2022

M.Sc.

2nd Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-201

FLUID MECHANICS

Full Marks : 50

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any *four* questions : 4×2

(a) Find the substantial derivative of the steady state velocity field represented by the velocity vector $\vec{V} = (-3x, -3y, 6z)$.

(b) Draw an infinitesimally small moving element and show all energy fluxes along y-direction associated with the above element.

(c) Write the physical principals used for the equations of continuity, Navier-Stokes and energy and then write the equations of continuity and Navier-Stokes for incompressible viscous two-dimensional flow.

(d) How many types of variable arrangement are there in the Computational Fluid Dynamics? Discuss them by arranging the x- and y-components of velocities and pressure.

(e) What do you mean by the grid independent study in the field of computational fluid dynamics? Also show graphically in the plane : Error versus number of grid.

(f) Using the general technique, derive the expression for approximation of temperature gradient $\frac{dT}{dX}$ at $x = x_i$ in terms of y_i, y_{i-1} & y_{i-2}.

2. Answer any *four* questions : 4×4

(a) Write all the four forms of the continuity equation: Integral-Conservation, Integral-Nonconservation, Differential-Conservation and Differential-Nonconservation. Finally convert the Differential-Conservation form to that of Differential-Nonconservation.

(b) Write all the possible boundary conditions for tangential and normal components of velocity and temperature.

(c) Derive the expression for the substantial derivative of z-component of the velocity and hence discuss its physical significance. Also derive the above substantial derivative using the chain rule.

(d) An incompressible velocity fields is given by $u = a(x^2 - y^2)$, $v = -2axy$ and $w = 0$. Determine under what conditions it is a solution to the Navier-Stokes momentum equation for the case of without any body forces. Assuming that these conditions are met, determine the resulting pressure distribution.

(e) Write the set of governing equations for the boundary layer flow along a flat plate. Show that the x-component of the momentum equation applied at the edge of the boundary layer reduces to the Bernoulli equation. Finally write the governing equations for the outside of the Boundary layer.

(f) Discretize the one dimensional transport equation $\frac{\partial T}{\partial t} + a\frac{\partial T}{\partial x} = \alpha\frac{\partial^2 T}{\partial t^2}$, where a and α are constants, using Crank-Nicolson scheme and hence write the algebraic expression in a matrix form for the case of Neumann boundary conditions.

3. Answer any *two* questions : 2×8

(a) (i) An incompressible velocity fields is given by $u = 2(x^3 - 2xz)$, $v = c$ and w is unknown, where c is any constant. What must be the form of velocity component w be ?

(ii) Write the algebric formula for $\frac{dy}{dx}$ using forward, backward, central and three points asymmetry for forward as well as backward schemes. Also write the order of accuracy of these schemes. 4+4

(b) (i) Write the assumptions of boundary layer theory.

(ii) Based on the above assumptions, derive the set of governing equations for the boundary layer flow along a flat plate. Also write the proper boundary conditions for the above set of equations. 2+6

(c) (i) Write the y-component of Navier-Stokes equation and the energy equation for Newtonian, incompressible, viscous fluid flow with negligible gravity and radiation effects.

(ii) Make the y-component of Navier-Stokes equation for Newtonian, incompressible and viscous fluid flow with negligible gravity in non-dimensional form (in terms of Reynolds number $Re = \frac{Va}{\gamma}$). Use the characteristics length, velocity and pressure as a, V and ρV^2, respectively, where symbols have their usual meaning. 8

(d) Discretize the one-dimensional heat conduction equation using DuFort-Frankel scheme with leading term of truncation error. Discuss the consistency of this scheme and stability restriction. 8

[Internal Assessment – 10]
