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An Application of Interval-Valued Intuitionistic Fuzzy Relation Predicting Scores in Game Theory

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ABSTRACT

Portend the scores of a match is inquisitive about match lovers and, owing to its complexity, becomes very interesting and meaningful as a research topic. In this paper, our work motive is to predict scores in a game from multi-observer data based on the knowledge of the min-max, max-product and max-average composition of interval-valued intuitionistic fuzzy relations (IIFR). First, we accumulated an IIFR chart regarding an explicit emergency to obtain the required result and inspected it using the min-max, max-product and max-average composition of IIFR's. Finally, we weigh up the three outcomes and discern that max product composition is more reliable among them.

Keywords: Fuzzy set, intuitionistic fuzzy set, interval-valued intuitionistic fuzzy set

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1. Introduction

The hypothesis of fuzzy set and fuzzy relation was brought out by American Mathematician Zadeh in 1965. After that, the notion of the intuitionistic fuzzy set was brought out through Atanassov. Later, Turksen invented the Interval-valued fuzzy set based on his idea Atanassov and Gargov established the concept of interval-valued intuitionistic fuzzy set. Most of our real-life issues draw on hazy facts. Nowadays, many postulations have been designated to analyse hazy situations in a workable way. To conduct the hazy situation, the typical method of set theory and numbers is inadequate and has to be expanded. The idea of a fuzzy set and fuzzy relation is one of the answers for this purpose. Fuzzy relations are worthy of conviction of fuzzy theory and extensively apply in numerous areas such as fuzzy group, fuzzy logic, decision making, fuzzy diagnosis, fuzzy modelling, etc. In 1969, Zadeh proposed an application of fuzzy set in the medical science field. Sanchez was the first one to invent a fully relationships modelling theory of symptoms and disease, employing the idea of his proposed method. This paper proposed a score, loss or profit-predicting method based on FS, IFS, IIFS.

Aiming to this, we first proposed a methodology and then, with an illustrative example, we try to explain it practically.

2. Preliminary

In the sequent part, we will shortly recapitulate the elementary thought of FS, IFS and IIFS, which would be imperative for subsequent discussion.

Fuzzy Set [13]

Presume V to be the Universe of discourse. A mapping numbers a fuzzy set A in V

$$\mu_A : V \to [0,1].$$

Here, μ_A narrate the grade of membership. Thus A can be defined by the set of ordered pair

$$A = \{(a, \mu_A(a)) \mid a \in V\}.$$

Intuitionistic Fuzzy Set [5]

Presume V to be the Universe of discourse. A IFS A in V is numbered by two mapping

$$\mu_A : V \to [0,1] \text{ and } \gamma_A : V \to [0,1]$$

Here, μ_V and γ_V narrate the grade and non-grade of membership gradually. Thus A can be defined by the set of ordered triplet

$$A = \{(a, \mu_A(a), \gamma_A(a)) \mid a \in V, (\mu_A(a) + \gamma_A(a)) \leq 1\}.$$

Interval-valued Fuzzy Set [6]

Presume V to be the Universe of discourse. A IVFS A in V is given by $A = \{(a, M_A(a)) \mid a \in V\}$, where $M_A(a) = [M_{AL}, M_{AU}]$ is acquainted as interval-valued fuzzy number with membership value.

Interval-valued Intuitionistic Fuzzy Set [6]

Presume V to be the Universe of discourse. An IIFS A in V is given by $A = \{(a, M_A(a), N_A(a)) \mid a \in V\}$, where the degree of membership and non-membership are evolve gradually as $M_A : V \to [0,1]$ and $N_A : V \to [0,1]$ with condition. $0 \leq \sup(M_A(a)) + \sup(N_A(a)) \leq 1$.

Intuitionistic Fuzzy Relation (IFR) [12]

For two sets V and W, an IFR R, denoted by $R(V \to W)$ is an IFS on $V \times W$ and narrated by the membership and non-membership functions. γ_R.

Max-Min and Min-Max Composition

Consider R_1 and R_2 two IFRs on $U \times V$ and $V \times W$. The membership function gives the max-min and min-max compositions.

$$\mu_{R_1 \circ R_2}(u,w) = \vee[\mu_{R_1}(u,v) \wedge \mu_{R2}(v,w)], \quad \mu_{R_1 \circ R_2}(u,w) = \wedge[\mu_{R_1}(u,v) \vee \mu_{R_2}(v,w)]$$

and the non-membership function

$$\gamma_{R_1 \circ R_2}(u,w) = \wedge[\gamma_{R_1}(u,v) \vee \gamma_{R2}(v,w)], \quad \gamma_{R_1 \circ R_2}(u,w) = \vee[\gamma_{R_1}(u,v) \wedge \gamma_{R_2}(v,w)],$$

for all $(u,w) \in U \times W$ and v$\in$ V.

