Chapter 2

Consensus measuring and reaching to consensus threshold in fuzzy soft set based group decision-making by using distance measure

2.1 Introduction

In 1999, the idea of soft set theory [118] has been introduced by Prof. D. Molodtsov. Basically, in soft set theory, all the associated alternatives are defined through some considered parameters instead of using a membership function likewise fuzzy set. Consequently, with a very small duration of time, soft set theory has achieved a huge popularity to the researchers. Several algebraic operations, algebraic structures and fundamental results have been initiated through soft set theory. First, Maji et al. [107] engaged on developing some basic algebraic operations including, union, intersection, AND, OR, etc. on soft sets. Then, researchers have introduced several algebraic structures like, soft group [16], soft mapping [110], soft relation [133], etc. on soft set theory. Further, soft set has been used [107] in solving real-life decision-making problems. In this regard, Çağman and Enginoğlu [33] initiated soft matrices and some of its properties to handle decision-making problems more easily.

Besides these theoretical developments, several new generalizations of soft set theory have been introduced by using different parameterizations. Fuzzy soft set theory [104] is one of the significant generalizations of soft set theory where, soft set has been incorporated with fuzzy set by considering all the parameters in fuzzy sense i.e, by using fuzzy parametrization. In a fuzzy soft set, rating of an alternative with respect to a parameter is in fuzzy membership where, no need to introduce a membership function to define a fuzzy membership of an alternative rather all the evaluations are defined based on human cognitive system. In solving various types of real-life problems, fuzzy soft set has been widely used by the researchers. For instance, Feng et al. [61], Kong et al. [92, 93], Roy and Maji [139], etc. have used fuzzy soft set theory in solving decision-making. Then, Basu et al. [22], Li et al. [99], Wang et al. [162], etc. have used fuzzy soft set in diagnosis decision-making. Moreover, some times, in a fuzzy soft set based decision-making, multiple experts have been engaged for evaluating the associated alternatives with respect to some parameters. Based on the existing literature, we have seen that, Roy and Maji [139], Basu et al. [22] and Alcantud [4] have addressed such type of multiple observer decision-making problems through fuzzy soft sets and in this regard, they have used different aggregation operators to construct a resultant fuzzy soft set from a multiple number of fuzzy soft sets. Actually, in these existing fuzzy soft set based decision-making, researchers have paid their focuses on recognizing the best alternative based on the opinions of the experts with respect to some selected parameters.

However, in reality, the opinions of all the experts about an alternative may not be correct or some of them may have a diverse opinion about the alternative. So, in that case, to get an exact result, consensus measuring of an associated decision maker is very necessary. Consensus measuring of a decision maker about an alternative means, determine the similarity degree of the decision maker with the other decision makers for his/her provided opinion about the alternative. So, before selecting the best alternative from a group decision-making problem, it is very necessary to measure the consensus of a decision maker because, if any one of the decision makers has low disagreement with the others, then the decision maker will get an opportunity to recheck his/her opinion about the alternative before selecting the best decision solution from the problem. But, in all the above existing approaches, researchers did not focus on this issue. So, there is a research gap in solving fuzzy soft set based group decision-making. Then, in order to fulfill this research gap, our main contributions of this chapter are as follows:

- Firstly, we have established a methodological approach for to handle group decision-making problems based on fuzzy soft set with the help of fuzzy distance and fuzzy soft distance containing mainly three major parts: comprehensive consensus level measuring part, comprehensive consensus level increasing part and best decision solution selection part.
  In comprehensive consensus level measuring part, we have provided an algorithm by which one can measured the comprehensive consensus of a decision maker about an alternative in a fuzzy soft set based group decision-making. Then, in comprehensive

consensus level increasing part, we have provided two suggestions by which a decision maker, who have less comprehensive consensus than the considered threshold value, can reformulate his/her primary opinion about the alternative so that, his/her comprehensive consensus level can be increased up to the considered threshold value. After that, for best decision solution selection part, we have provided two algorithms.

- Then, a sustainable supplier selection problem of a textile industry has been solved by using our proposed fuzzy soft set based group approach to show the real-life applicability of our proposed approach.

The outline of the chapter is as follows:
Section 2.2 recalls some basic notions. Then, in Section 2.3, we have provided a new methodological approach for solving fuzzy soft set based group decision-making. Then, in Section 2.4, an experimental analysis regarding the sustainable supplier selection problem of a textile industry has been illustrated and solved by applying our proposed methodology. In Section 2.5, we have discussed some comparative analysis to verify the validity and efficiency of our approach. Section 2.6 contains some conclusions of this chapter.

2.2 Some basic relevant notions

In this section, some basic concepts including, fuzzy soft set (FSS), fuzzy distance, fuzzy soft distance, etc. have been discussed which are related with our main interpretation.

(i) **Complement of a fuzzy set (FS) [183].**

Let us consider a fuzzy soft set over a universal set as, $A = \{(x, \mu_A(x_j))|x_j \in X\}$. Then, its complement is defined as follows:

$$A^c = \{(x_j, \mu_A^c(x_j))|x_j \in X\}$$

where, $\mu_A^c(x_j) = 1 - \mu_A(x_j)$.

(ii) **Euclidean distance of two FSs [84].**

Consider two fuzzy sets over X as, $A = \{(x_j, \mu_A(x_j))|x_j \in X\}$ and $B = \{(x_j, \mu_B(x_j))|x_j \in X\}$. Then, their Euclidean distance is as follows:

$$d_{Eu}(A, B) = \sqrt{\frac{1}{n}\sum_{j=1}^{n}(\mu_A(x_j) - \mu_B(x_j))^2}$$

(iii) **Mathematical representation of a FSS [22].**

Let a universal set as, $X = \{x_1, x_2, .., x_m\}$ and a parameter set as, $E = \{e_1, e_2, .., e_n\}$ where, each of them is in fuzzy sense. Then, mathematically, a fuzzy soft set $(\tilde{f}, E)$ over X can be defined as follows:

$$(\tilde{f}, E) = \left\{\left(e_1, \tilde{f}(e_1)\right), \left(e_2, \tilde{f}(e_2)\right), .., \left(e_n, \tilde{f}(e_n)\right)\right\}$$
$$= \{(e_1, ((x_1, \tilde{f}_{e_1}(x_1)), (x_2, \tilde{f}_{e_1}(x_2)), .., (x_m, \tilde{f}_{e_1}(x_m)))), (e_2, ((x_1, \tilde{f}_{e_2}(x_1)), (x_2, \tilde{f}_{e_2}(x_2)), ..., (x_m, \tilde{f}_{e_2}(x_m)))), .., (e_n, ((x_1, \tilde{f}_{e_n}(x_1)), (x_2, \tilde{f}_{e_n}(x_2)), .., (x_m, \tilde{f}_{e_n}(x_m))))\}$$

Here, $\tilde{f}_{e_j}(x_s)$ is the fuzzy-valued rating of an alternative x_s; $s = 1, 2, .., m$ over a parameter e_j; $j = 1, 2, .., n$. Tabular form of a fuzzy soft set $(\tilde{f}, E)$ has been illustrated in Table 2.1.

