# PG 1st Semester Examination, 2023

## MATHEMATICS

( *Complex Analysis* )

PAPER – MTM-102

*Full Marks* : 50

*Time* : 2 hours

*The figures in the right hand margin indicate marks*

*Candidates are required to give their answers in their own words as far as practicable*

**1.** Answer any *four* quesions : 2×4

(*a*) With necessary conditions, write the Homotopy form of Cauchy's theorem.

(*b*) Is it possible to evaluate the integral $\int_C f(z)dz$ where $f(z)=(5z+2)/(z(z-2))$ and

( Turn Over

$C : |z| = 1$ using the single residue of $\frac{1}{z^2} f\left(\frac{1}{z}\right)$ at z = 0 ? Justify.

(*c*) Under what condition/s the bilinear transformation $f(z) = \frac{az+b}{cz+d}$ has only one fixed point ? Justify your answer.

(*d*) Find the singular points of

$$f(z) = \frac{\log(z+2-3i)}{z^2+4}$$

and plots them in the complex plane.

(*e*) Find the Mobious transformation that maps 1, 0, −1 to the respective points *i*, ∞, 1.

(*f*) Find the order of the pole at $z = \frac{\pi}{4}$ of the function $f(z) = \frac{1}{\cos z - \sin z}$.

2. Answer any *four* questions : $4 \times 4$

(*a*) Write Taylor's and Laurent's series representation of a function $f(z)$ by stating necessary condition/s for each of the series. Hence discuss under what condition/s the Laurent's series of the said function reduced to Taylor's series of the said function.

(*b*) Find the inverse of the bilinear transformation $f(z) = \dfrac{az+b}{cz+d}$ and show that this inverse function is also a bilinear. Also show that the determinants of both the transformations are the same.

(*c*) Use an antiderivative and evaluate the integeral

$$\int_{-1-i\sqrt{3}}^{1+i\sqrt{3}} \left( \frac{5\pi}{z} + 3iz^{i-1} \right) dz$$

by taking any path of integration in region

$y < \sqrt{3x}$ from $z = -1 - \sqrt{3i}$ to $z = 1 + \sqrt{3i}$, except for its end points. Use principal branches of the required functions.

(*d*) Using the calculus of residue evaluate

$$\int_0^\infty \frac{x \sin 2x}{x^2 + 3} dx.$$

(*e*) With the help of residue, find the inverse Laplace transformation $f(t)$ of

$$F(s) = \frac{s}{(s^2 + a^2)^2} (a > 0)$$

(*f*) Prove that the zeros of an analytic function are ioslated.

3. Answer any *two* questions : 8 × 2

(*a*) (*i*) State and prove the argument principal theorem.

(*ii*) State the Cauchy Residue theorem. 6 + 2 (*b*) (*i*) Prove that if $\psi(u,v)$ satisfies the Laplace equation in the $(u,v)$ plane, then $\phi(x,y)$ satisfies the Laplace equation in the $(x,y)$ plane.

(*ii*) Write the Jordan's lemma. 6 + 2

(*c*) (*i*) Using the method of residues, evaluate

$$\int_0^\infty \frac{x^a}{(x^2+1)^2}\,dx$$

where $-1<a<3$ and $x^a = \exp(a \ln x)$.

(*ii*) Find the singular points of the function $z|z|$, if any. Justify your answer. 6 + 2

(*d*) (*i*) Define the direct analytic continuation of an analytic function.

(*ii*) State and prove the Casorati-Weierstrass theorem. 2 + 6

**[ Internal Assessment — 10 Marks ]**

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