DDE/II/A.MATH/X/13

M.Sc. Part-II Examination, 2013

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—X(OR/OM)

The figures in the right-hand margin indicate marks

(*OR*)

Special Paper : (*Advance Optimization and Operations Research*-II)

[*Marks* : 100]

Time : 4 hours

Answer Q. No. 11 and any six from the rest

1. (*a*) Find the optimum order level which minimizes the total expected cost under the following assumptions :

   (*i*) Demand is deterministic and uniform.

   (*ii*) Production is instantaneous.

(*iii*) Shortages are allowed and fully back logged.

(*iv*) Lead time is zero.

(*v*) Inventory planning horizon is infinite.

(*vi*) The carrying cost, shortage cost and ordering cost are known and constant. 8

(*b*) The demand for an item is 18000 units per year. The holding cost is Rs. 1·20 per unit item and the shortage cost is Rs. 5·00. The ordering cost is Rs. 400·00. Assuming the replenishment rate as instantaneous, determine the optimum order quantity along with total minimum cost of the system. 8

2. (*a*) (*i*) What is PERT ? What information is revealed by PERT analysis ?

(*ii*) Distinguish between PERT and CPM.

(*iii*) Define expected time and variance in terms of optimistic, pessimistic and most likely time.

(*iv*) How do you calculate the earliest starting time and the earliest finishing time ? 2 × 4

(*b*) The following are the details of estimated times of activities of a certain project :

Activity	:	*A*	*B*	*C*	*D*	*E*	*F*
Immediate Predecessor	:	—	*A*	*A*	*B*,*C*	—	*E*
Estimated Time (Weeks)	:	2	3	4	6	2	8

(*i*) Find the critical path and the expected time of the project.

(*ii*) Calculate the earliest starting time and earliest finishing time for each activity.

(*iii*) Calculate the float for each activity. 8

3. (*a*) A man is engaged in buying and selling identical items. He operates from a warehouse that can hold 500 items. Each month he can

sell any quantity that he chooses upto the stock at the beginning of the month. Each month, he can buy as much as he wishes for delivery at the end of the month so long as his stock does not exceed 500 items. For the next four months, he has the following error-free forecasts of cost sales prices :

Month	:	i	1	2	3	4
Cost	:	c_i	27	24	26	28
Sale Price	:	p_i	28	25	25	27

If he currently has a stock of 200 units, what quantities should he sell and buy in next four months ? Find the solution using dynamic programming. 10

(*b*) Solve the following problem using dynamic programming technique :

Minimize $Z = y_1^2 + y_2^2 + y_3^2$

subject to $y_1 y_2 y_3 \geq 4$, where y_1, y_2 and y_3 are positive integers. 6

4. (*a*) Obtain the expected waiting time for a customer in the queue for the queuing model (M | M | 1) : (N | FCFS | ∞). 10

(*b*) Define the following terms in connection with queueing theory : 2 × 3

(*i*) Service mechanism

(*ii*) Service discipline

(*iii*) Mean servicing rate and traffic intensity.

5. (*a*) Define entropy. Prove that

$$H(p_1, p_2, \ldots, p_n) \leq \log_2 n,$$

and the equality holds iff

$$p_k = \frac{1}{n}; k = 1, 2, .., n.$$ 3

(*b*) State the fundamental theorem of information theory. Define memory less channel and channel matrix. 3

(*c*) Let x_n be a particular event with probability p_n divided into m mutually exclusive subsets, say $E_1, E_2, .., E_m$ with probabilities $q_1, q_2, .., q_m$ respectively such that $p_n = q_1 + q_2 + .. + p_m$, then prove that

$$H(p_1, p_2, .., p_{n-1}, q_1, q_2, .., q_m) = H(p_1, p_2, .., p_{n-1}, p_n) + p_n H(q_1/p_n, q_2/p_n, .., q_m/p_n).$$ 6

(*d*) Evaluate the entropy associated with the following probability distribution : 4

Event	:	A	B	C	D
Probability	:	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

6. (*a*) The following table gives the arrival pattern at a coffee counter for 'one minute' intervals. The service is taken as 2 person in one minute in one counter :

No. of person arriving	:	0	1	2	3	4	5	6	7
Probability	:	0·05	0·10	0·15	0·30	0·20	0·10	0·05	0·05

Using Monte Carlo simulation technique and the following random numbers, generate the pattern of arrivals and queue formed when the following 20 random numbers are given :

5, 25, 16, 80, 35, 48, 67, 79, 90, 92, 9, 14, 1, 55, 20, 71, 30, 42, 60 and 85.

Find the queue length if two counters are used, i.e. 4 persons in one minute. 8

(*b*) Use Monte-Carlo simulation to find the area of a unit circle. 4

(*c*) Explain the meaning of six parameters $(a/b/c) : (d/e/f)$ used in queuing theory. 4

7. (*a*) Solve the following problem by geometric programming.

