Total Pages—13 **DDE/II/A.MATH/IX/13**

M.Sc. Part-II Examination, 2013

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER— IX (OR/OM)

Full Marks : 100

Time : 4 hours

The figures in the right-hand margin indicate marks

Special Paper : OR (*Advanced Optimization and Operations Research* - I)

Answer **Q. No. 11** and any **six** from the rest

1. (*a*) Apply Wolfe's method for solving the following problem : 7

Maximize $Z = -4x_1 + x_1^2 - 2x_1x_2$

subject to the constraints

$$2x_1 + x_2 \leq 6$$
$$x_1 - 4x_2 \leq 0$$
$$x_1, x_2 \geq 0$$

(*Turn Over*)

(*b*) State and prove Wolfe's duality theorem. 5

(*c*) Define the following : 4

(*i*) Minimization problem involving differentiable function

(*ii*) Local Minimization problem for differentiability.

2. (*a*) Apply Beale's method for solving the following quadratic programming problem :

Maximize $Q(x) = x_1^2 + 3x_2^2$

subject to

$$x_1 + 3x_2 \geq 5$$
$$5x_1 + 2x_2 \geq 2$$
$$x_1, x_2 \geq 0$$

8

(*b*) State and prove Fritz-John saddle point necessary optimality theorem. When does the theorem fail ? 6 + 2

3. (*a*) Using decomposition principle reduce the following problem to an elegant form of LPP which can be solved by simplex or revised simplex method.

Maximize $Z = 8x_1 + 3x_2 + 8x_3 + 6x_4$

subject to

$$4x_1 + 3x_2 + x_3 + 3x_4 \leq 16$$
$$4x_1 - x_2 + x_3 \leq 12$$
$$x_1 + 2x_2 \leq 8$$
$$3x_1 + x_2 \leq 10$$
$$2x_3 + 3x_4 \leq 9$$
$$4x_3 + x_4 \leq 12$$

and $x_j \geq 0,\ j = 1, 2, 3, 4$ 7

(*b*) What do you mean by theorems of the alternative ? State and prove Slater's theorem of the alternative. 6

(*c*) What are the basic differences between first existence and second existence theorems concerned with non-linear programming ? 3

4. (*a*) State Farkas' theorem and give its geometrical interpretation of non-linear programming. 6

(*b*) Let x^o be an open set in R^n, let Q and g be defined on x^o. Find the conditions under which a solution $(\bar{x}, \bar{r}_0, \bar{r})$ of the Fritz-John saddle point problem is a solution of the Fritz-John stationary point problem and conversely. 2

(c) Solve the following LPP using bounded variable simplex method :

$$\text{Maximize } Z = 2x_1 + x_2$$

subject to the constraints

$$x_1 + 2x_2 \le 40$$
$$x_1 + x_2 \le 6$$
$$x_1 - x_2 \le 2$$
$$x_1 - 2x_2 \le 1$$
$$0 \le x_1 \le 3 \text{ and } 0 \le x_2 \le 2.$$ 8

5. (a) Derive the Kuhn-Tucker conditions for quadratic programming problem. Under what condition, these conditions will be necessary and sufficient ? 8

(b) Let θ be a numerical function defined on an open set $\Gamma \subset R^n$ and let θ be differentiable of $\bar{x} \in \Gamma$. If θ is concave at $\bar{x} \in \Gamma$, then prove that

$$\theta(x) - \bar{\theta}(x) \leqq \nabla\theta(\bar{x})(x - \bar{x}),$$

for each $x \in \Gamma$. 6

(c) What are the basic differences between Wolfe's method and Beale's method ? 2

6. (a) Use revised simplex method to solve the following LPP :

$$\text{Maximize } Z = 6x_1 - 2x_2 + 3x_3$$

subject to

$$2x_1 - x_2 + 2x_3 \le 2$$
$$x_1 + 4x_3 \le 4$$
$$\text{and } x_1, x_2, x_3 \ge 0$$ 8

(b) Discuss the effect of discrete change in the requirement vector b to the LPP

$$\text{Maximize } Z = Cx$$
$$\text{subject to } \mathscr{A}x = b,\ x \ge 0$$

where $C, x^T \in R^n$, $b^T \in R^m$ and $\mathscr{A}$ is an $m \times n$ matrix. 8

7. (a) Using Branch and Bound technique solve the following IPP :

$$\text{Max. } Z = x_1 + x_2$$

sub. to

$$3x_1 + 2x_2 \le 12$$
$$x_2 \le 2$$

and $x_1, x_2 \ge 0$ and are integers. 8

(*b*) Using graphical method solve the following Goal Programming Problem :

Minimize $z = P_1d_1^+ + P_1d_2^- + 30P_2d_3^- + 40P_2d_4^- + P_3d_5^-$

subject to $2x_1 + 4x_2 + d_1^- - d_1^+ = 80$

$3x_1 + 3x_2 + d_2^- - d_2^+ = 80$

$x_1 + d_3^- - d_3^+ = 12$

$x_2 + d_4^- - d_4^+ = 12$

$30x_1 + 40x_2 + d_5^- - d_5^+ = 1200$

$x_1, x_2, d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+, d_4^-, d_4^+, d_5^-, d_5^+ \geq 0$ 8

8. (*a*) Using Davidon-Fletcher Powell method minimize the function

$$f(x_1, x_2) = 8x_1^2 + 4x_2^2 - 24x_1 + 16x_2 + 35$$

with $\begin{bmatrix} 1/2 \\ 1 \end{bmatrix}$ as the starting point. 8

(*b*) Discuss the Golden section method to find the minimum point of a unimodal function defined on $[a, b]$. How this method differs from the Fibonacci method ? 8

