PG 1st Semester Examination, 2023

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Classical Mechanics and Non-linear Dynamics*)

PAPER – MTM-105

Full Marks : 50

Time : 2 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

1. Answer any *four* questions : 2 × 4

(*a*) Establishe the relationship between the Poisson bracket and Hamilton's equations in classical mechanics.

(*b*) What is the Coriolis force, and how does it relate to a rotating frame ?

(*c*) How does the Lagrangian differ from the Hamiltorian in classical mechanics ?

(*d*) State the basic postulates of the special theory of relativity.

(*e*) State Hamilton's principle and write its significance.

(*f*) Suppose a rigid body is rotating about a fixed point. Find its kinetic energy in terms of moment of inertia and product of inertia.

2. Answer any *four* questions : 4×4

(*a*) State and prove the Poincaré theorem.

(*b*) A Lagrangian for a particular physical system can be written as

$$L = \frac{m}{2}\left(\dot{x}^2 + 2b\dot{x}\dot{y} + c\dot{y}^2\right) - \frac{k}{2}\left(ax^2 + 2bxy + cy^2\right),$$

where a, b and c are arbitrary constants satisfying $b^2 - ac \neq 0$. Determine the

Lagrange's equations of motion. Examine particularly the case $a = 0 = c$.

(*c*) What do you mean by canonical transformation ? If $Q = ap + bq$, $P = cp + dq$ is a canonical transformation, then find the relation among a, b, c, d.

(*d*) Prove that

$$J = \int_{x_0}^{x_1} F(y_1, y_2, \ldots, y_n, y_1', y_2', \ldots, y_n', x)dx$$

will be stationary only if

$$\frac{d}{dx}\left(\frac{\partial F}{\partial y_j'}\right) - \frac{\partial F}{\partial y_j} = 0 \text{ for all } j = 1,2,\ldots,n.$$

(*e*) Given a mass-spring system consisting of a mass m and a linear spring of stiffness k hanging from a fixed point. Find the equation of motion using the Hamiltonian procedure, and assume that the displacement x is measured from the unstretched position of the spring.

(*f*) Let $G = G_1(q_1, q_2, ..., q_n, Q_1, Q_2, ..., Q_n, t)$ be a generating function of a canonical transformation. Prove that

$$p_j = \frac{\partial G_1}{\partial q_j}, P_j = -\frac{\partial G_1}{\partial Q_1}$$

for all j. Hence, prove that if the canonical transformation is given, then one can determine the generating function.

3. Answer any *two* questions : 8×2

(*a*) For any vector R, derive the following equation

$$\left(\frac{dR}{dt}\right)_{fix} = \left(\frac{dR}{dt}\right)_{rot} + w \times R$$

the symbols have their usual meanings.

(*b*) Suppose a particle of mass m_0 is moving with a velocity v then show that its mass at any time is given by

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}},$$

where c is the speed of the light.

(*c*) Deduce Hamilton's equations of motion starting from Lagrange's equation of motion for an unconnected holonomic system. Hence, show that $H + L$ is explicitly independent of time.

(*d*) Consider the following nonlinear dynamical system, $\dot{x} = x^2 y - x^5, \dot{y} = -y + x^2$. Study the stability at the origin and draw the phase diagram.

[Internal Assessment — 10 Marks]
