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A Parametric Non Linear Programming Approach to Fuzzy Decision Problem of a Queuing System with Finite Capacity

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ABSTRACT

This paper purposes, a mathematical non linear programming method to construct the membership function of a total cost in a queuing decision problem with cost co-efficient and the arrival rate being fuzzy trapezoidal numbers The basic idea is to transform a fuzzy queuing cost problem with finite capacity to a family of conventional crisp queues cost problem with finite capacity by applying the a cut approach and Zadeh's extension principle A set of parametric non linear programs are developed to calculate the lower and upper bound of the minimal expected total cost per unit time at a, through which the membership function of the total cost is constructed. A numerical example is given by specifying two queuing models with finite capacity and among the models, the total minimal cost is obtained by using the proposed method.

1. Introduction

Queuing decision problem play an important role in the queuing system design that involves one or many decision such as the number of servers at a service facility, the efficiency of the servers. A queuing cost based decision model is to determine a suitable service rate such that the sum of the cost of offering the service and cost of delay in offering the service is minimized. Many researchers like Papadopoulos [14], Waller Shih [16], Larsen [12], Arnott et al [2], Kerbache [10] and Smith Johnson [9] and Kwik and Tielemans [11] have published many papers on this topic.

Queuing decision problem can be solved when the cost co-efficient and the arrival or service pattern are known exactly. There are cases that these parameters may not be presented precisely. In many practical situations, the statistical information are described by linguistic terms such as fast moderate slow, rather than probability distributions Obviously when the cost co-efficient, arrival rates are fuzzy, the minimal expected total cost per unit time will be fuzzy. Therefore the minimal expected total cost should be described by the membership function rather than by a crisp value.

In this paper we developed a mathematical non-linear parametric programming [17] approach for the queuing decision problem by the basic idea of Zadeh's extension principle and a cut representation [3, 4]. A set of non linear programming problems are formulated to calculate the upper and lower bound of a cut of the minimal expected total cost and consequently membership function of the minimal expected total cost is derived.

2. Fuzzy Queuing Decision Problem with Finite Capacity

We consider an (FM/M/1) : (N/FCFS) queuing model in which customers arrive at the service facility at rate $\overline{\lambda}$ where $\overline{\lambda}$ is a fuzzy number and at service rate μ . If a queuing system has limited capacity of N customer, the cost per service per unit time is $\widetilde{C}_1$, cost of waiting per customer per unit time is $\widetilde{C}_2$ the cost of serving each additional customer per unit of time is C_3 and the cost of lost customer is C_4 where $\widetilde{C}_1$ and $\widetilde{C}_2$, are fuzzy numbers.

Let $\mu_{\overline{\lambda}}^{(x)}, \mu_{\widetilde{c}_1}^{(u)}$, and $\mu_{\widetilde{c}_2}^{(v)}$ denoted the membership functions of $\overline{\lambda}, \widetilde{C}_1$ and $\widetilde{C}_2$ respectively.

We have the following fuzzy sets.

$$\overline{\lambda} = \left\{\left(x, \mu_{\overline{\lambda}}^{(x)}\right) / x \in X\right\} \tag{1a}$$

$$\widetilde{C}_1 = \left\{\left(u, \mu_{\widetilde{c}_1}^{(u)}\right) / u \in C_1\right\} \tag{1b}$$

$$\widetilde{C}_2 = \left\{\left(v, \mu_{\widetilde{c}_2}^{(v)}\right) / v \in C_2\right\}, \tag{1c}$$

where X, C_1, C_2 are the crisp universal sets of arrival and cost co-efficient. Let f(x, u, v) denote the system characteristics of interest. Since $\overline{\lambda}$, $\widetilde{C}_1$ and $\widetilde{C}_2$ are fuzzy numbers, f($\overline{\lambda}$, $\widetilde{C}_1$, $\widetilde{C}_2$) is also a fuzzy number.