Table 2.1: Tabular form of a FSS $(\tilde{f}, E)$ (in general case)

	e_1	e_2	$\dots$	e_n
x_1	$\tilde{f}_{e_1}(x_1)$	$\tilde{f}_{e_2}(x_1)$	$\dots$	$\tilde{f}_{e_n}(x_1)$
x_2	$\tilde{f}_{e_1}(x_2)$	$\tilde{f}_{e_2}(x_2)$	$\dots$	$\tilde{f}_{e_n}(x_2)$
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
x_m	$\tilde{f}_{e_1}(x_m)$	$\tilde{f}_{e_2}(x_m)$	$\dots$	$\tilde{f}_{e_n}(x_m)$

(iv) **Euclidean distance of two FSSs. [63].**

Let us consider two fuzzy soft sets over the universe X as follows:

$$(\tilde{f}, E) = \left\{\left(e_1, \tilde{f}(e_1)\right), \left(e_2, \tilde{f}(e_2)\right), .., \left(e_n, \tilde{f}(e_n)\right)\right\}$$
$$= \{(e_1, ((x_1, \tilde{f}_{e_1}(x_1)), (x_2, \tilde{f}_{e_1}(x_2)), .., (x_m, \tilde{f}_{e_1}(x_m)))), (e_2, ((x_1, \tilde{f}_{e_2}(x_1)), (x_2, \tilde{f}_{e_2}(x_2)), ..., (x_m, \tilde{f}_{e_2}(x_m)))), .., (e_n, ((x_1, \tilde{f}_{e_n}(x_1)), (x_2, \tilde{f}_{e_n}(x_2)), .., (x_m, \tilde{f}_{e_n}(x_m))))\}$$

$$(\tilde{g}, E) = \{(e_1, \tilde{g}(e_1)), (e_2, \tilde{g}(e_2)), .., (e_n, \tilde{g}(e_n))\}$$
$$= \{(e_1, ((x_1, \tilde{g}_{e_1}(x_1)), (x_2, \tilde{g}_{e_1}(x_2)), .., (x_m, \tilde{g}_{e_1}(x_m)))), (e_2, ((x_1, \tilde{g}_{e_2}(x_1)), (x_2, \tilde{g}_{e_2}(x_2)), ..., (x_m, \tilde{g}_{e_2}(x_m)))), .., (e_n, ((x_1, \tilde{g}_{e_n}(x_1)), (x_2, \tilde{g}_{e_n}(x_2)), .., (x_m, \tilde{g}_{e_n}(x_m))))\}$$

Then, their Euclidean distance is derived by using the following equation:

$$D_{Eu}((\tilde{f}, E), (\tilde{g}, E)) = \frac{1}{n}\sum_{j=1}^{n} d_{Eu}(\tilde{f}(e_j), \tilde{g}(e_j))$$

where, $d_{Eu}(\tilde{f}(e_j), \tilde{g}(e_j)) = \sqrt{\frac{1}{m}\sum_{s=1}^{m}\left(\tilde{f}_{e_j}(x_s) - \tilde{g}_{e_j}(x_s)\right)^2}$.

2.3 Fuzzy soft set based group decision-making

Basically, group decision making problems over fuzzy soft sets are humanistic and subjective in nature. Consequently, researchers have used it widely in practice. In a fuzzy soft set based group decision-making, associated alternatives are evaluated by some decision makers with respect to some considered parameters in terms of fuzzy membership. In the existing literature it has been observed that, the researchers Roy and Maji [139], Basu et al. [22] and Alcantud [4] applied fuzzy soft sets in solving group decision-making problems to recognize the best alternative from a real-life problem where, multiple decision makers have been involved. But, however, employed decision makers can have diverse opinion about the decision alternatives due to their different choices, different knowledge backgrounds and individual satisfaction levels. So, in that case, there may be an error at the final stage. Then, to deal with such an unworthy situation in a practical group decision-making problem, measuring the consensus of the opinion of an decision maker with the others is an effective issue. Therefore, now we have given a new algorithmic approach for solving group decision-making problems over fuzzy soft sets which exceeds the above difficulty. The main goals of our proposed approach are, *to measure the consensus level of the associated decision makers about every alternative and then provide some suggestions to increase the consensus level of a decision maker, who have low consensus with the other decision makers for his/her provided opinion about the alternative, up to the considered threshold value. After that, to recognize the best alternative.*

Now, the mathematical illustration of our considered fuzzy soft set based group decision-making problem has been given in the next subsection.

2.3.1 Mathematical illustration of the problem

Assume a set of m alternatives as, $X = \{x_1, x_2, .., x_m\}$ and a set of n parameters as, $E = \{e_1, e_2, .., e_n\}$ which are in fuzzy sense. Now consider k decision makers as, $D = \{d_1, d_2, .., d_k\}$ and k fuzzy soft sets over X as, $(\tilde{f}^1, E), (\tilde{f}^2, E), .., (\tilde{f}^k, E)$ provided k decision makers. The description of the l^{th} fuzzy soft set $(\tilde{f}^l, E)$ which has been given by d_l decision maker is as follows:

$$\begin{aligned}(\tilde{f}^l, E) \quad &= \{(e_1, \{(x_1, \tilde{f}^l_{e_1}(x_1)), (x_2, \tilde{f}^l_{e_1}(x_2)), .., (x_m, \tilde{f}^l_{e_1}(x_m))\}), (e_2, \{(x_1, \tilde{f}^l_{e_2}(x_1)),\\ &(x_2, \tilde{f}^l_{e_2}(x_2)), .., (x_m, \tilde{f}^l_{e_2}(x_m))\}), .., (e_n, \{(x_1, \tilde{f}^l_{e_n}(x_1)),\\ &(x_2, \tilde{f}^l_{e_n}(x_2)), .., (x_m, \tilde{f}^l_{e_n}(x_m))\})\}; l = 1, 2, .., k.\end{aligned}$$

The k fuzzy soft sets have been given in Table 2.2.

