2022

M.Sc.

4th Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-404

Full Marks : 50

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

MTM–404A COMPUTATIONAL OCEANOLOGY

1. Answer any *four* questions : 4×2

(a) Derive the expression for u_e for two points upwind scheme in non-uniform grids and hence simply this for uniform grid.

(Turn Over)

(b) Write the advantages of use of finite volume method.

(c) Apply the finite volume method to the continuity equation of incompressible viscous flow.

(d) Write down the kinematical condition for the wave propagation at the free surface.

(e) Define the terms circulation and vorticity in a fluid rotation.

(f) Write down a short note on "Sverdrup wave".

2. Answer any *four* questions : 4×4

(a) (i) Draw the control volume for u-velocity and place the variables (velocity and pressure) on the respective faces for Quadratic Upwind Interpolation for Convective Kinematics (QUICK).

(ii) Then write the expressions for u-velocity at the east and west faces of the said control volume for both negative and positive fluxes.

(iii) Also with help of appropriate symbol, compose two expressions for negative and positive fluxes into one for both east and west faces separately.

(b) Discuss about the closed boundary conditions for three unknowns u, v & h at the bottom of ocean for three grids Grid-A, B and C.

(c) Using the implicit Euler scheme for time derivative and center differencing for space derivative, discretise the one-dimensional heat conduction equation $\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$.

(d) Derive depth-averaged continuity equations for shallow water theory.

(e) Prove that the horizontal velocity expression for linear waves in the absence of rotation is $u = \frac{g}{2c_0}\{F(x + c_0 t) - F(x - c_0 t)\}$ where symbols have their usual meaning.

(f) Prove that the path of the particle is an ellipse for the progressive wave on the surface of the cannel of finite depth.

3. Answer any *two* questions : 2×8

(a) (i) Write the non-dimensional continuity and Navier-Stokes equations for laminar two dimensional incompressible fluid flow.

(ii) Draw the control volume for v-velocity and arrange the variables (velocity and pressure).

(iii) Hence apply the finite volume method to the y-momentum equation. 1+2+5

(b) (i) Draw the grids for Grid-A (cell centered) and Grid-C (staggered grid) and arrange the variables (u, v & h) in both the grids.

(ii) Discretize the x- and y-momentum equations of two-dimensional gravity waves with centred differencing for space derivative and backward for time derive on the Grid-C.

3+5

(c) Derive Klein-Gordon equation for long surface wave and hence, prove that geostrophic velocity in y-direction is given by $\bar{v} = \frac{gh}{2C_0}\exp(-|x|/a)$, where symbols have their usual meaning.

(d) Prove that the total energy of progressive wave is $\frac{1}{2}\rho g a^2 \lambda$ where a, λ is the wave amplitude and wave length respectively.

[Internal assessment – 10]

MTM-404B NON-LINEAR OPTIMIZATION

1. Answer any *four* questions : 4×2

(a) Define posynomial and polynomial in connection with geometric programming with an example.

(b) Let X^0 be an open set in R^n, let θ and g be defined on X^0. Find the conditions under which a solution $(\bar{x}, \bar{r_0}, \bar{r})$ of the Fritz-John saddle point problem is a solution of the Fritz-John stationary point problem and conversely.

(c) Define bi-matrix game with an example.

(d) State Dorn's duality theorem in connection with duality in quadratic programming.

(e) Write the basic difference(s) between Beale's and Wolfe's method for solving quadratic programming problem.

(f) Under what condition(s) the Kuhn-Tucker conditions for quadratic programming problem are necessary and sufficient.

2. Answer any *four* questions : 4×4

(a) Define the following :

(i) Minimization problem ;

(ii) Local minimization problem ;

(iii) Kuhn-Tucker stationary point problem ;

(iv) Fritz-John stationary point problem.

(b) State and prove Weak duality theorem in connection with duality in non-linear programming.

(c) Minimize the following using geometric programming :

$$f(x) = 16x_1x_2x_3 + 4x_1x_2^{-1} + 2x_2x_3^{-2} + 8x_1^{-3}x_2$$
$$x_1, x_2, x_3 > 0.$$

(d) State and prove Motzkin's theorem of alternative.

(e) Define multi-objective non-linear programming problem. Define the following in terms of multi-objective non-linear programming problem :

(i) Complete optimal solution;

(ii) Pareto optimal solution;

(iii) Local Pareto optimal solution;

(iv) Weak Pareto optimal solution.

(f) State and prove Fritz-John saddle point sufficient optimality theorem. What are the basic differences between the necessary criteria and sufficient criteria of FJSP.

3. Answer any *two* questions : 2×8

(a) (i) Use the chance constrained programming to find an equivalent deterministic problem to following stochastic programming problem, when c_j is a random variable :

$$\text{Minimize } F(x) = \sum_{j=1}^{n} c_j x_j$$

$$\text{subject to } \sum_{j=1}^{n} a_{ij} x_j \le b_i$$

$$x_j \ge 0, \ i, j = 1, 2, \ldots, n.$$

(ii) Define the following terms :

The (primal) quadratic minimization problem (QMP).

The quadratic dual (maximization) problem (QDP). 6+2

(b) (i) Prove that a pair $\{y^*, z^*\}$ constitutes a mixed stategy Nash equilibrium solution to a bimatrix game (A, B) if and only if, there exists a pair $\{p^*, q^*\}$ such that $\{y^*, z^*, p^*, q^*\}$ is a solution of the following bilinear programming problem :

Minimize $[y'Az + y'Bz + p + q]$

subject to $Az \geq -p l_m$

$B'y \geq -q l_n$

$y \geq 0,\ z \geq 0,\ y' l_m = 1,\ z' l_n = 1.$

(ii) Give the geometrical interpretations of differentiable convex function and concave function. 5+3

(c) (i) Let X be an open set in R^n and θ and g be differential and convex on X and let $\bar{x}$ solve the minimization problem and let g satisfy the Kuhn-Tucker constraint qualification. Show that there exists a $\bar{u} \in R^m$ such that $(\bar{x}, \bar{u})$ solves the dual maximization problem and $\theta(\bar{x}) = \psi(\bar{x}, \bar{u})$.

(ii) Prove that all strategically equivalent bimatrix game have the Nash equilibria.

5+3

(d) (i) Solve the following quadratic problem by using Beale's method :

Maximize $Z = 10x_1 + 25x_2 - 10x_1^2 - x_2^2 - 4x_1x_2$

Subject to the constraints $x_1 + 2x_2 \leq 10$

$x_1 + x_2 \leq 9$

$x_1, x_2 \geq 0.$

(ii) Write short note on complementary slackness principle. 6+2

[I nternal assessment – 10]