Following Zadeh's extension principle [19] the membership function of expected total cost is defined as

$$\mu_{f\left(\overline{\lambda}, \widetilde{C}_1, \widetilde{C}_2\right)}^{(z)} = \sup \min \left\{\mu_{\overline{\lambda}}^{(x)}, \mu_{\widetilde{C}_1}^{(u)}, \mu_{\widetilde{C}_2}^{(v)} / z = f\left(x, u, v\right)\right\} \tag{2}$$

The minimal expected total cost of a crisp queuing system [7, 8] with finite capacity is given by

$$E(C) = \mu c_1 + L c_2 + N c_3 + \lambda P_N c_4 \tag{3}$$

The membership function for the minimal cost is

$$\mu_{E(\widetilde{C})}^{(z)} = \sup\min\left\{\mu_{\bar{\lambda}}^{(x)}, \mu_{\widetilde{C}_1}^{(u)}, \mu_{\widetilde{C}_2}^{(v)} / z = \mu u + Lv + Nc_3 + cP_N c_4 \right\} \tag{4}$$

In this paper we approach the representation problem using a mathematical programming technique parametric NLPs are developed to find the α cut f ($\overline{\lambda}, \widetilde{C}_1, \widetilde{C}_2$) based on the extension principle.

3. Solution Procedure

Definitions for the α cuts of $\overline{\mu}$, $\widetilde{C}_1$, $\widetilde{C}_2$ as crisp intervals [20] are as follows.

$$\lambda(\alpha) = \left[x_\alpha^L, x_\alpha^U\right] = \left[\min\left\{x/\mu_{\bar{\lambda}}(x) \geq \alpha\right\} \max\left\{x/\mu_{\bar{\lambda}}(x) \geq \alpha\right\}\right] \tag{5a}$$

$$u(\alpha) = \left[u_\alpha^L, u_\alpha^U\right]$$

$$u(\alpha) = \left[\min\left\{u/\mu_{\widetilde{c}_1}^{(u)} \geq \alpha\right\}, \max\left\{u/\mu_{\widetilde{c}_1}^{(u)} \geq \alpha\right\}\right] \tag{5b}$$

$$v(\alpha) = \left[v_\alpha^L, v_\alpha^U\right]$$

$$= \left[\min\left\{v/\mu_{\widetilde{c}_2}^{(v)} \geq \alpha\right\} \max\left\{v/\mu_{\widetilde{c}_2}^{(v)} \geq \alpha\right\}\right] \tag{5c}$$

As a result, the bound of these intervals can be described as functions of α and can be obtained as

$$x_\alpha^L = \min \mu_{\bar{\lambda}}^{-1}(\alpha) \quad x_\alpha^U = \max \mu_{\bar{\lambda}}^{-1}(\alpha)$$
$$u_\alpha^L = \min \mu_{\widetilde{c}_1}^{-1}(\alpha) \quad u_\alpha^U = \max \mu_{\widetilde{c}_1}^{-1}(\alpha)$$
$$v_\alpha^L = \min \mu_{\widetilde{c}_2}^{-1}(\alpha) \quad v_\alpha^U = \max \mu_{\widetilde{c}_2}^{-1}(\alpha)$$

Therefore we can use the α cuts of to construct its membership function. Since the membership function defined in (4) is parameterized by α.

Using Zadeh's extension principle, $\mu_{E(\widetilde{c})}$ is minimum of $\mu_{\bar{\lambda}}^{(x)}$, $\mu_{\widetilde{c}_1}^{(u)}$ and $\mu_{\widetilde{c}_2}^{(v)}$. To derive the $\mu_{E(\widetilde{c})}$, we need at least one of the following cases to hold such that

$$Z = \mu u + Lv + Nc_3 + x P_N c_4$$

Satisfies $\mu_{E(\tilde{c})}(z) = \alpha$

Case i : $\mu_{\tilde{\lambda}}^{(x)} = \alpha, \quad \mu_{\tilde{c}_1}^{(u)} \geq \alpha, \quad \mu_{\tilde{c}_2}^{(v)} \geq \alpha$

Case ii : $\mu_{\tilde{\lambda}}^{(x)} \geq \alpha, \quad \mu_{\tilde{c}_1}^{(u)} = \alpha, \quad \mu_{\tilde{c}_2}^{(v)} \geq \alpha$

