PG/IS/MTM/106/23

[redacted] 1st Semester Examination, 2023

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Graph Theory*)

PAPER – MTM-106

Full Marks : 20

Time : 1 hour

Anawer **all** questions

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answer whereve necessary

1. Answer any *two* questions : 2×2

(*a*) Prove that any connected graph with n vertices and $(n-1)$ edges is a tree.

(*Turn Over*)

(*b*) Draw the undirected graph G corresponding to adjacency matrix A :

$$A = \begin{vmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{vmatrix}$$

(*c*) Write down a short note on "Spanning tree".

(*d*) Show that the maximum number of edges in a complete biparite graph of n vertices is $\frac{n^2}{4}$.

2. Answer any *two* questions : 4×2

(*a*) Prove that every tree has either one or two centre.

(*b*) If G is a simple connected planar graph with $n\ (\geq 3)$ vertices and e edges, then prove that $e \leq 3n - 6$ and using this, verify that $K_{3,3}$ (Kuratowski's second graph) is planar or not.

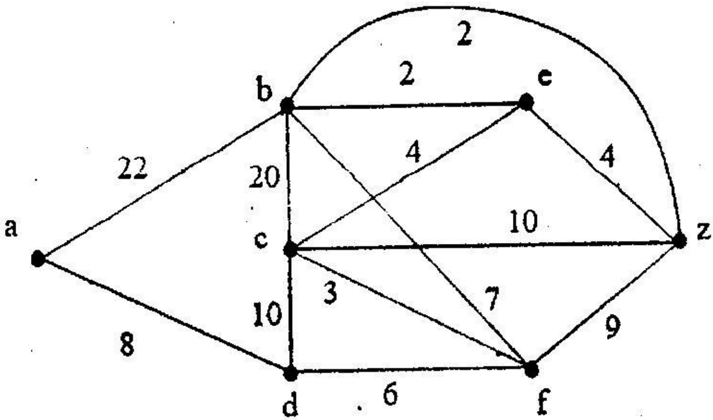
(*c*) Show that a simple graph with n vertices and k components cannot have more than $\frac{(n-k)(n-k+1)}{2}$ edges.

(*d*) Define the term "Chromatic number" for graph colouring. Find the chromatic number of Peterson graph.

3. Answer any *one* question : 8×1

(*a*) (*i*) Show that a simple connected planar graph with 6 vertices and 12 edges, each of the face is bounded by 3 edges. 3

(*ii*) Apply Dijkstra's algorithm to the graph given below and find the shortest path from *a* to *z*. 5



(*b*) (*i*) Prove that the relation $\chi(G) \leq \Delta(G) + 1$ for any graph *G*, where $\chi(G)$ is the chromatic number and $\Delta(G)$ is the maxium degree of a vertex in *G*. 3

(*ii*) Show that the chromatic number of a cycle with *n* vertices (C_n) is 2 if *n* is even and 3 if *n* is odd. 4

(*iii*) Explain why the following polynomial cannot be a chromatic polynomial,
$$\lambda^3 + 5\lambda^2 - 3\lambda + 5.$$ 1

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