Minimize

$$Z = 2x_1^3 x_2^{-3} + 4x_1^{-2} x_2 + x_1 x_2 + 8x_1 x_2^{-1}$$

$$x_1, x_2 \geq 0.$$ 8

(*b*) Find an optimal sequence for the following sequencing problem of four jobs and five machines, when passing is not allowed. Its processing time (in hours) is given below :

Job	Machine				
	M_1	M_2	M_3	M_4	M_5
A	7	5	2	3	9
B	6	6	4	5	10
C	5	4	5	6	8
D	8	3	3	2	6

Also find the total elapsed time. 8

8. (*a*) In a system, there are n number of components connected in parallel with reliability $R_j(t), j = 1, 2, ..., n$. Find the reliability of the system. If $R_1(t) = R_2(t) = \cdots = R_n(t) = e^{-\lambda t}$, then determine the expression of system reliability. 6

(*b*) Derive Johnson's algorithm for processing n jobs through three machines in a sequence. 5

(*c*) A linear second order differential equation is described by $\dot{x}_1 = x_2$, $x(0) = 1$, $\dot{x}_2 = u$, $x_2(0) = 1$. Using Euler's equations find the optimal control $u(t)$ which minimizes

$$J = \int_0^1 u(t)\,dt$$

with the conditions $x_1(1) = x_2(1) = 0$. 5

9. (*a*) Define a non-cooperative game with n players. What is equilibrium situation ? When two games are said to be strategically equivalent ? Show that strategically equivalent games obey symmetric and transitive properties. 8

(*b*) Machine *A* costs Rs. 45,000 and its operating costs are estimated to be Rs. 1,000 for the first year and increasing by Rs. 10,000 per year in the second and subsequent years. Mechine *B* costs Rs. 50,000 and operating costs are Rs. 2,000 for the first year, increasing by Rs. 4,000 in the second and

subsequent years. If at present we have a machine of type A, should we replace it with B? If so when? Assume that both machines have no resale value and their future costs are not discounted. 8

10. (a) If $f(t)$ is the failure density function of reliability, $R(t)$ is the reliability, $Q(t)$ is the unreliability and $Z(t)$ is the Hazard rate, then find the relations between

(i) $f(t)$ and $R(t)$

(ii) $f(t)$ and $Q(t)$

(iii) $Z(t)$ and $R(t)$

(iv) $Z(t)$ and $f(t)$ 8

(b) At time zero, all items in a system are new. Each item has a probability p of failing immediately before the end of the first month of life, and a probability $q = 1-p$ of failing immediately before the end of the second month (i.e, all times fail by the end of the second month). If all items are

replaced as they fail, then show that the expected number of failures $f(x)$ at the end of month x is given by

$$f(x) = \frac{N}{1+q}\left[1-(-q)^{x+1}\right]$$

where N is the number of items in the system. 8

11. Answer any *one* question : 4 × 1

(a) Write short note on "optimal control".

(b) Explain the terms 'shortage' and 'lead time' in connection with inventory.

(*OM*)

[*Marks* : 75]

Time : 3 hours

Answer any **five** questions

1. (a) Obtain the momentum equation of an air parcel in the atmosphere in cartesian co-ordinate system. 7

(*b*) What do you mean by adiabatic process ? Deduce the Poisson's equation. 6

(*c*) Show that the potential temperature is invariant during adiabatic motion. 2

2. (*a*) Derive the vorticity equation of an air parcel in the atmosphere. 7

(*b*) Derive the saturated adiabatic lapse rate of moist air and hence show that it is less than dry adiabatic lapse rate. 6

(*c*) What is equivalent temperature and equivalent potential temperature ? 2

3. (*a*) What is the concept of grid point in numerical weather forecasting ? 4

(*b*) Derive the formula to predict the potential temperature due to advection using finite difference method. 5

(*c*) Derive the effect of ascent and descent of an air parcel on lapse rate in terms of changes in pressure. 6

4. (*a*) Derive the relation from which saturation temperature can be calculated when saturation of air will be done by adiabatic ascent and in this case estimate the height at which saturation will be attend. 9

(*b*) Show that the sum of kinetic energy, potential energy and enthalpy of an air parcel is constant when the flow is steady, adiabatic and frictionless. 6

5. (*a*) What is hurricane ? Find the relation between the pressure difference at the top and the bottom of the hurricane. Show that the tangential wind is a hurricane vary with altitude and hence show that the hurricane has a warm core. 2 + 3 + 6

(*b*) What is CAPE ? Derive the expression for CAPE. 1 + 3

6. (*a*) Show that the relative increase in dew-point temperature is about 5% of the sum of the relative increases in mixing ratio and the pressure during isobaric cooling of an air parcel. 5

(*b*) Deduce the equation of state for moist air in the following form :

$$p\alpha = R_d T\left[1-\left(1-\epsilon\right)\frac{e}{p}\right].$$

Hence define the virtual temperature and show that the virtual temperature is always higher than the actual temperature. 6

(*c*) Derive the expression of the pressure gradient force in the atmosphere. 4

7. (*a*) What is frontal surface and front ? Derive the angle between the frontal surface and earth's surface. 2 + 5

(*b*) Find the rate of change of circulation in the atmosphere and interpret each term. 8

8. (*a*) What is geopotential surface ? Derive the geopotential thickness between two pressure levels in the atmosphere. 8

(*b*) Explain the geo-dynamical paradox. 2

(*c*) Explain the convergence and divergence in the atmosphere. 3

(*d*) Derive the hydro-static equation in the atmosphere. 2
