2022

M.Sc.

2nd Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-205

GENERAL THEORY OF CONTINUUM MECHANICS

Full Marks : 50

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any *four* questions : 4×2

(a) The components of the stress dyadic at a certain point of a continuous medium are given by

$$(T_{ij}) = \begin{pmatrix} 200 & 400 & 300 \\ 400 & 0 & 0 \\ 300 & 0 & -100 \end{pmatrix}.$$

Determine the maximum shear stress.

(Turn Over)

(b) Show that Lagrangian linear strain tensors are identical with Eulerian linear strain tensors when the deformation is small.

(c) If the deformation of a body is defined by the displacement components $u_1 = k(3X_1^2 + X_2)$, $u_2 = k(X_2^2 + X_3)$ and $u1 = k(X_3 + X_1)$ where $k > 0$. Compute the extension of a line element that passes through the point (1, 1, 1) in the direction

$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$.

(d) Find the complex potential due to a source.

(e) The velocity (u, v, w) of a fluid at a point P(x, y, z) is given by $u = \frac{-2xyz}{x^2 + y^2}$, $v = \frac{yz}{x^2 + y^2}$ and

$w = \frac{z}{x^2 + y^2}$. Find the rate at which density of

the fluid at point P is decreasing in the flow field.

(f) The diameters of a pipe at the sections A and B are 200 mm and 300 mm respectively. If the velocity of water following through for pipe at section A is 4 m/s, find the velocity of water at section B.

(g) Define source and sink.

(h) State Newton's law of viscosity.

2. Answer any *four* questions : 4×4

(a) For the deformation defined by the equations

$$X_1 = \frac{1}{2}\left(x_1^2 + x_2^2\right),\ X_2 = \tan^{-1}\left(\frac{x_2}{x_2}\right),\ X_3 = x_3,\ x_1 \neq 0$$

find the deformation gradient tensors in material forms. Hence show that the deformation is isochoric.

(b) Define principal strain and principal direction of strain. Prove that all principal strains are *real*.

(c) The stress matrix at a point $P(x_i)$ in a material is given by

$$(T_{ij}) = \begin{pmatrix} x_1x_3 & x_3^2 & 0 \\ x_3^2 & 0 & -x_2 \\ 0 & -x_2 & 0 \end{pmatrix}.$$

Find the stress vector at the point Q(1, 0, –1) on the surface

$$x_1 = x_2{}^2 + x_3{}^3.$$

(d) The velocity components in a fluid flow are given by
$u = x^2 + z^2$, $v = y^2 + z^2$ and $w = -2z(x + y)$. Show that the flow is possible with these velocity components. Examine whether the motion is rotational or not.

(e) The velocity field at a point P(x, y, z) in a fluid motion is given by

$$u = \frac{x}{t}, v = y, w = 0 .$$

Find the path lines and stream lines.

(f) State and prove Kelvin's circulation theorem.

(g) Show that the principal directions of strain at each point in a linearly elastic isotropic body must be coincident with the principal directions of stress.

(h) Show that $\frac{x^2}{a^2}\phi(t)+\frac{y^2}{b^2}\frac{1}{\phi(t)}=1$ is a possible form of boundary surface of a liquid.

3. Answer any *two* questions : 2×8

(a) What is strain quadric? Explain the geometric interpretation of infinitesimal strain tensors. 2+6

(b) The stress tensor at a point is given by

$$\left(T_{ij}\right)=\begin{pmatrix}1 & 2 & 1\\ 2 & 1 & 1\\ 1 & 1 & 1\end{pmatrix}.$$

Determine the principal stresses and corresponding principal directions. Also check on the invariance of θ, θ_2 and θ_3. 2+4+2

(c) (i) If any portion of the moving fluid once becomes irrotational, then show that it will remain so for all subsequent times provided that the external body forces are conservative and pressure is a function of density alone.

(ii) Derive the Bernoulli's equation of motion for a perfect fluid. 2+6

(d) Derive the basic elastic constants for isotropic elastic solid.

[Internal Assessment – 10]

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