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# M.Sc. Part-I Examination, 2013

## APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER— IV (A & B)

*Full Marks* : 100

*Time* : 4 hours

*The figures in the right-hand margin indicate marks*

## GROUP — A

( *Principle of Mechanics* )

[ *Marks* : 50 ]

Answer **Q. No. 1** and any **three** questions from the rest

**1.** Answer any *one* question : 2

(*a*) State and prove conservation law of linear momentum.

(*b*) Define generalized coordinates with example.

( *Turn Over* )

2. (*a*) What do you mean by Euler angles ? Suppose a rigid body is rotating about a fixed point. Deduce the relation between the coordinates $(x, y, z)$ (in fixed set of axes) and $(x', y', z')$ (in rotating set of axes) in terms of Euler angles). 8

(*b*) Deduce Euler's dynamical equations when a rigid body is rotating about a fixed point. 8

3. (*a*) Deduce Lagrange's equations of motion for a conservative unconnected holonomic system. 8

(*b*) If $L$ is a Lagrangian for a system of $n$ degrees of freedom satisfying Lagrange's equation, show by direct substitution that

$$L' = L + \frac{dF(q_1, q_2, \ldots, q_n, t)}{dt}$$

also satisfies Lagrange's equations where $F$ is arry arbitrary but differentiable function of its arguments. 4





(*c*) The Hamiltonian

$$H = p_1 q_1 - p_2 q_2 - a q_1^2 - b q_2^2,$$

solves the Hamilton's equations of motion and prove that $(p_2 - bq_2)/q_1$ = constant and $q_2 q_2$ = constant, $a$, $b$ are constants $p_1, p_2$, $q_1, q_2$ are generalised momenta and coordinates. 4

4. (*a*) Prove that

$$J = \int_{x_0}^{x_1} F(y, y', x)\, dx$$

will be minimum only when

$$\frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) - \frac{\partial F}{\partial y} = 0.$$ 8

(*b*) Define Poisson bracket. State and prove Jacobi's identity. 4

(*c*) If $H$ is the Hamiltonian and $f$ is any function depending on position, momenta and time show that

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + [f, H].$$ 4

5. (*a*) Discuss a method to determine the eigen frequencies and normal modes of small oscillation of a dynamical system. 8

(*b*) A top has an axis of symmetry $OG$, where $G$ is the centre of mass, and it spins with the end $O$ on a rough horizontal table. The mass of the top is $m$ and its moment of inertia about $OG$ and any axis through $O$ perpendicular to $OG$ are $C$ and $A$ respectively. Initially, $OG$ is vertical and the top is set spinning with spin $n$ about its axis. It is then slightly displaced. If in the subsequent motion $\theta$ is the angle $OG$ makes with the vertical and $\phi$ is the angular velocity about the vertical, show that

$$A\,\dot{\varphi}\sin^2\theta = Cn(1-\cos\theta) \text{ and}$$

$$A\,(\dot{\varphi}^2\sin^2\theta + \dot{\theta}^2) = 2mgh(1-\cos\theta)$$

where $OG = h$. 8

6. (*a*) Derive the Lorentz transformation equations. 8

(*b*) Under Lorentz transformation, show that the expression $x^2 + y^2 + z^2 - c^2t^2$ is invariant. 4





(*c*) Show that the transformation

$$Q = \log(\sin p/q),\ P = q\cot p$$

is canonical. Find the generating function $G(q, Q)$. 4

GROUP – B

( *Partial Differential Equation* )

[ *Marks* : 50 ]

Answer **Q. No. 1** and any **three** questions from the rest

1. What do you mean by a Partial Differential Equations (PDE) ? Give an example. 2

*Or*

Define a semi-linear and a quasi-linear partial differential equation.

2. (*a*) Show that Lagrange equation

$$(2z - y)p + (x + z)q + (2x + y) = 0$$

has the complete integral $x^2 + y^2 + z^2 = \alpha(x - 2y - z) + \beta$, a family of sphere. Find the envelope of the one parameter family by substituting $\beta = 1 - \frac{3\alpha^2}{2}$ and show that the envelope is a part of the given integral. 8

(*b*) Derive Charpit's method to solve a non-linear PDE of order one for two independent variables. 8

**3.** (*a*) Solve the partial differential equation

$$(D^2 - 3DD' + 2D'^2 - D + 2D')z = (2 + 4x)e^{-y}$$

where $D \equiv \frac{\partial}{\partial x}$, $D' \equiv \frac{\partial}{\partial y}$. 8

(*b*) Reduce the partial differential equation to its canonical form

$$(n-1)^2 z_{xx} - y^{2n} z_{yy} = ny^{2n-1} z_y,$$

$n$ is any positive integer. 8





**4.** (*a*) Derive the equation of transverse vibration of a stretched string of length '$l$' with fixed end points. 8

(*b*) A tightly stretched string with fixed end points at $x = 0$ and $x = l$ is initially in a position

$$y(x, 0) = a \sin^3 \frac{\pi x}{l}.$$

If it is released from rest from this position find the displacement $y$ at a distance $x$ from one end at any time $t$. 8

**5.** (*a*) If a region $G$ is two dimensional and simply -connected, show that it is possible to reduce Neumann problem to Dirichlet problem. 8

(*b*) Find the function $\phi(x, y)$ which satisfies Laplace equation $\phi_{xx} + \phi_{yy} = 0$ in the rectangle $0 < x < a$, $0 < y < b$ and which also satisfies the boundary conditions $\phi(x, 0) = 0$, $\phi(x, b) = 0$, $\phi(0, y) = 0$, $\phi(a, y) = f(y)$, where $f(y)$ is a given function of $y$ for $0 < y < b$. 8

6. (*a*) Solve the diffusion equation $\theta_t = \theta_{xx}$ in the range $0 \le x \le 2\pi$, $t \ge 0$ subject to the following conditions $\theta(x, 0) = \sin^3 x$ for $0 \le x \le 2\pi$, $\theta(0, t) = \theta(2\pi, t) = 0 \ \forall t \ge 0$. 8

(*b*) Solve the initial value problem described by the inhomogeneous wave equation

$$u_{tt} - c^2 u_{xx} = f(x, t)$$

subject to the initial conditions $u(x, 0) = \phi_0(x)$, $u_t(x, 0) = \phi_1(x)$. 8

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