2.3.2 Optimality criteria

Now, in order to handle a fuzzy soft set based group decision-making, we follow the following optimal criteria.

(i) The optimal decision alternative must satisfy all the corresponding parameters with a maximum evaluation rate over all the decision makers.

(ii) The optimal decision alternative do not have huge difference of satisfaction from one decision maker to other decision makers with respect to the considered parameters.

(iii) The optimal decision alternative should have a minimum distance from ideal decision alternative of this group decision-making problem.

Table 2.2: Tabular form of k fuzzy soft sets

	$(\tilde{f}_1, E)$				$(\tilde{f}_2, E)$			
	e_1	e_2	$\ldots$	e_n	e_1	e_2	$\ldots$	e_n
x_1	$\tilde{f}^1_{e_1}(x_1)$	$\tilde{f}^1_{e_2}(x_1)$	$\ldots$	$\tilde{f}^1_{e_n}(x_1)$	$\tilde{f}^2_{e_1}(x_1)$	$\tilde{f}^2_{e_2}(x_1)$	$\ldots$	$\tilde{f}^2_{e_n}(x_1)$
x_2	$\tilde{f}^1_{e_1}(x_2)$	$\tilde{f}^1_{e_2}(x_2)$	$\ldots$	$\tilde{f}^l_{e_n}(x_2)$	$\tilde{f}^2_{e_1}(x_2)$	$\tilde{f}^2_{e_2}(x_2)$	$\ldots$	$\tilde{f}^2_{e_n}(x_2)$
			$\ldots$				$\ldots$	
x_m	$\tilde{f}^1_{e_1}(x_m)$	$\tilde{f}^1_{e_2}(x_m)$	$\ldots$	$\tilde{f}^l_{e_n}(x_m)$	$\tilde{f}^2_{e_1}(x_m)$	$\tilde{f}^2_{e_2}(x_m)$	$\ldots$	$\tilde{f}^2_{e_n}(x_m)$

	$(\tilde{f}_k, E)$			
	e_1	e_2	$\ldots$	e_n
..	$\tilde{f}^k_{e_1}(x_1)$	$\tilde{f}^k_{e_2}(x_1)$	$\ldots$	$\tilde{f}^k_{e_n}(x_1)$
..	$\tilde{f}^k_{e_1}(x_2)$	$\tilde{f}^k_{e_2}(x_2)$	$\ldots$	$\tilde{f}^k_{e_n}(x_2)$
..			$\ldots$	
..	$\tilde{f}^k_{e_1}(x_m)$	$\tilde{f}^k_{e_2}(x_m)$	$\ldots$	$\tilde{f}^k_{e_n}(x_m)$

2.3.3 Solution framework

In this Subsection, we have proposed a solution approach to solve our considered fuzzy soft set based group decision-making which follows the above defined optimality criteria. Our proposed methodology consists mainly three parts as given in Figure 2.1. In first part, we have measured the comprehensive consensus level of every decision maker for every alternative over the every associated fuzzy soft set. In second part, we have given two suggestions to the decision maker, who have less comprehensive consensus than the considered threshold value λ for a certain alternative, to reformulate his/her opinion about the alternative so that, his/her comprehensive consensus level about the alternative can be increased up to the considered threshold value λ before selecting the best decision alternative from the problem. In third part, best decision alternative recognition process has been provided.

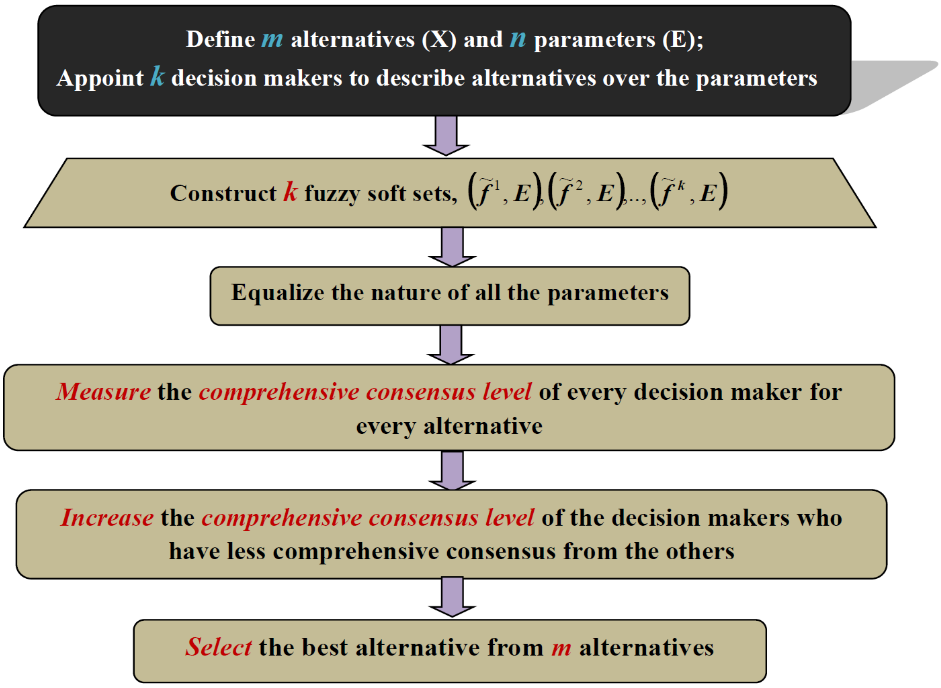

Figure 2.1: Schematic framework of our proposed approach

For each of the different parts, we have provided individual algorithms. Algorithm I is for first part (to measure the comprehensive consensus level). Algorithm II is for second part (to increase the comprehensive consensus level). For third part (best alternative selection), we have provided two different algorithms, Algorithm III and Algorithm IV.

