M.Sc. 3rd Semester Examination, 2022

APPLIED MATHEMATICS

(Partial Differential Equations and Generalized Functions)

PAPER – MTM-301

Full Marks : 50

Time : 2 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Symbols have their usual meaning

A. Answer any *four* questions from the following : 2×4

1. State the Basic existence theorem for Cauchy problem.

(Turn Over)

2. Give an example of an ill-posed partial differential equation.

3. Define characteristic curve and base characteristics of a first order quasi linear PDE.

4. Give an example of a harmonic function in a domain D which has neither a maximum value nor a minimum value in D.

5. Define domain of dependence of the Cauchy problem for the wave equation.

6. Show that $\delta(-t) = \delta(t)$ where δ is the Dirac delta function.

B. Answer any *four* questions from the following : 4×4

7. Use method of characteristic to solve $u_x + 2u_y = 1 + u$ such that $u = \sin x$ on the line $y = 3x + 1$.

8. Solve the equation $\Delta u = 0$ in the disc $D = \{(x, y) : x^2 + y^2 < a^2\}$ with the boundary condition $u = 1 + 3\sin\theta$ on the circle $r = a$.

9. Show that for a continuous function $u : D \to \mathbb{R}$, the two mean value properties are equivalent.

10. Find the general solution of the problem $u_{ttx} - u_{xxx} = 0$, $u_x(x, 0) = 0$, $u_{xt}(x, 0) = \sin x$, in the domain $\{(x, t) : -\infty < x < \infty, t > 0\}$.

11. Using the method of separation of variables, find the solution of the following problem :

$$u_t = 12u_{xx} \text{ in } 0 < x < \pi,\ t > 0,$$
$$u_x(0, t) = u_x(\pi, t) = 0,\ t \geq 0,$$
$$u(x, 0) = 1 + \sin^3 x,\ 0 \leq x \leq \pi.$$

12. Solve :

$$(x^2D^2 - xyDD' - 2y^2D'^2 + xD - 2yD')u = \log\left(\frac{y}{x}\right) - \frac{1}{2}.$$

C. Answer any *two* questions from the following : 8 × 2

13. (*i*) Find the derivative of the Heaviside unit step function.

(*ii*) Establish the Poisson's formula for the solution of a Dirichlet problem for the Laplace equation in a disk of radius '*a*'. 2 + 6

14. (*i*) State and prove the weak maximum principle.

(*ii*) Establish the Laplace equation in polar coordinates. 4 + 4

15. (*i*) Prove that $\delta(at) = \frac{1}{a}\delta(t), a > 0$.

(*ii*) Solve the following initial boundary value problem using parallelogram identity:

$$u_{tt} - u_{xx} = 0,\ 0 < x < \infty,\ 0 < t < 2x$$

$u(x, 0) = f(x), 0 \leq x < \infty$
$u_t(x, 0) = g(x), 0 \leq x < \infty,$
$u(x, 2x) = h(x), \ x \geq 0,$
where $f, g, h \in C^2[(0, \infty)]$. 3 + 5

16. Reduce the following equation to a canonical form and hence solve it :

$3u_{xx} + 10 u_{xy} + 3u_{yy} = 0.$ 6 + 2

[*Internal Assessment* – 10 Marks]