Case iii : $\mu_{\tilde{\lambda}}^{(x)} \geq \alpha, \quad \mu_{\tilde{c}_1}^{(u)} \geq \alpha, \quad \mu_{\tilde{c}_2}^{(v)} = \alpha$

This can be accomplished using parametric NLP techniques. The NLP to find the lower and upper bounds of the cut of $\mu_{E(\tilde{c})}$ for case (i) are

$$[E(c)]_{\alpha}^{L_1} = \min[\mu u + Lv + Nc_3 + x P_N c_4] \quad (6a)$$

$$[E(c)]_{\alpha}^{U_1} = \max[\mu u + Lv + Nc_3 + x P_N c_4] \quad (6b)$$

for case ii. Are

$$[E(c)]_{\alpha}^{L_2} = \min[\mu u + Lv + Nc_3 + x P_N c_4] \quad (6c)$$

$$[E(c)]_{\alpha}^{U_2} = \max[\mu u + Lv + Nc_3 + x P_N c_4] \quad (6d)$$

and for case (iii) are

$$[E(c)]_{\alpha}^{L_3} = \min[\mu u + Lv + Nc_3 + x P_N c_4] \quad (6e)$$

$$[E(c)]_{\alpha}^{U_3} = \max[\mu u + Lv + Nc_3 + x P_N c_4]. \quad (6f)$$

From the definitions of $\lambda(\alpha), u(\alpha), v(\alpha)$ in $(5a, 5b, 5c)\, x \in \lambda(\alpha), u \in C_1(\alpha), v \in C_2(\alpha)$ can be replaced by $x \in [x_{\alpha}^L, x_{\alpha}^U]\ u \in [u_{\alpha}^L, u_{\alpha}^U] v \in [v_{\alpha}^L, v_{\alpha}^U]$. The α cut form a nested structure with respect to α {6a, 6b, 6c, 6d, 6e, 6f]. For given $0 < \alpha_2 < \alpha_1 < 1$ we have

$$[x_{\alpha 1}^L, x_{\alpha 1}^U] \subseteq [x_{\alpha 2}^L, x_{\alpha 2}^U]$$

$$[u_{\alpha 1}^L, u_{\alpha 1}^U] \subseteq [u_{\alpha 2}^L, u_{\alpha 2}^U]$$

$$[v_{\alpha 1}^L, v_{\alpha 1}^U] \subseteq [v_{\alpha 2}^L, v_{\alpha 2}^U]$$

Therefore ((6a), (6c), (6e)) have the same smallest element and ((6b), (6d), (6f)) have the same largest element. To find the lower and upper board of E(c),

$$[E(c)]^L = \min[\mu u + Lv + Nc_3 + xP_N c_4] \quad (7a)$$

Such that $x_{\alpha}^L \leq x \leq x_{\alpha}^U$, $u_{\alpha}^L \leq u \leq u_{\alpha}^U < v_{\alpha}^L \leq v \leq v_{\alpha}^U$

$$\left[E(c)_\alpha\right]^U = \max\left[\mu u + Lv + Nc_3 + xP_N c_4\right] \quad (7b)$$

Such that $x_\alpha^L \le x \le x_\alpha^U$, $u_\alpha^L \le u \le u_\alpha^U < v_\alpha^L \le v \le v_\alpha^U$

At least any one of x, u, v must hit the boundaries of their α cut satisfying $\mu_{E(\tilde{c})}(z) = \alpha$.

Applying the results of Zimmerman and Kaufmann [20] and convexity properties, we have

$[E(c)]_{\alpha 1}^L \ge [E(c)]_{\alpha 2}^L$ and $[E(c)]_{\alpha 1}^U \le [E(c)]_{\alpha 2}^U$

where $0 \le \alpha_2 \le \alpha_1 \le 1$

If both $[E(c)]_\alpha^L$ and $[E(c)]_\alpha^U$ are invertible with respect to, then a left shape function $L(Z) = \left[E(c)_\alpha^L\right]^{-1}$ and right shape function $R(Z) = \left[E(c)_\alpha^U\right]^{-1}$ can be derived, such that

$$\mu_{E(\bar{c})}^z = \begin{cases} L(z), & [E(c)]_{\alpha=0}^L \le z \le [E(c)]_{\alpha=1}^L \\ 1, & [E(c)]_{\alpha=1}^L \le z \le [E(c)]_{\alpha=1}^U \\ R(z), & [E(c)]_{\alpha=1}^U \le z \le [E(c)]_{\alpha=0}^U \end{cases} \quad (8)$$