Algorithm I (Measurement of comprehensive consensus level)

Step 1. Input all the alternatives ($X = \{x_1, x_2, .., x_m\}$) and all the corresponding parameters ($E = \{e_1, e_2, .., e_n\}$). Input k fuzzy soft sets $(\tilde{f}^1, E)$, $(\tilde{f}^2, E)$,..,$(\tilde{f}^k, E)$ as given in Table 2.2.

Step 2. *Equalization of all the decision evaluations.*
In a fuzzy soft set based group decision-making, in the considered parameter set, there may have a conflicting disposition i.e., some of them may have positive character (positive parameter means, with respect to such kind of parameter, high evaluation of a decision alternative is good) and some of them may have negative character (negative parameter means, with respect to such kind of parameter, low evaluation of a decision alternative is good). Then, to deal with this equivocal situation, we have equalized of all the corresponding decision evaluations before solving the problem as follows:

- Firstly, we have classified positive parameters (suppose its collection set is A) and negative parameters (suppose its collection set is B) from the set E where, $A \cup B = E$ and $A \cap B = \phi$.

- Then, we have taken fuzzy complement (given in point (i) in Section 2.2) of the evaluations of all the decision alternatives over each of the negative parameters for every fuzzy soft set.

Step 3. *Measurement of the consensus level of every pair of decision makers.*
The consensus level or closeness level between two decision makers d_l and $d_{l'}$ with respect to their associated fuzzy soft sets $(\tilde{f}^l, E)$ and $(\tilde{f}^{l'}, E)$ for an alternative x_s is denoted $\Theta^s(l, l')$ and is derived as follows:

$$\Theta^s(l, l') = 1 - d(\tilde{f}^l(x_s), \tilde{f}^{l'}(x_s)) \tag{2.1}$$

where, d is the fuzzy distance (given in point (ii) in Section 2.2) and $\tilde{f}^l(x_s)$, $\tilde{f}^{l'}(x_s)$ are the fuzzy approximations of an alternative x_s over all the parameters provided by the experts d_l and $d_{l'}$ as follows:
$\tilde{f}^l(x_s) = \{(e_1, \tilde{f}^l_{e_1}(x_s)), (e_2, \tilde{f}^l_{e_2}(x_s)), .., (e_n, \tilde{f}^l_{e_n}(x_s))\}$;
$\tilde{f}^{l'}(x_s) = \{(e_1, \tilde{f}^{l'}_{e_1}(x_s)), (e_2, \tilde{f}^{l'}_{e_2}(x_s)), .., (e_n, \tilde{f}^{l'}_{e_n}(x_s))\}$.

Step 4. *Derivation of the comprehensive consensus level of a decision maker d_l for an alternative x_s.*
The comprehensive consensus level of a decision maker d_l for an alternative x_s with respect to his/her associated fuzzy soft set $(\tilde{f}^l, E)$ is denoted by, $\Theta^s(l)$ and is defined by the following equation:

$$\Theta^s(l) = \sum_{l'=1, l \neq l'}^{k} \frac{\Theta^s(l, l')}{k-1} \tag{2.2}$$

Step 5. Let, λ $(0 < \lambda \leq 1)$ be the threshold value for the comprehensive consensus level of this fuzzy soft set based group decision-making. If, comprehensive consensus level of each of the decision makers for every alternative is greater than or equals to the considered threshold value λ then, we can go to the selection process i.e., if $\Theta^s(l) \geq \lambda; \forall s = 1, 2, .., m; l = 1, 2, .., k$ then, we will start the selection process to recognize the best decision alternative. But, if, the comprehensive consensus level of some decision makers for some alternatives is less than the consider threshold value λ then, we will go to the Algorithm II to increase the comprehensive consensus level of these decision makers before selecting the best decision alternative.

Algorithm II (Increasing process of comprehensive consensus level)

Suppose that, at T^{th} decision round ($T = 1, 2, 3, ...$), the comprehensive consensus level ($\Theta^s(l)$) of a decision maker d_l for an alternative x_s is less than the considered threshold value λ. Then, to increase the comprehensive consensus level of the decision maker d_l for the alternative x_s at the next decision round $(T + 1)$, we will follow the following steps. Suppose, at T^{th} decision round, the primary evaluation of an alternative x_s over a parameter e_j given by the decision maker d_l is $\tilde{f}^l_{e_j,T}(x_s)$.

Step 1. Select the decision maker (d_l) and the associated alternative (x_s) whose corresponding comprehensive consensus level at the T^{th} decision round is less than the considered threshold value λ.

Step 2. Then, detect the corresponding parameter e_j for which the comprehensive consensus of the decision maker d_l for the alternative x_s is less than the considered threshold value λ as follows:

- First, evaluate the total deviation of a decision maker d_l with the other decision makers for the alternative x_s over the parameter e_j. It can be denoted by $D^s(l)(e_j)$ and can be derived by the following equation:

$$D^s(l)(e_j) = \frac{1}{k-1} \sum_{l \neq l', l'=1}^{k} |\tilde{f}^l_{e_j,T}(x_s) - \tilde{f}^{l'}_{e_j,T}(x_s)| \tag{2.3}$$

  Suppose, λ' be the the threshold value in deriving the total deviation of a decision maker d_l.

- Select the parameter (e_j) for which the total deviation of the decision maker d_l for the alternative x_s is greater than the consider threshold value λ'. i.e., $D^s(l)(e_j) > \lambda'$.

Step 3. Then, for the next $(T + 1)^{th}$ decision round, apply appropriate suggestion (as provided below) to reformulate the evaluation of the alternative x_s for the decision maker d_l over the parameter e_j.

Step 4. After applying our proposed suggestion at the $(T + 1)^{th}$ decision round, we will check the comprehensive consensus level ($\Theta^s(l)$) of the decision maker d_l for the alternative x_s at this decision round by using Equations 2.1 and 2.2.

Step 5. If, at this decision round, the value of $\Theta^s(l)$ is greater than or equals to the considered threshold value λ, then we will go to the selection process. Otherwise, we have to go to Step 1 of this algorithm and we have to repeat this algorithm again. This process will repeat until the comprehensive consensus level ($\Theta^s(l)$) of every decision maker d_l for every alternative x_s is greater than or equals to the considered threshold value λ.

The suggestions to reformulate the primary evaluation of the expert d_l for x_s alternative over the parameter e_j are as follows:

Suggestion 1.
$\tilde{f}^l_{e_j,(T+1)}(x_s) = \gamma \left(\tilde{f}^l_{e_j,T}(x_s)\right) \circledS_\gamma (1-\gamma) \left(\cup^k_{l=1} \tilde{f}^l_{e_j,T}(x_s)\right)$.

