DDE/I/A.MATH/III/13

M.Sc. Part-I Examination, 2013

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER – III

Full Marks : 100

Time : 4 hours

The figures in the right-hand margin indicate marks

GROUP – A

(*Probability and Statistics*)

[*Marks* : 30]

Answer any **two** questions

1. (*a*) Deduce the equation of the plane of regression of x_1 on $x_2, x_3, \ldots, x_n$. 5

(*b*) What is the concept of transition probability matrix ? 3

(*c*) Show that the state *j* is persistent or transient according as

$$\sum_{n=o}^{\infty} p_{jj}^{(n)} = \infty \text{ or } < \infty$$

where $p_{jj}^{(n)}$ is the nth step transition probability. 5

(*d*) What is the concept of Brownian motion involving Stochastic process ? 2

2. (*a*) Consider a Markov chain with state space {0, 1, 2 } and transition probability martix

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix} \end{array}.$$

(*i*) Show that the states are persistent.

(*ii*) Show that the states are periodic and calculate the periodicity. 6

(*b*) Derive the differential equations for pure birth process and then solve these by certain conditions to be stated by you. Also find the mean population size under this process. 9

3. (*a*) State and prove first Entrance theorem. 3

(*b*) If $\{x_n, n = 0, 1, 2 \ldots\}$ is a Galton-Watson branching process and if

$$m = E[x_1] = \sum_{k=0}^{\infty} kpk \text{ and } \sigma^2 = \text{var}(x_1)$$

then prove that

(*i*) $E[x_n] = m^n$

(*ii*) $$\text{Var}(x_n) = \begin{cases} \dfrac{m^{n-1}(m^n - 1)}{m-1}\sigma^2, & \text{if } m \neq 1 \\ n\sigma^2 & \text{if } m = 1 \end{cases}$$

what do you mean by Galton-Watson Branching process ? 9

(*c*) What is the necessity of studying the multiple correlation and multiple regression plane in statistics ? 3

GROUP – B

(*Numerical Analysis*)

[*Marks* : 40]

Answer **Q. No. 4** and any **three** from the rest

4. Prove the following relations :

(*a*) $\frac{\Delta\vec{E}}{2}+\frac{\Delta}{2}\equiv\mu\delta$, the symbols have their usual meanings. 2

(*b*) $\Delta\log f(x)=-\log\left[1-\frac{\nabla f(x)}{f(x)}\right]$ 2

5. (*a*) Deduce Stirling's central difference interpolation formula. State its limitations. 8

(*b*) Using inverse interpolation find a real root of the equation $x^3-2x-4=0$. 4

6. (*a*) Describe a suitable method to approximate the function $y=f(x)$ using orthogonal polynomials. What is the advantage to use orthogonal polynomials ? 6

(*b*) The three-point Gauss-Legendre formula is

$$\int_{-1}^{1} f(x)dx=\frac{1}{9}\left[5f\left(\sqrt{-0\cdot6}\right)+8f(0)+5f\left(\sqrt{0\cdot6}\right)\right]$$

show that the formula is exact for $f(x)=1, x, x^2, x^3, x^4, x^5$. 6

7. (*a*) Describe LU-decomposition method to solve the system of equations

$$\underset{\sim}{A}\underset{\sim}{x}=\underset{\sim}{b}$$

with necessary conditions. 8

(*b*) Find the value of $|A|$ using partial pivoting, where

$$A=\begin{bmatrix}0 & 1 & 2\\ 1 & 5 & 3\\ -1 & 8 & 6\end{bmatrix}$$ 4

8. (*a*) Describe Jacobi's method to find all eigenvalues and eigenvectors of a real symmetric matrix. 8

(*b*) Describe Runge-Kutta method to solve a pair of differential equations

$$\frac{dy}{dx} = f(x,y,z) \text{ and } \frac{dz}{dx} = g(x,y,z)$$

with initial conditions

$$x = x_0,\ y(x_0) = y_0,\ z(x_0) = z_0.$$ 4

9. (*a*) Describe Milne's predictor-corrector method to solve

$$\frac{dy}{dx} = f(x,y) \text{ with } y(x_0) = y_0$$ 6

(*b*) Solve the wave equation by a suitable numerical method

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2},\ 0 \le x \le 1,\ t \ge 0$$

with initial conditions $u(x, 0) = f(x)$ and

$$\left(\frac{\partial u}{\partial t}\right)(x,0) = g(x),\ 0 < x < 1$$

and boundary conditions

$$u(0, t) = \phi(t) \text{ and } u(1, t) = \psi(t),\ t \ge 0.$$ 6

GROUP – C

(*Introduction to Computing*)

[*Marks* : 30]

10. Answer any *six* questions : 5×6

(*a*) Write a program in C to find the product of two matrics using pointer.

(*b*) Explain 'if', 'if-else', 'if-else if' statement with an appropriate example for each.

(*c*) Differentiate between coding and conversion of a number. Explain the self-complementing codes.

(*d*) What do you understand by 'normalised

floating point representation' of a real number ? Explain it. Evaluate $0{\cdot}3265 \times 10^7 - 0{\cdot}4312 \times 10^4$ using normalised floating point representation.

(*e*) How can the 'getchar' and 'putchar' functions be used to read and write multicharacter strings ?

(*f*) Explain the different types of storage class available in programming language C.

(*g*) What is structure in C ? Using structure to define a complex number write a program to multiply two complex numbers.

(*h*) Write the prototype for :

(*i*) function returning a pointer to a function and a void argument.

(*ii*) function returning a pointer to an array and take a pointer to an integer as character.

(*i*) Write a program in C to find the sum of the digits of an integer number using a function.

(*j*) Write short notes on the following :

(*i*) printf function,

(*ii*) for statement.