Max Product composition

The max product compositions of two IFRs R_1 and R_2 on $U \times V$ and $V \times W$ is formed by the membership function

$$\mu_{R_1 \circ R_2}(u,w) = \vee[\mu_{R_1}(u,v) \bullet \mu_{R2}(v,w)],$$

and the non-membership function

$$\gamma_{R_1 \circ R_2}(u,w) = \wedge[\gamma_{R_1}(u,v) \bullet \gamma_{R2}(v,w),$$

for all $(u,w) \in U \times W$ and v∈ V.

Max Average composition

Consider R_1 and R_2 be two IFRs on $U \times V$ and $V \times W$. The max average compositions is an IFR and is formed by the membership function

$$\mu_{R_1 \circ R_2}(u,w) = \max\left[\frac{\mu_{R_1}(u,v) + \mu_{R2}(v,w)}{2}\right],$$

and the non-membership function

$$\gamma_{R_1 \circ R_2}(u,w) = \min\left[\frac{\gamma_{R_1}(u,v) + \gamma_{R2}(v,w)}{2}\right],$$

for all $(u,w) \in U \times W$ and v∈ V.

3. Model for predicting score in cricket

This section contains infliction of IIFS for predicting scores in the game. For prediction let us assume three crisp sets (bowling speed) $B = \{b_i\}$, (*pitches* condition) $P = \{p_i\}$, and (run rate) $R = \{r_i\}$, where $i = 1,2,3,\cdots,n$. Two fuzzy relations R_1 and R_2 are defined as:

$$R_1 = \{((b,p), \mu_{R_1}(b,p), \gamma_{R_1}(b,p)) \mid (b,p) \in B \times P\} \text{ and}$$
$$R_2 = \{((p,r), \mu_{R_2}(p,r), \gamma_{R_2}(p,r)) \mid (p,r) \in P \times R\},$$

where $\mu_{R_1}(b,p)$ and $\gamma_{R_1}(b,p)$ notify the membership degree and the non-membership degree of bowling speed with refers to the pitches condition. Similarly $\mu_{R_2}(p,r)$ and $\gamma_{R_2}(p,r)$ notify the membership degree and the non-membership degree of pitches condition refers to the run rate. Let T denote the composition relation of R_1 and defined by the membership and non-membership functions as:

For max-min composition:

$$\mu_T = \mu_{R_1 \circ R_2}(b,r) = \vee[\mu_{R_1}(b,p) \wedge \mu_{R_2}(p,r)],$$
$$\gamma_T = \gamma_{R_1 \circ R_2}(b,r) = \wedge[\gamma_{R_1}(b,p) \vee \gamma_{R_2}(p,r)].$$

For max product composition:

$$\mu_T = \mu_{R_1 \circ R_2}(b,r) = \vee[\mu_{R_1}(b,p) \bullet \mu_{R_2}(p,r)],$$
$$\gamma_T = \gamma_{R_1 \circ R_2}(b,r) = \wedge[\gamma_{R_1}(b,p) \bullet \gamma_{R_2}(p,r)].$$

For max average composition:

$$\mu_T = \mu_{R_1 \circ R_2}(b,r) = \vee \left[\frac{\mu_{R_1}(b,p) + \mu_{R_2}(p,r)}{2} \right],$$

$$\gamma_T = \gamma_{R_1 \circ R_2}(b,r) = \wedge \left[\frac{\gamma_{R_1}(b,p) + \gamma_{R_2}(p,r)}{2} \right].$$

To perceive the implementation of the method, we compose a hypothesis case study below:

4. Case study

Let, B, P, R express the class of bowling speed, condition of pitches and run rate gradually and is constructed as follows:

A = bowling speed = {fast bowling, medium bowling, spin bowling}

B = Condition of pitches = {fast pitches, slow and low pitches, flat pitches, spinning pitches, seaming pitches, dead pitches, green pitches}

C = Run rate = {low run, average, high}.