Suggestion 2.
$\tilde{f}^l_{e_j,(T+1)}(x_s) = \gamma \left(\tilde{f}^l_{e_j,T}(x_s)\right) \circledS_\gamma (1-\gamma)\ \tilde{A} \left((\tilde{f}^1_{e_j,T}(x_s)), (\tilde{F}^2_{e_j,T}(x_s)), .., (\tilde{F}^k_{e_j,T}(x_s))\right)$.

where, $\cup$ is the standard fuzzy union, $\tilde{A}$ is the fuzzy aggregation, $\circledS_\gamma$ is the convex linear sum, and γ is the influence factor.

Remark 1: *Suggestion 1* will be preferable for the cases, when the opinion of a decision maker d_l about an alternative x_s over a parameter e_j is smaller than the other decision makers.
Remark 2: *Suggestion 2* will be preferable for the cases, when the opinion of a decision maker d_l about an alternative x_s over a parameter e_j is larger than the other decision makers or when the opinion of a decision maker d_l about an alternative x_s is lies between the larger evaluation and smaller evaluation over the opinions of all the decision makers.

Selection process for choosing the best alternative

After checking comprehensive consensus every expert for every alternative, now we have evaluated the best decision alternative from m corresponding alternatives for this fuzzy soft set based group decision-making. Suppose $\mathfrak{T}$ is the final decision round where, comprehensive consensus level of every decision maker for every alternative is greater than or equals to the considered threshold value λ. At $\mathfrak{T}^{th}$ decision round, the corresponding k fuzzy soft sets has been given in Table 2.3.

Now, to recognize the best decision alternative based on all the k fuzzy soft sets $(\tilde{f}^1_{\mathfrak{T}}, E)$, $(\tilde{f}^2_{\mathfrak{T}}, E)$,..$(\tilde{f}^k_{\mathfrak{T}}, E)$ at the final $\mathfrak{T}^{th}$ decision round, we have provided two algorithms: Algorithm III and Algorithm IV.

- *In Algorithm III,* firstly, we have aggregated k fuzzy soft sets $(\tilde{f}^1_{\mathfrak{T}}, E)$, $(\tilde{f}^2_{\mathfrak{T}}, E)$,..,$(\tilde{f}^k_{\mathfrak{T}}, E)$ by using AND operator to construct a single resultant fuzzy soft set $(\tilde{f}_{\mathfrak{T}}, E)$. Then, from this resultant fuzzy soft set $(\tilde{f}_{\mathfrak{T}}, E)$, we have obtained the raking order of the alternatives to recognize best decision alternative.

- On the other hand, *in Algorithm IV*, firstly, we have derived the ranking indices of every alternative from each of the individual fuzzy soft sets $(\tilde{f}^1_{\mathfrak{T}}, E)$, $(\tilde{f}^2_{\mathfrak{T}}, E)$,..,$(\tilde{f}^k_{\mathfrak{T}}, E)$ and then, we have aggregated these ranking index values over all the k fuzzy soft sets to get overall ranking index of an alternative. After that, we have obtained the ranking order of every alternative to recognize best decision alternative.

So, the backgrounds of these two algorithms are little bit different as given in Figure 2.2. Algorithm III is suitable for the problems where, integrated opinion of the decision makers is significant and Algorithm IV is suitable for the problems where, individual opinion of a decision maker is significant. In the following, we have given the detail descriptions of these two algorithms.

Table 2.3: Tabular form of k fuzzy soft sets at final decision round $\mathfrak{T}$

	$(\tilde{f}^1_{\mathfrak{T}}, E)$				$(\tilde{f}^2_{\mathfrak{T}}, E)$			
	e_1	e_2	$\dots$	e_n	e_1	e_2	$\dots$	e_n
x_1	$\tilde{f}^1_{e_1,\mathfrak{T}}(x_1)$	$\tilde{f}^1_{e_2,\mathfrak{T}}(x_1)$	$\dots$	$\tilde{f}^1_{e_n,\mathfrak{T}}(x_1)$	$\tilde{f}^2_{e_1,\mathfrak{T}}(x_1)$	$\tilde{f}^2_{e_2,\mathfrak{T}}(x_1)$	$\dots$	$\tilde{f}^2_{e_n,\mathfrak{T}}(x_1)$
x_2	$\tilde{f}^1_{e_1,\mathfrak{T}}(x_2)$	$\tilde{f}^1_{e_2,\mathfrak{T}}(x_2)$	$\dots$	$\tilde{f}^l_{e_n,\mathfrak{T}}(x_2)$	$\tilde{f}^2_{e_1,\mathfrak{T}}(x_2)$	$\tilde{f}^2_{e_2,\mathfrak{T}}(x_2)$	$\dots$	$\tilde{f}^2_{e_n,\mathfrak{T}}(x_2)$
			$\dots$				$\dots$	
x_m	$\tilde{f}^1_{e_1,\mathfrak{T}}(x_m)$	$\tilde{f}^1_{e_2,\mathfrak{T}}(x_m)$	$\dots$	$\tilde{f}^l_{e_n,\mathfrak{T}}(x_m)$	$\tilde{f}^2_{e_1,\mathfrak{T}}(x_m)$	$\tilde{f}^2_{e_2,\mathfrak{T}}(x_m)$	$\dots$	$\tilde{f}^2_{e_n,\mathfrak{T}}(x_m)$

	$(\tilde{f}^k_{\mathfrak{T}}, E)$			
	e_1	e_2	$\dots$	e_n
..	$\tilde{f}^k_{e_1,\mathfrak{T}}(x_1)$	$\tilde{f}^k_{e_2,\mathfrak{T}}(x_1)$	$\dots$	$\tilde{f}^k_{e_n,\mathfrak{T}}(x_1)$
..	$\tilde{f}^k_{e_1,\mathfrak{T}}(x_2)$	$\tilde{f}^k_{e_2,\mathfrak{T}}(x_2)$	$\dots$	$\tilde{f}^k_{e_n,\mathfrak{T}}(x_2)$
..			$\dots$	
..	$\tilde{f}^k_{e_1,\mathfrak{T}}(x_m)$	$\tilde{f}^k_{e_2,\mathfrak{T}}(x_m)$	$\dots$	$\tilde{f}^k_{e_n,\mathfrak{T}}(x_m)$

