DDE/II/A.MATH/VIII/13

M.Sc. Part-II Examination, 2013

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—VIII

Full Marks : 100

Time : 4 hours

The figures in the right-hand margin indicate marks

GROUP – A

(*Mathematical Methods*)

[*Marks* : 50]

Answer **Q. No. 1** and any **three** from the rest

1. Answer any *two* questions : 4 × 2

(*a*) Write down the Laplace transform $F(p)$ of a function $f(x)$ defined for $x \geq 0$, and its inverse. In the inverse transform formula,

explain all the terms in some detail. Does the Laplace transform of the function $f(x) = e^{x^2} (x \geq 0)$ exist ? Give reasons for your answer.

(*b*) If $f(t)$ is continuous and is of exponential order at $t \to \infty$ and $f'(t)$ is piecewise continuous in any finite interval, then show that

$$\underset{p \to \infty}{\text{Lt}}\ p\, f(p) = f(\infty)$$

(*c*) Write down the Hankel transform of order 0 of a function $f(r)$ defined for $r > 0$ and its inverse. Find the Hankel transform of order 0 of the function $f(r) = \frac{1}{r}$.

2. (*a*) Solve

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, y \geq 0, |x| < \infty$$

subject to the following conditions

$$u(x, 0) = f(x), |x| < \infty,$$

$$u, \frac{\partial u}{\partial x} \to 0 \text{ as } x \to \pm \infty$$ 7

(*b*) Using Green's function, solve the boundary value problem

$$y''(x) - y(x) = x,$$
$$y(0) = y(1) = 0.$$ 7

3. (*a*) Find the exponential Fourier transform of the function $f(x)$, where

$$f(x) = \begin{cases} 1 - x^2, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$$

Hence evaluate the integral

$$\int_0^\infty \frac{\sin w - w \cos w}{w^3} \cdot \cos \frac{w}{2}\, dw$$ 7

(*b*) Use the method of Laplace transform to solve the integral equation

$$\int_0^x \frac{\varphi(t)}{(x-t)^{1/2}}\, dt = f(x), x > 0 \left(f(0) = 0\right).$$ 7

4. (*a*) State and prove Browmich integral formula on Laplace transform. Using it, evaluate

$$L^{-1}\left\{\frac{1}{(p+1)(p-2)^2}\right\}$$

where L^{-1} stands for an inverse Laplace transform. 5 + 3

(*b*) Reduce the differential equation

$$y''(x) - y = 0,\ x \geq 0$$

with $y(0) = 0, y'(0) = 0$
to an equivalent volterra integral equation of the second kind. 6

5. (*a*) Find the characteristic values and characteristic functions of Sturm-Liouville problem

$$\frac{d}{dx}\left[x\frac{dy}{dx}\right] + \frac{\lambda}{x}y = 0;$$

$$y'(1) = 0,\ y'(e^{2\pi}) = 0.$$

Also verify that the four properties for the Sturm-Liouville problem. 8

(*b*) Using Laplace transform, solve the ODE

$$y''(x) + 4y(x) = 3\sin x$$

with $y(0) = 1, y'(0) = 1.$ 6

6. (*a*) If the integral

$$\int_0^\infty f(r)\,dr$$

is absolutely convergent and $f(r)$ is continuous in the neighbourhood of r, then prove that

$$f(r) = \int_0^\infty \alpha\, F_0(\alpha)\, J_0(\alpha r)\, d\alpha, \text{ where } F_0(\alpha)$$

is the Hankel transform of order zero of the function $f(r)$ and $J_0(\alpha r)$ is the Bessel's function of order zero. 6

(*b*) Define regular sequence. When two regular sequences are equivalent ? When do you mean by generalised function ? 3

(c) Let $F(k)$ and $G(k)$ be the Fourier transforms of $f(x)$ and $g(x)$ respectively defined in $(-\infty, \infty)$. Show that the Fourier transform of

$$\int_{-\infty}^{\infty} f(u)\,g(x-u)\,du$$

can be expressed in terms of the product $F(k)\,G(k)$. Hence prove the Parseval's relation

$$\int_{-\infty}^{\infty} |F(k)|^2\,dk = \int_{-\infty}^{\infty} |f(x)|^2\,dx \qquad 5$$

GROUP – B

(Elements of Optimization and Operations Research)

[For the students whose special paper is OM]

[*Marks* : 50]

Answer **Q. No. 7** and any **three** from the rest

7. How do you compute reduced costs for non-basic variables in revised simplex method ? 2

Or

What is the importance of integer programming problem to solve non-linear programming problem ? 2

8. (a) Derive the order-level lot size system with finite replenishment rate, constant demand, and zero lead time. 8