In most cases, the values of $[E(c)]_\alpha^L$ and $[E(c)]_\alpha^U$ cannot be solved analytically. Consequently, a closed form membership function for $E(\tilde{c})$ cannot be obtained. However, the numerical solution for $[E(c)]_\alpha^L$ and $[E(c)]_\alpha^U$ at different possibility levels can be collected to approximate the shape of L(Z) and R(Z).

4. Solution Algorithm

Inputs the arrival rate, service cost, waiting cost are trapezoidal fuzzy numbers represented by (x_1, x_2, x_3, x_4) (u_1, u_2, u_3, u_4), (v_1, v_2, v_3, v_4) Out the numbers $x_\alpha^L, x_\alpha^U, u_\alpha^L, u_\alpha^U$, $v_\alpha^L, v_\alpha^U, f_\alpha^L, f_\alpha^U$.

Step 1 : For $\alpha = 0$ to 1 Step 0.1

Step 2 : $x_\alpha^L = (x_2 - x_1)\alpha + x_1$

$$x_\alpha^U = (x_4) - (x_4 - x_3)\alpha$$

$$u_\alpha^L = (u_2 - u_1)\alpha + u_1$$

$$u_\alpha^U = (u_4) - (u_4 - u_3)\alpha$$

$$v_\alpha^L = (v_2 - v_1)\alpha + v_1$$

$$v_\alpha^U = (v_4) - (v_4 - v_3)\alpha$$

Step 3 : For x = x_α^L to x_α^U

For u = u_α^L to u_α^U

For v = v_α^L to v_α^U

Step 4 : $f_\alpha^L = \arg\{\min(f(x,u,v))\}$

$f_\alpha^U = \arg\{\max(f(x,u,v))\}$

Step 5 : Out put x_α^L , x_α^U , u_α^L , u_α^U , v_α^L , v_α^U , f_α^L , f_α^U

Step 6 : Stop

The numerical solutions of f_α^L , f_α^U at different α levels can be gathered to approximate the shape of L(Z) and R(Z) from which the membership function can be constructed.

5. Numerical Example

To demonstrate how the proposed approach can be applied to analyze fuzzy decision questing problem with finite capacity, we present some examples often encountered in real fuzzy environment

A pizza unlimited restaurants has two franchises Model A has a capacity of 20 groups of customers and model B can seat 30 groups of customers. The monthly operating cost of model A is a frizzy trapezoidal number $\widetilde{C}_1$ = [10,000 11,500 12,500 14,000] An investor wants to set up a pizza restaurant and estimates that group of customers each occupying one table arrive according to a Poisson distribution at the rate of $\widetilde{\lambda}$ = [20 23 27 30] per hour. It all the tables are occupied, customers will go elsewhere. Model A will serve 26 groups per hour and model B will serve 29 groups per hour. Because of the variation in group sizes and in the types of orders, the service time is exponential. The investor estimates that the average cost of lost business per customer group per hour is Rs.15 The cost of serving additional Customer Rs.5. A delay in serving waiting customers is estimated as fuzzy trapezoidal number $\widetilde{C}_2$ = [5, 7, 12, 16] per customer group per hour. The Manager of restaurant wants to determine the optimum model so that the total expected cost per unit time is minimized.

