the barrier width. To comprehend the role of the number of barriers in the superlattice, the well width, barrier width and mole fraction in the barrier material which determines the barrier height on the number of resonant energy states, we have compiled our results on the basis of different figures.
4. The width of the allowed band reduces significantly with increase in barrier width. Increase in barrier width causes decrease in the overlap interaction among the states of adjacent wells resulting in the decrease of band width. As the number of resonant energy states in a band remain constant at (N-1) for an N barrier superlattice, the energy difference between the states inside the band decreases for wide barrier systems. The width of barriers do not have any affect on the number of bands for 0<E<V.

5. As the number of bands increases in the same energy range, both the allowed minibands and the forbidden gaps become narrower for wider wells. Investigation of tunneling through MBS on relativistic and non-relativistic footing demands the fact that the energies of the bound states of quantum wells move to lower energy values which conforms to the quantum size effect.




6. It is seen that the variation of RTL with resonance energy has a special kind of minima and these minima take place around the centre of the allowed bands. The occurrence of the minima indicates the fact that the group velocity of an electron is highest at the middle of the allowed band. Consequently an electron at the resonant state in the middle of any allowed band would tunnel out faster than the ones with other values of resonant energies in the same band. As the band gap is proportional to the barrier height and the group velocity is inversely proportional to the bandgap, RTL always assumes higher values for larger y. So, an increase in the mole fraction accompanies a decrease in the peak width of the transmission spectrum at the resonant level of the superlattice.

7. Due to the quantum size effect the broader wells will have lower energies and will accommodate more number of minibands in the same energy range 0<E<V. It is depicted that increase in barrier width causes the decrease in overlap interaction and leads to narrower mini bands and wider forbidden gaps. This effect is largely responsible for the observed increase in tunneling time for systems with wide barriers. Consequently, the narrower bands results in narrower troughs in each mini band.

## 5. **Conclusions**

In this work, a model for computation of transmission coefficient for multibarrier semiconductor heterostructure is proposed. The resonant energy values are found to be dependent on the number of barriers, number of cells in the well region and number tunneling states. Also the fundamental physics of the RTL in superlattices has been investigated. The dependence of the RTL on the mole fraction of barrier layer, barrier width, and well width has been explored in some detail. The studies reveal that the variation of RTL with resonance energy exhibits a special kind of trough in the $\tau$ versus $E_m$ relation with minima in each allowed band occurring around the center of the allowed band. The lifetime of resonant states increases with the increase in the mole fractions.

## REFERENCES

1. R. Tsu and L. Esaki, Appl. Phys. Lett. **22** (1973) 562.
2. J. W. Choe, H. J. Hwang, A. G. U. Perera, S. G. Matsik, M. H. Frnacombe, J. Appl. Phys. **79** (1996) 7510.
3. A. Khan, C. L. Roy and P. K. Mahapatra, Phys. Lett. A **249** (1998) 512.
4. P. Pereyra, Phys. Rev. Lett. 84 (2000) 1772.
5. P. J. Price, Phys. Rev. B **38** (1988) 1994.
6. C. J. Arsenault, M. Meunier, Phys. Rev. B **39** (1989) 8739.
7. A.F.M. Anwar, A. N. Khondker, M. R. Khan, J. Appl. Phys. **65** (1989) 2761.
8. H. Z Xu, Y. Okada, Physica B **305** (2001) 113.
9. D. Mukherjee, B. R. Nag, Phys. Rev. B **12** (1975) 4338.