(b) The demand for an item in a company is 18,000 units per year and the company can produce the item at a rate 3,000 per month. The cost of one set up is INR 500 and the holding cost of one unit per month is INR 0·15. The shortage cost of one unit is INR 20·00 per month. Determine the optimum manufacturing quantity and the number of shortages. Also determine the manufacturing time and time gap between two set ups. 8

9. (a) Solve the LPP by revised simplex method

$$\text{Max } Z = 2x_1 + 3x_2 - x_3 + 4x_4 + x_5 - 3x_6$$

subject to the constraints

$$x_1 - 2x_2 + x_4 + 4x_5 + \frac{1}{2}x_6 \leq 10$$
$$x_1 + x_2 + 3x_3 + 2x_4 + x_5 - x_6 \leq 16$$
$$2x_1 + \frac{1}{2}x_2 - x_3 - x_4 + 2x_5 + 5x_6 \leq 8$$
$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$ 8

(*b*) What are the basic differences between the simplex method and the revised simplex method ? 4

(*c*) Derive the algorithm to solve an LPP by revised simplex method. 4

10. (*a*) Derive the effects for variation in the cost vectors of an LPP

$$\text{Max } Z = CX$$
subject to
$$AX = b$$
$$X \geq 0,$$

such that the costs, in basic variables, remain unaltered. 8

(*b*) Using simplex method, solve the linear programming problem

$$\text{Max } Z = 3x_1 + 5x_2$$
subject to
$$x_1 + x_2 \leq 1,$$
$$2x_1 + 3x_2 \leq 1,$$
$$x_1, x_2 \geq 0.$$

Obtain the variations in $C_j (j = 1, 2)$ which are permitted without changing the optimal solutions. 8

11. (*a*) Derive the dynamic programming algorithm to solve a non-linear programming problem. 8

(*b*) Divide a positive quantity C into n parts in such a way that their product maximum. Solve this problem by using dynamic programming problem. 8

12. (*a*) Using Wolfe's method, solve the quadratic programming problem

$$\text{Max } Z = 2x_1 + x_2 - x_1^2$$
subject to
$$2x_1 + 3x_2 \leq 6,$$
$$2x_1 + x_2 \leq 4,$$
$$x_1, x_2 \geq 0.$$ 8

(*b*) Use dynamic programming to show that

$$Z = p_1 \log p_1 + p_2 \log p_2 + \cdots + p_n \log p_n$$

subject to the constraints

$$p_1 + p_2 + \cdots + p_n = 1$$
$$p_j \geq 0 \ ; (j = 1, 2, \cdots n)$$

is a minimum when $p_1 = p_2 = \cdots = p_n = 1/n$. 8

GROUP – B

(*Dynamical Oceanology and Meteorology*)

[For students whose special paper is OR]

[*Marks* : 50]

Answer **Q. No. 12** and any **three** from the rest

7. (*a*) What do you mean by adiabatic process ? Deduce the Poisson's equation. 6

(*b*) Derive the equation of motion of an air parcel in Cartesian form. 7

(*c*) Find the relation between mixing ratio and specific humidity. 3

8. (*a*) What is the thermal wind in the atmosphere ? Derive the thermal wind components in the atmosphere. 7

(*b*) Show that the pressure tendency at the earth surface is zero when the air motion at all levels in the atmosphere is geostrophic. 5

(*c*) Explain the convergence and divergence in the atmosphere. 4

9. (*a*) Derive the adiabatic lapse rate of moist, unsaturated air in the atmosphere. 6

(*b*) Show that given volume of moist air is lighter than an equal volume of dry air at the same pressure and temperature. 3

(*c*) Assuming that the mass exchange process across the free ocean surface $F(\vec{r}, t) = 0$ amount to a flux, b, of pure water in unit time per unit area, obtain the boundary conditions at the free ocean surface. 7

10. Define salinity of sea-water. Derive the following relations :

(*i*) $c_p = c_v + T\left\{\left(\frac{\partial \tau}{\partial T}\right)^2 \Big/ \frac{\partial \tau}{\partial p}\right\}$

(*ii*) $\Gamma = \frac{T}{c_p} \cdot \frac{\partial \tau}{\partial T}$

(*ii*) $K_\eta = K_T - \Gamma \cdot \alpha = K_T\left(c_v / c_p\right)$

where symbols have their usual meanings. 16

11. Show that under usual notations

$$T = -\frac{1}{\lambda},\ \mu_s = -U - \frac{\lambda_s}{\lambda} + \frac{\vec{q}^2}{2}$$

$$\mu_w = -U - \frac{\lambda_w}{\lambda} + \frac{\vec{q}^2}{2},\ \vec{q} = -\frac{\vec{a}}{\lambda} - \frac{1}{\lambda}\left(\vec{b} \times \vec{r}\right)$$

are the necessary conditions of thermo-dynamical equilibrium of a finite volume of sea-water. Hence deduce the hydrostatic pressure quations (symbols have their usual meanings). 16

12. Define adiabatic temperature gradient. 2