For Model A, it is easy to find

$$\left[x_\alpha^L , x_\alpha^U\right] = [20 + 3\alpha, 30 - 3\alpha]$$

$$\left[u_\alpha^L, u_\alpha^U\right] = [10{,}000 + 1500\alpha, 14000 - 1500\alpha]$$

$$\left[v_\alpha^L, v_\alpha^U\right] = [5 + 2\alpha, 16 - 4\alpha]$$

Next it obvious

$x = x_\alpha^U$ $u = x_\alpha^U$ and $v = v_\alpha^U$ the expect total cost of queuing decision problem attains its maximum value and when $x = x_\alpha^L$, $u = u_\alpha^L$ $v = x_\alpha^L$, the expected total cost attains its minimum value. According to the cut of expected total cost of a queuing systems are

$$[E(c)]_\alpha^U = 26(14000 - 1500\alpha) + (16 - 4\alpha)\left[\frac{(30-3\alpha)/26}{1-\left(\frac{30-3\alpha}{26}\right)} - \frac{21\left(\frac{30-3\alpha}{26}\right)^{21}}{1-\left(\frac{30-3\alpha}{26}\right)^{21}}\right]$$

$$+ 20 \times 5 + 15(30 - 3\alpha)\left[\frac{1-\left(\frac{30-3\alpha}{26}\right)}{1-\left(\frac{30-3\alpha}{26}\right)^{21}}\right]\left(\frac{30-3\alpha}{26}\right)^{20}$$

$$[E(c)]_\alpha^L = 26(10000 + 1500\alpha) + 20 \times 5 + 15(20 + 3\alpha)\left(\frac{30+3\alpha}{26}\right)^{20}\left[\frac{1-\left(\frac{20+3\alpha}{26}\right)}{1-\left(\frac{20+3\alpha}{26}\right)^{21}}\right]$$

$$+ (5 + 2\alpha)\left[\frac{\left(\frac{20+3\alpha}{26}\right)}{1-\left(\frac{20+3\alpha}{26}\right)} - \frac{21\left(\frac{20+3\alpha}{26}\right)}{1-\left(\frac{20+3\alpha}{26}\right)^{21}}\right]$$

With the help of MATLAB 6.0, in used to solve the above mathematical programs and then the shape of can be found for given. Here we enumerate 11 value of 0, 0.1, 0.2, …, 1.0. The figures depict the rough shape μ from 22 values

$\left[[E(c)]_{\alpha}^{L}, [E(c)]_{\alpha}^{U}\right]$ for these values. The present the corresponding α cuts of μ.

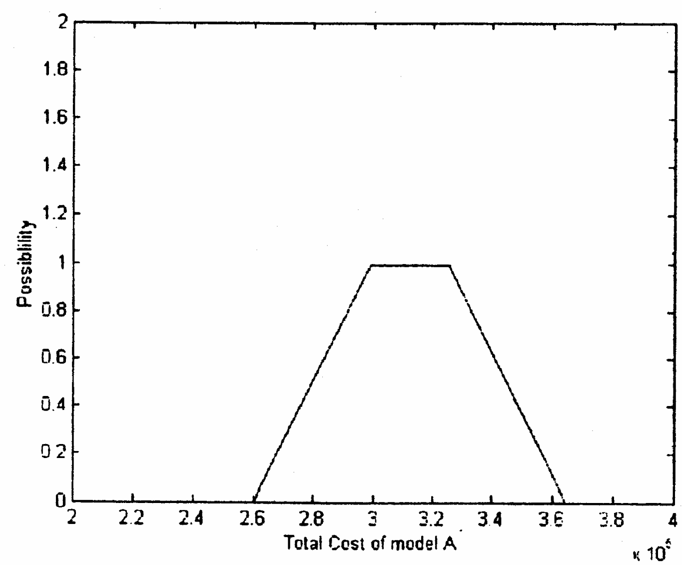

Figure 1

Table 1 : The α cuts of Total Expected Minimum cost per unit time at eleven α values in Model A.