Algorithm III:

Step 1. ***Construction of the ideal fuzzy soft set.***
Construct the ideal fuzzy soft set $(\tilde{f}^I, E)$ form k fuzzy soft sets $(\tilde{f}^1_{\mathfrak{T}}, E)$, $(\tilde{f}^2_{\mathfrak{T}}, E)$,..,$(\tilde{f}^k_{\mathfrak{T}}, E)$ by using fuzzy soft union operation as follows:

$$(\tilde{f}^I, E) = (\tilde{f}^1_{\mathfrak{T}}, E) \cup (\tilde{f}^2_{\mathfrak{T}}, E) \cup .. \cup (\tilde{f}^k_{\mathfrak{T}}, E)$$

where, $\forall e_j \in E$, and $x_s \in X$,

$$\begin{aligned} \tilde{f}^I_{e_j}(x_s) &= \tilde{f}^1_{e_j,\mathfrak{T}}(x_s) \cup \tilde{f}^2_{e_j,\mathfrak{T}}(x_s) \cup .. \cup \tilde{f}^k_{e_j,\mathfrak{T}}(x_s) \\ &= max\{\tilde{f}^1_{e_j,\mathfrak{T}}(x_s), \tilde{f}^2_{e_j,\mathfrak{T}}(x_s), .., \tilde{f}^k_{e_j,\mathfrak{T}}(x_s)\} \end{aligned}$$

Step 2. ***Derivation of the similarity degree of an individual*** $(\tilde{f}^l_{\mathfrak{T}}, E)$ ***with the ideal fuzzy soft set*** $(\tilde{f}^I, E)$**.**

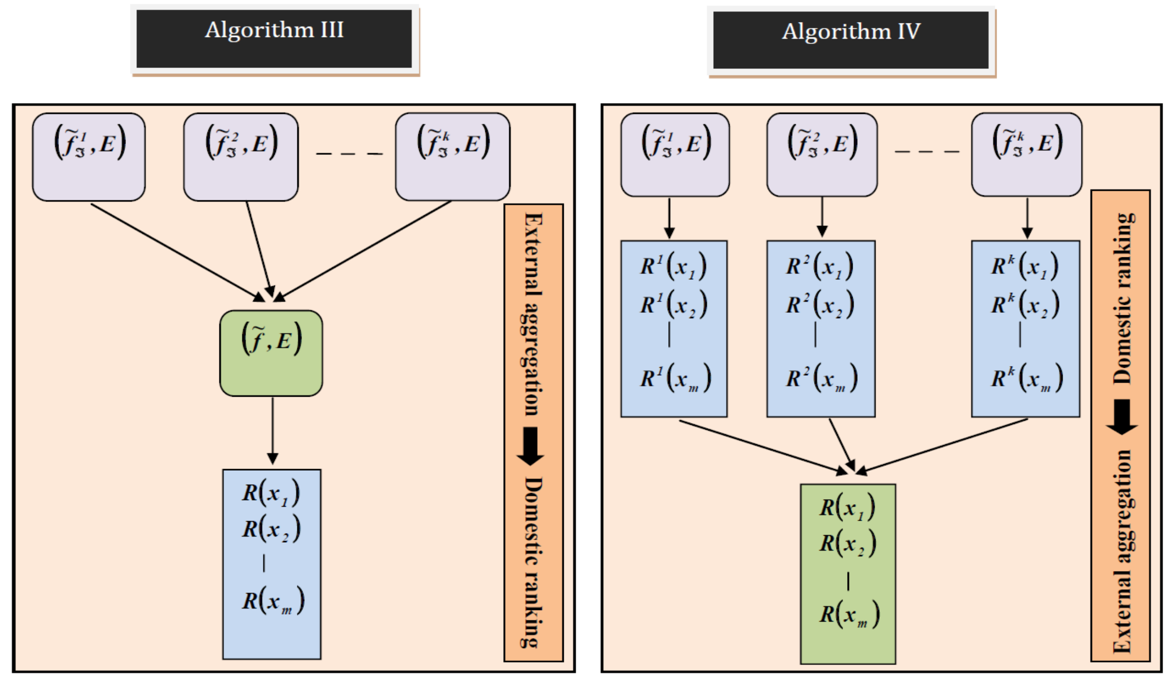

Figure 2.2: Key features of Algorithm III and Algorithm IV

The similarity degree of a fuzzy soft set $(\tilde{f}^l_{\mathfrak{I}}, E)$ with the ideal fuzzy soft set $(\tilde{f}^I, E)$ is denoted by ς_l and is defined as follows:

$$\varsigma_l = 1 - D((\tilde{f}^l_{\mathfrak{I}}, E)(\tilde{f}^I, E)); l = 1, 2, .., k \tag{2.4}$$

where, D is the fuzzy soft distance measure as given in point (iv) in Section 2.2.

Now to satisfy the condition that, $\sum\limits_{l=1}^{k} \varsigma_l = 1$, we have taken the similarity degree of an individual $(\tilde{f}^l_{\mathfrak{I}}, E)$ as, $\rho_l = \frac{\varsigma_l}{\sum\limits_{l=1}^{k} \varsigma_l}; \forall l = 1, 2, .., k$.

Step 3. ***Construction of the weighted fuzzy soft set*** $(\tilde{f}^l_{\mathfrak{I}\rho}, E)$ ***from the individual*** $(\tilde{f}^l_{\mathfrak{I}}, E)$**.**
Now, construct the weighted fuzzy soft set $(\tilde{f}^l_{\mathfrak{I}\rho}, E)$ from the individual fuzzy soft set $(\tilde{f}^l_{\mathfrak{I}}, E)$ by multiplying the corresponding similarity degree ρ_l as follows:

$$\begin{aligned}(\tilde{f}^l_{\mathfrak{I}\rho}, E) &= \rho_l \times (\tilde{f}^l_{\mathfrak{I}}, E) \\ &= \{(e_j, (x_s, \rho_l \tilde{f}^l_{e_j,\mathfrak{I}}(x_s)))|\forall e_j \in E, x_s \in X\}\end{aligned} \tag{2.5}$$

Step 4. ***Computation of the resultant fuzzy soft set*** $(\tilde{f}_{\mathfrak{I}}, E)$**.**
By using 'AND' operator, now compute resultant fuzzy soft set $(\tilde{f}_{\mathfrak{I}}, E)$ from k fuzzy soft

sets $(\tilde{f}^1_{\mathfrak{T}\rho}, E)$, $(\tilde{f}^2_{\mathfrak{T}\rho}, E)$,.., $(\tilde{f}^k_{\mathfrak{T}\rho}, E)$. Tabular form of the resultant fuzzy soft set has been given in Table 2.4.

