

2022

1st Semester Examination
APPLIED MATHEMATICS
WITH OCEANOLOGY AND
COMPUTER PROGRAMMING

Paper : MTM - 106

(Graph Theory)

Full Marks : 20

Time : One Hour

The figures in the margin indicate full marks.
Candidates are required to give their answers
in their own words as far as practicable.

1. Answer any *two* questions : 2×2=4

- (a) Prove that every connected graph has at least one spanning tree.
- (b) Draw the digraph graph G corresponding to adjacency matrix

$$A = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

- (c) Prove that every tree with two or more vertices is 2-chromatic.

P.T.O.

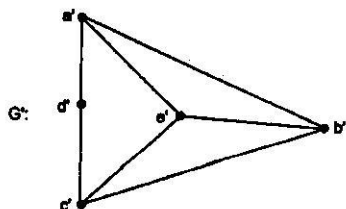
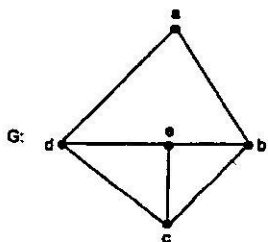
(d) Describe cut-set and cut-vertex in a connected graph.

2. Answer any *two* questions :

4×2=8

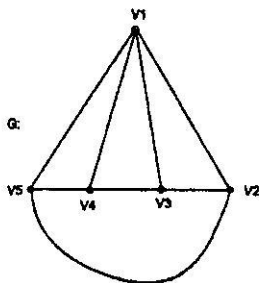
(a) Let T is a tree with n vertices. Prove that it has precisely $(n - 1)$ edges.

(b) Show that the graphs G and G' are isomorphic.



(c) Define a binary tree. Find the number of pendant vertices in a binary tree with n vertices.

(d) Find the chromatic polynomial of the graph G

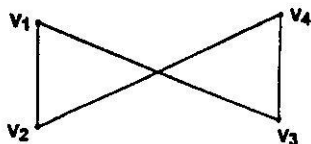


3. Answer any *one* question :

8×1=8

(a) (i) Does there exist a 4-regular graph on 6 vertices? If so construct a graph. 2

(ii) Consider the graph shown in figure, find the number of walks of length three from V_2 to V_4 and also check the connectedness of the graph :



4

(iii) Write down the statement of four-colour problem in graph theory. 2

(b) (i) State and prove Euler's theorem for a connected planar graph. 4

(ii) Prove that the chromatic polynomial of any cycle C_n of length n is

$$p_n(\lambda) = (\lambda - 1)^n + (-1)^n (\lambda - 1). \quad 4$$