α	$[E(c)]_{\alpha}^{L}$	$[E(c)]_{\alpha}^{U}$
0	2,60,120	3,64,400
.1	2,64,020	3,60,480
.2	2,67,920	3,56,570
.3	2,71,820	3,2,660
.4	2,75,720	3,48,740
.5	2,79,630	3,44,830
.6	2,83,530	3,40,910
.7	2,87,430	3,87,000
.8	2,91,340	3,33,090
.9	2,95,240	3,29,180
1	2,99,150	3,25,260

For Model B restaurant

$$\left[x_{\alpha}^{L}, x_{\alpha}^{U}\right] = [20+3\alpha, 30-3\alpha]$$

$$\left[u_\alpha^L, u_\alpha^U\right] = \left[10000+5000\alpha, 20000-1000\alpha\right]$$

$$\left[v_\alpha^L, v_\alpha^U\right] = \left[5+2\alpha, 16-4\alpha\right]$$

$$[\mathrm{E(c)}]_\alpha^{\mathrm{L}} = 29(10000-5000\alpha)+(5+2\alpha)\left[\frac{\left(\frac{20+3\alpha}{29}\right)}{1-\left(\frac{20+3\alpha}{29}\right)} - \frac{31\left(\frac{20+3\alpha}{29}\right)^{31}}{1-\left(\frac{20+3\alpha}{29}\right)^{31}}\right]$$

$$+15(20+3\alpha)\left[\frac{1-\left(\frac{20+3\alpha}{29}\right)}{1-\left(\frac{20+3\alpha}{29}\right)^{31}}\right]\left(\frac{20+3\alpha}{29}\right)^{30} + 30\times 5$$

and

$$[\mathrm{E(c)}]_\alpha^{\mathrm{U}} = 29(20000+1000\alpha)+(16-4\alpha)\left[\frac{\left(\frac{30-3\alpha}{29}\right)}{1-\left(\frac{30-3\alpha}{29}\right)} - \frac{31\left(\frac{30-3\alpha}{29}\right)^{31}}{1-\left(\frac{30-3\alpha}{29}\right)^{31}}\right]$$

$$+15(30-3\alpha)\left[\frac{1-\left(\frac{30-3\alpha}{29}\right)}{1-\left(\frac{30-3\alpha}{29}\right)^{31}}\right]\left[\left(\frac{30-3\alpha}{29}\right)^{30}\right] + 30\times 5$$

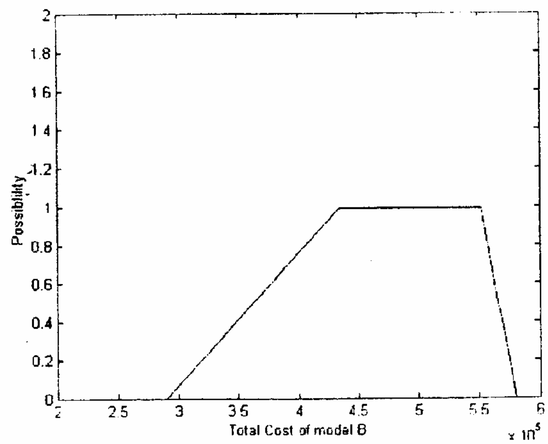

Figure 2

Table 2 : The α cuts of Total Expected Minimum cost per unit time at eleven α values in Model B.

α	$[E(c)]_{\alpha}^{L}$	$[E(c)]_{\alpha}^{U}$
0	2,90,160	5,80,410
.1	3,04,660	5,77,490
.2	3,19,160	5,74,580
.3	3,33,660	5,71,670
.4	3,48,170	5,68,760
.5	3,62,670	5,65,840
.6	3,77,170	5,62,930
.7	3,91,670	5,60,020
.8	4,06,170	5,57,110
.9	4,20,670	5,54,200
1	4,35,180	5,51,290

When the arrival rate, operating cost per unit of time a cost of delay in serving waiting customer are fuzzy the minimal expected total cost per unit time is also fuzzy.

Specifically the cut of α = 1 shows the minimal expected total cost per unit time that is most likely to be and the cut of α = 0 shows the range that the minimal expected total cost per unit time could appear In model A, the minimal total cost like between 2,60,120 and 3,64,400 lakhs. For Model B, the minimal total cost lies between 2, 90,160 and 5, 80,410.

6. Conclusion

Note that the membership function of the minimal expected total cost per unit time is in Model A Clearly if the obtained results of crisp values, then it may lose some useful information. In this paper since the fuzzy minimal expected, total cost per unit time is expressed by a membership function, it completely conserve the fuzziness.