Table 2.4: Resultant fuzzy soft set $(\tilde{f}_{\mathfrak{T}}, E)$ at final decision round $\mathfrak{T}$

	e_1	e_2	$\dots$	e_n
x_1	$\tilde{f}_{e_1,\mathfrak{T}}(x_1)$	$\tilde{f}_{e_2,\mathfrak{T}}(x_1)$	$\dots$	$\tilde{f}_{e_n,\mathfrak{T}}(x_1)$
x_2	$\tilde{f}_{e_1,\mathfrak{T}}(x_2)$	$\tilde{f}_{e_2,\mathfrak{T}}(x_2)$	$\dots$	$\tilde{f}_{e_n,\mathfrak{T}}(x_2)$
			$\dots$	
x_m	$\tilde{f}_{e_1,\mathfrak{T}}(x_m)$	$\tilde{f}_{e_2,\mathfrak{T}}(x_m)$	$\dots$	$\tilde{f}_{e_n,\mathfrak{T}}(x_m)$

Step 5. ***Determination of the positive ideal solution and negative ideal solution from the resultant fuzzy soft set.***
Evaluate positive ideal and negative ideal solutions from the resultant fuzzy soft set $(\tilde{f}_{\mathfrak{T}}, E)$ as follows:

$$x_{I^+} = \Big\{(e_1, max^m_{s=1}\tilde{f}_{e_1,\mathfrak{T}}(x_s)), (e_2, max^m_{s=1}\tilde{f}_{e_2,\mathfrak{T}}(x_s)), .., (e_n, max^m_{s=1}\tilde{f}_{e_n,\mathfrak{T}}(x_s))\Big\}$$
$$x_{I^-} = \Big\{(e_1, min^m_{s=1}\tilde{f}_{e_1,\mathfrak{T}}(x_s)), (e_2, min^m_{s=1}\tilde{f}_{e_2,\mathfrak{T}}(x_s)), .., (e_n, min^m_{s=1}\tilde{f}_{e_n,\mathfrak{T}}(x_s))\Big\}$$

Step 6. ***Derivation of the distance of an alternative x_s from x_{I^+} and x_{I^-}.***
Now, by using fuzzy distance, evaluate the positive $(d(x_s, x_{I^+}))$ and negative distances $(d(x_s, x_{I^-}))$ of an alternative x_s $(s = 1, 2, .., m)$ form positive ideal (x_{I^+}) and negative ideal (x_{I^-}) solutions over the resultant fuzzy soft set $(\tilde{f}_{\mathfrak{T}}, E)$.

Step 7. ***Determination of the confidence grade of an alternative x_s.***
By using positive distance and negative distance, derive the confidence grade (CG_s) or ranking value $(R(x_s))$ of the alternative x_s as follows:

$$R(x_s) = CG_s = \frac{d(x_s, x_{I^+})}{d(x_s, x_{I^-}) + d(x_s, x_{I^+})} \tag{2.6}$$

Step 8. ***Selection of the best alternative.***
The alternative having minimum confidence grade or ranking value is the best decision alternative among the corresponding m alternatives for this fuzzy soft set based group decision-making problem.

If, the best decision alternative is not unique i.e., suppose if, x_{m_1}, x_{m_2},..,x_{m_M} have the same confidence grade where, $m_1, m_2, .., m_M \in \{1, 2, .., m\}$, then we will go to the next steps.

Step 8.1 Evaluate the total fluctuation rate $FG_{s'}$ of an alternative $x_{s'}$; $s' = m_1, m_2, .., m_M$ with respect to every pair of parameters as follows:

$$FG_{s'} = \sum_{j=1, j\neq j'}^{n} \left| \tilde{f}_{e_j,\mathfrak{T}}(x_{s'}) - \tilde{f}_{e_{j'},\mathfrak{T}}(x_{s'}) \right|$$

Step 8.2 From the set of alternatives, who have same confidence grade at Step 8, recognize the best decision alternative based on their minimum fluctuation rate i.e.,

$$Best\ decision\ alternative = min_{s'=m_1}^{m_M} FG_{s'} \tag{2.7}$$

If, the best decision alternative is not unique, then we will select any one of them.

Algorithm IV:

Step 1. ***Construction of the ideal fuzzy soft set.***
Construct the ideal fuzzy soft set $(\tilde{f}^I, E)$ likewise Step 1 of Algorithm III.

Step 2. ***Derivation of the similarity degree of an individual*** $(\tilde{f}^l_{\mathfrak{T}}, E)$ ***with the ideal*** $(\tilde{f}^I, E)$.
By using Equation 2.4, derive the similarity degree ρ_l of an individual fuzzy soft set $(\tilde{f}^l_{\mathfrak{T}}, E)$ with the ideal fuzzy soft set $(\tilde{f}^I, E)$.

Step 3. ***Construction of the weighted fuzzy soft set*** $(\tilde{f}^l_{\mathfrak{T}\rho}, E)$ ***from an individual*** $(\tilde{f}^l_{\mathfrak{T}}, E)$.
Now, construct the weighted fuzzy soft set $(\tilde{f}^l_{\mathfrak{T}\rho}, E)$ from an individual fuzzy soft set $(\tilde{f}^l_{\mathfrak{T}}, E)$ by using Equation 2.5.