9. (*a*) Using steepest descent method to minimize the function

$$f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$$

starting from $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Also compare this optimum point with exact optimum point determine by solving $\nabla f(x) = 0$. 6 + 2

(*b*) Using Golden section method minimize the function

$$f(x) = \begin{cases} (x^2 - 6x + 13)/4, & x \leq 4 \\ x - 2, & x > 4 \end{cases}$$

in the interval [3, 5] upto six experiments. 8

10. (*a*) Derive the expression for Gomory-cut in the case of IPP. Apply it to obtain the initial iterate to the following problem :

Max. $Z = 9x_1 + 7x_2$

sub. to

$$3x_1 - x_2 \le 6$$

$$x_1 + 7x_2 \le 35$$

and $x_1, x_2 \ge 0$ and are integers. 4 + 4

(*b*) The optimum solution of the LPP :

Max. $Z = x_1 + 4x_2 - 2x_3 + 3x_4 - x_5$

sub. to

$$x_1 - 3x_2 + x_3 + 2x_4 + 6x_5 \le 3$$

$$2x_1 + x_2 + 3x_4 + 2x_5 \le 6$$

$$4x_1 + x_2 - x_4 + x_5 \le 2$$

and $x_1, x_2, x_3, x_4, x_5 \ge 0$.

is contained in the table :

		C_j	1	4	-2	3	-1	0	0	0
Basis	C_B	b	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8
x_6	0	10	$\frac{25}{2}$	0	1	0	$\frac{37}{4}$	1	$\frac{1}{4}$	$\frac{11}{4}$
x_4	3	1	$-\frac{1}{2}$	0	0	1	$\frac{1}{4}$	0	$\frac{1}{4}$	$-\frac{1}{4}$
x_2	4	3	$\frac{7}{2}$	1	0	0	$\frac{5}{4}$	0	$\frac{1}{4}$	$\frac{3}{4}$
$Z_j - L_j$		15	$\frac{23}{2}$	0	2	0	$\frac{27}{4}$	0	$\frac{7}{4}$	$\frac{9}{4}$

find the range over which c_1, c_3, c_4 and b_2 can be changed one at a time, so that the obtimality of the current solution remains undisturbed, where c_1, c_3 and c_4 are 1st, 3rd and 4th cost components of the objective function and b_2 is the 2nd component of the requirement vector. 2+2+2+2

11. Answer any *one* :

(*a*) What are the differences between regular simplex method and dual simplex method ? What are the advantages of regular simplex method ? 2 + 2 (*b*) Write a short note on any *one* of the following : 4

Differentiable Convex and Concave functions.

Special Paper : OM

Answer Q. No. 11 and any six from the rest

1. Give a definition of salinity of sea-water. Derive the following relations :

(*i*) $C_v = C_p + T\left\{\left(\frac{\partial\tau}{\partial p}\right)^2 \Big/ \left(\frac{\partial\tau}{\partial p}\right)\right\}$

(*ii*) $\Gamma = \left(\frac{T}{C_p}\right).\frac{\partial\tau}{\partial p}$

(*iii*) $\Gamma_\eta = K_T - \Gamma.\alpha = K_T(C_v / C_p)$

where symbols have their usual meanings. 16

2. (*a*) Assuming that sea-water is a two-component mixture of salt and pure water, show that the

principle of conservation of mass leads to the pair of equations

$$\frac{D\rho}{Dt} + \rho \operatorname{div} \vec{q} = 0, \ \rho\frac{Ds}{Dt} = -\operatorname{div} \vec{I}_s$$

is usual notations. 8

(*b*) Assuming that the mass exchange process across the free ocean surface $F(\vec{r}, t) = 0$ amount to a flux, b, of pure water in unit time per unit area, obtain the boundary conditions at the free ocean surface. 8

3. (*a*) Find the condition of stability of equilibrium of a stratified fluid and hence explain the significance of the Brünt-Väisälä frequency. 8

(*b*) Obtain an expression of the Brünt-Väisälä frequency for the following cases : 8

(*i*) In layers where temperature and salinity variation with depth are large.

(*ii*) In a homogeneous layer where salinity and temperature vary little with depth.

4. Assuming the sea-water to be viscous compressible heat-conducting fluid, derive the energy equation in the form

$$\frac{\partial}{\partial t}(\rho E_m) = -\text{div}\,\vec{I}_E.$$

Hence deduce the equation of entropy evolution. 16

5. Stating assumptions, give a mathematical formulation of the Ekman-model of wind driven current in a homogeneous ocean. Solve the system of equation to explain the vertical structure of the flow. Find the volume flux for such flows. 16

6. Explain β-plane approximation. Assuming the sea-water to be a non-viscous stratified fluid, deduce the β-plane equations and examine the range of validity of these equations. 16

7. In two-dimensional model of ocean currents, solve the problem for viscous boundary layer and show that a weak back flow appears close to the external edge of the boundary of Western-shore. 16

8. Deduce the momentum equation of motion of the fluid on rotating earth. Also explain the physical interpretation of each term. 11 + 5

9. Deduce the equations of inertia currents under some assumptions stated by you. Hence find the inertial period at an equator and at pole of the earth. 8 + 8

10. Discuss the Poincare' and Kelvin waves when

$$\alpha = \frac{n\pi}{L}, \; n = 1, 2, 3, \ldots$$

where L is the width of the channel and α have its usual meanings. 16

11. Define adiabatic temperature. 4

Or

Define Ekman number and Rossby number. 2 + 2