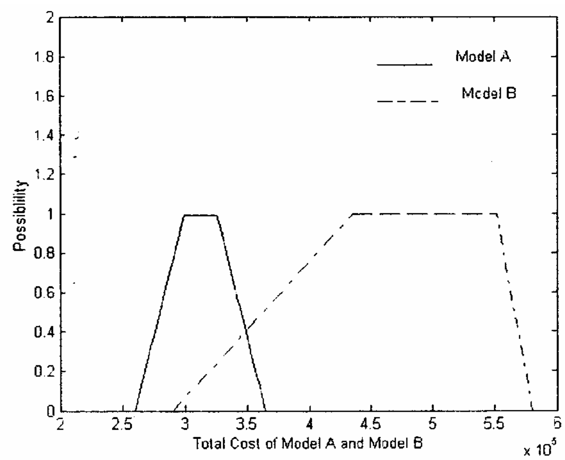

Figure 3

From the above figure, the Manager decides to prefer Model A, since the minimal total cost is obtained from the specified Model in which minimal total cost of the model is 2, 60, 116

REFERENCES

[1] Andradottir, S., 1995 A stochastic approximation algorithm with varying bounds Operations Research 43, 1037-1048.

[2] Arnott, R., de Palma, A., Lindsey, R., 1999. Information and time-of-usage decisions in the bottleneck model with stochastic capacity and demand. European Economic Review 43, 525-548.

[3] Buckley, J.J., 1990. Elementary queuing theory based on possibility theory. Fuzzy Sets and Systems 37, 43-52.

[4] Buckley, J.J., Qu, Y., 1990. On using a-cuts to evaluate fuzzy equations. Fuzzy Sets and Systems 38, 309-312.

[5] Buckley, J J., Feuring, T., Hayashi, Y., 2001. Fuzzy queuing theory revisited International Journal of Uncertainty, Fuzziness and Knowledge-based Systems 9, 527-537.

[6] Chen, S.H , 1985. Ranking fuzzy numbers with maximizing set and minimizing set Fuzzy Sets and Systems 17, 113-129.

[7] Gross, D, Harris, CM., 1998. Fundamentals of Queuing Theory, third ed. Wiley, New York

[8] Hillier, F S., Lieberman, GJ., 2001. Introduction to Operations Research, seventh ed. McGraw-Hill, Singapore

[9] Johnson, S.G., Larsen. C. 2001. Computation of a near-optimal service policy for a single-server queue with homogenous jobs. European Journal of Operational Research 134,648-663

[10] Kerbache. L. Smith. J. M., 2000. Multi-objective routing within large scale facilities using open finite queuing networks. European Journal of Operational Research 121, 105- 123

[11] Kwik. R , Tielemans, P F J., 2004 Expected time in system analysis of a single-machine multi-item processing centre European Journal of Operational Research 156, 287-304

[12] Larsen, C, 1998. Investigating sensitivity and the impact of information on pricing decisions in an M/M/l/ α queuing model International Journal of Production Economics 56-57, 365-377

[13] Law, A M , Kelton, W D, 2000 Simulation Modeling and Analysis, third ed McGraw- Hill, Singapore

[14] Papadopoulos, H.T., 1996. A field service support system using a queuing network model and the priority MVA algorithm. Omega 24, 195-203.

[15] Ross, S M, 2003. Introduction to Probability Models, eighth ed Academic Press, New York

[16] Shih, H., Park, J., Lee, C, Park, J., 1997. A decision support model for the initial design of FMS. Computers and Industrial Engineering 33, 549-552,

[17] Taha, H.A., 2003. Operations Research : An Introduction, seventh ed. Prentice-Hall, New Jersey.

[18] Yager, R.R., 1986. A characterization of the extension principle. Fuzzy Sets and Systems 18,205-217.

[19] Zadeh, LA., 1978. Fuzzy sets as a basis for a theory of possibility. Fuzzy Sets and Systems 1, 3-28.

[20] Zimmermann, H. J., 2001. Fuzzy Set Theory and its Applications, fourth ed. Kluwer-Nijhoff, Boston.