2022

M.Sc.

2nd Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-202

NUMERICAL ANALYSIS

Full Marks : 50

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any *four* questions : 4×2

(a) Discretize the PDE $\frac{\partial u^2}{\partial x^2}+\frac{\partial u^2}{\partial y^2}=0$.

(b) Discuss Gauss-Seidel iteration method to solve a system of non-linear equations.

(c) State the limitations of LU-decomposition method.

(d) Express the polynomial $x^4 - 5x^3 + 7x$ in terms of Chebyshev polynomials.

(e) State the minimax principle of polynomial interpolation.

(f) What do you mean by the terms single-step and multi-step methods ?

2. Answer any *four* questions : 4×4

(a) Discussed Newton-Rapson's method to solve the following non-linear equations :

$$f(x, y) = 0, \quad g(x, y) = 0.$$

(b) Explain the least square method to solve a system of linear equation.

(c) What do you mean by tri-diagonal system of linear equations ? Discuss a method to solve this type of equations.

(d) Using Milne's predictor-corrector formula find $y(0.4)$ for the following IVP $\frac{dy}{dx} = x^2 - y$; $y(0) = 1$ with step length $h = 0.1$.

(e) Describe the solution of 2nd order BVP by finite difference method.

(f) Using six points Gauss-Legendre quadrature formula evaluate

$$\int_1^2 \frac{x}{1+2x^2} dx .$$

3. Answer any *two* questions : 2×8

(a) Let us consider the wave equation

$$u_{tt} = c^2 u_{xx}, \qquad t > 0,\ 0 < x < 1$$

where initial conditions $u(x, 0) = f(x)$ and $u_t = g(x)$, $0 < x < 1$ at $(x, 0)$ and boundary conditions

$$u(0, t) = \phi(t) \text{ and } u(1, t) = \psi(t), \qquad t \geq 0.$$

Describe a finite difference method to solve the above problem.

(b) Discussed successive over-relaxation method to solve a system of linear equations.

(c) Describe power method for least eigenvalue. Find the largest eigenvalue in magnitude and corresponding eigenvector of the matrix

$$A = \begin{pmatrix} 1 & 4 & 2 \\ -2 & 0 & 2 \\ 3 & 2 & 1 \end{pmatrix}.$$ 3+5

(d) Define spline interpolation. Derive natural cubic spline interpolation of a continuous function $y = f(x)$ in $[a, b]$. 2+6

[Internal Assessment – 10]

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