The IIFRs R_1 and R_2 are represented in tabular form in Table 1 and Table 2, subsequently.

	fast pitches	slow & low pitches	flat pitches	spinning pitches	seaming pitches	dead pitches	green pitches
Fast	[0.8, 0.9] [0.05, 0.1]	[0.1, 0.2] [0.6, 0.7]	[0.2, 0.3] [0.5, 0.6]	[0.5, 0.6] [0.2, 0.3]	[0.7, 0.8] [0.1,0.15]	[0.05,0.1] [0.8,0.85]	[0.1,0.15] [0.7, 0.8]
Medium	[0.7, 0.8] [0.1, 0.15]	[0.2, 0.3] [0.6, 0.7]	[0.7, 0.8] [0.1,0.15]	[0.6, 0.7] [0.2,0.25]	[0.8, 0.9] [0.05,0.1]	[0.1, 0.2] [0.6, 0.7]	[0.2, 0.3] [0.6,0.65]
Spin	[0.6, 0.7] [0.2, 0.25]	[0.1, 0.2] [0.6, 0.7]	[0.2, 0.3] [0.6, 0.7]	[0.7, 0.8] [0.1,0.15]	[0.5, 0.6] [0.2, 0.3]	[0.05,0.1] [0.8,0.85]	[0.1, 0.2] [0.7,0.75]

Table 1: Relation between bowling speed and pitches condition

	low run	average run	high run
fast pitches	[0.2, 0.3] [0.6, 0.7]	[0.5, 0.6] [0.2, 0.3]	[0.4, 0.5] [0.4, 0.45]
slow & law pitches	[0.3, 0.4] [0.52, .55]	[0.4, 0.5] [0.4, 0.45]	[0.5, 0.6] [0.2, 0.3]
flat pitches	[0.6, 0.7] [0.2, 0.25]	[0.7, 0.8] [0.1, 0.15]	[0.8, 0.9] [0.02, .05]
spinning pitches	[0.7, 0.8] [0.1, 0.15]	[0.5, 0.6] [0.2, 0.3]	[0.6, 0.7] [0.2, 0.25]
seaming pitches	[0.3, 0.4] [0.52, .55]	[0.6, 0.7] [0.2, 0.25]	[0.7, 0.8] [0.1, 0.15]
dead pitches	[0.5, 0.6] [0.2, 0.3]	[0.7, 0.8] [0.1, 0.15]	[0.3, 0.4] [0.52,.55]
green pitches	[0.6, 0.7] [0.2, 0.25]	[0.2, 0.3] [0.6, 0.7]	[0.4, 0.5] [0.4, 0.45]

Table 2: Relation between pitches condition and run rate

Performing "max-min", "max-product" and "max-average" compositions for R_1 and R_2, we get three new relations T_1, T_2 and T_3, represented in Table 3, Table 4, Table 5 subsequently.

	low run	average run	high run
fast bowling	[0.5, 0.6] [0.2, 0.3]	[0.5, 0.7] [0.2, 0.3]	[0.7, 0.8] [0.1, 0.15]
medium bowling	[0.6, 0.7] [0.2, 0.25]	[0.7, 0.8] [0.1, 0.15]	[0.7, 0.8] [0.1, 0.15]
spin bowling	[0.7, 0.8] [0.1, 0.15]	[0.5, 0.6] [0.2, 0.3]	[0.6, 0.7] [0.2, 0.25]

Table 3: Max-min composition

	low run	average run	high run
fast bowling	[0.35,0.48] [0.02, 0.045]	[0.42, 0.56] [0.01, 0.03]	[0.49,0.64] [0.01, 0.023]
medium bowling	[0.42,0.50] [0.02, 0.038]	[0.49,0.64] [0.01, 0.023]	[0.56,0.72] [0.002,0.008]
spin bowling	[0.49,0.64] [0.01, 0.023]	[0.35,0.48] [0.02, 0.045]	[0.42,0.56] [0.012,0.035]

Table 4: Max-product composition

	low run	average run	high run
fast bowling	[0.6, 0.7] [0.15, .23]	[0.65, 0.75] [0.13,.2]	[0.7, 0.8] [0.1, 0.15]
medium bowling	[0.65, 0.75] .15,0.2]	[0.7, 0.8] [0.1, 0.15]	[0.75, 0.85] [.06,0.1]
spin bowling	[0.7, 0.8] [0.1, 0.15]	[0.6, 0.7] [0.15, 0.23]	[0.65, 0.75] [.15,0.2]

Table 5: Max-average composition

Table 6, Table 7 and Table 8 gradually illustrated the comparison table for "Table 3 and Table 4 ", "Table 3 and Table 5", and "Table 5 and Table 4". Finally, we observe that max product composition is more reliable than then other two.

	low run	average run	high run
low run	3	3	3
average run	3	3	3
high run	3	3	3

Table 6: Comparison table for "Table 3 and Table 4"

	low run	average run	high run
low run	0	1	1
average run	1	0	0
high run	2	1	0

Table 7: Comparison table for "Table 3 and Table 5"

	low run	average run	high run
low run	3	3	3
average run	3	3	3
high run	3	3	3

Table 8: Comparison table for "Table 5 and Table 4"

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