Step 4. ***Determination of the positive ideal and negative ideal solutions from an individual*** $(\tilde{f}^l_{\mathfrak{T}\rho}, E)$.
The positive ideal $(I^+(\tilde{f}^l_{\mathfrak{T}\rho}))$ and negative ideal $(I^-(\tilde{f}^l_{\mathfrak{T}\rho}))$ solutions from a weighted fuzzy soft set $(\tilde{f}^l_{\mathfrak{T}\rho}, E)$ are as follows: $\forall l = 1, 2, .., k$,

$$\begin{aligned} I^+(\tilde{F}^l_{\mathfrak{T}\rho}) &= \left\{ \cup_{s=1}^{m} \tilde{F}^l_{e_1,\mathfrak{T}\rho}(x_s), \cup_{s=1}^{m} \tilde{F}^l_{e_2,\mathfrak{T}\rho}(x_s), .., \cup_{s=1}^{m} \tilde{F}^l_{e_n,\mathfrak{T}\rho}(x_s) \right\} \\ &= \left\{ max_{s=1}^{m} \tilde{F}^l_{e_1,\mathfrak{T}\rho}(x_s), max_{s=1}^{m} \tilde{F}^l_{e_2,\mathfrak{T}\rho}(x_s), .., max_{s=1}^{m} \tilde{F}^l_{e_n,\mathfrak{T}\rho}(x_s) \right\} \\ I^-(\tilde{F}^l_{\mathfrak{T}\rho}) &= \left\{ \cap_{s=1}^{m} \tilde{F}^l_{e_1,\mathfrak{T}\rho}(x_s), \cap_{s=1}^{m} \tilde{F}^l_{e_2,\mathfrak{T}\rho}(x_s), .., \cap_{s=1}^{m} \tilde{F}^l_{e_n,\mathfrak{T}\rho}(x_s) \right\} \\ &= \left\{ min_{s=1}^{m} \tilde{F}^l_{e_1,\mathfrak{T}\rho}(x_s), min_{s=1}^{m} \tilde{F}^l_{e_2,\mathfrak{T}\rho}(x_s), .., min_{s=1}^{m} \tilde{F}^l_{e_n,\mathfrak{T}\rho}(x_s) \right\} \end{aligned}$$

Step 5. ***Derivation of the distance of an alternative*** x_s.
By utilizing fuzzy distance, derive the positive distance $d^l(x_s, I^+(\tilde{f}^l_{\mathfrak{T}\rho}))$ and negative distance $d^l(x_s, I^-(\tilde{f}^l_{\mathfrak{T}\rho}))$ of an alternative x_s from the positive ideal solution and negative ideal solution

Table 2.5: Positive distances of the alternatives

	$(\tilde{f}^1_{\mathfrak{T}\rho}, E)$	$(\tilde{f}^2_{\mathfrak{T}\rho}, E)$	$\dots$	$(\tilde{f}^k_{\mathfrak{T}\rho}, E)$
x_1	$d^1(x_1, I^+(\tilde{f}^l_{\mathfrak{T}\rho}))$	$d^2(x_1, I^+(\tilde{f}^l_{\mathfrak{T}\rho}))$	$\dots$	$d^k(x_1, I^+(\tilde{f}^l_{\mathfrak{T}\rho}))$
x_2	$d^1(x_2, I^+(\tilde{f}^l_{\mathfrak{T}\rho}))$	$d^2(x_2, I^+(\tilde{f}^l_{\mathfrak{T}\rho}))$	$\dots$	$d^k(x_2, I^+(\tilde{f}^l_{\mathfrak{T}\rho}))$
			$\dots$	
x_m	$d^1(x_m, I^+(\tilde{f}^l_{\mathfrak{T}\rho}))$	$d^2(x_2, I^+(\tilde{f}^l_{\mathfrak{T}\rho}))$	$\dots$	$d^k(x_m, I^+(\tilde{f}^l_{\mathfrak{T}\rho}))$

Table 2.6: Negative distances of the alternatives

	$(\tilde{f}^1_{\mathfrak{T}\rho}, E)$	$(\tilde{f}^2_{\mathfrak{T}\rho}, E)$	$\dots$	$(\tilde{f}^k_{\mathfrak{T}\rho}, E)$
x_1	$d^1(x_1, I^-(\tilde{f}^l_{\mathfrak{T}\rho}))$	$d^2(x_1, I^-(\tilde{f}^l_{\mathfrak{T}\rho}))$	$\dots$	$d^k(x_1, I^-(\tilde{f}^l_{\mathfrak{T}\rho}))$
x_2	$d^1(x_2, I^-(\tilde{f}^l_{\mathfrak{T}\rho}))$	$d^2(x_2, I^-(\tilde{f}^l_{\mathfrak{T}\rho}))$	$\dots$	$d^k(x_2, I^-(\tilde{f}^l_{\mathfrak{T}\rho}))$
			$\dots$	
x_m	$d^1(x_m, I^-(\tilde{f}^l_{\mathfrak{T}\rho}))$	$d^2(x_2, I^-(\tilde{f}^l_{\mathfrak{T}\rho}))$	$\dots$	$d^k(x_m, I^-(\tilde{f}^l_{\mathfrak{T}\rho}))$

over a weighted fuzzy soft set $(\tilde{f}^l_{\mathfrak{T}\rho}, E)$. In Table 2.5 and Table 2.6, these values have been presented.

Step 6. ***Determination of the confidence grade of an alternative x_s over a weighted fuzzy soft set $(\tilde{f}^l_{\mathfrak{T}\rho}, E)$.***

Now, we have evaluated the confidence grade of an alternative x_s over a weighted fuzzy soft set $(\tilde{f}^l_{\mathfrak{T}\rho}, E)$ by the following equation. In Table 2.7, these values have been presented.

$$CG^l_s = \frac{d^l(x_s, I^+(\tilde{F}^l_{\mathfrak{T}\rho}))}{d^l(x_s, I^+(\tilde{F}^l_{\mathfrak{T}\rho})) + d^l(x_s, I^-(\tilde{F}^l_{\mathfrak{T}\rho}))} \tag{2.8}$$

Table 2.7: Confidence grades of the alternatives

	$(\tilde{f}^1_{\mathfrak{T}\rho}, E)$	$(\tilde{f}^2_{\mathfrak{T}\rho}, E)$	$\dots$	$(\tilde{f}^k_{\mathfrak{T}\rho}, E)$
x_1	CG^1_1	CG^2_1	$\dots$	CG^k_1
x_2	CG^1_2	CG^2_2	$\dots$	CG^k_2
			$\dots$	
x_m	CG^1_m	CG^2_m	$\dots$	CG^k_m

Step 7. ***Construction of the ranking value of an alternative x_s.***

Now, the ranking value of an alternative x_s over all the k weighted fuzzy soft sets $(\tilde{f}^1_{\mathfrak{T}\rho}, E)$, $(\tilde{f}^2_{\mathfrak{T}\rho}, E)$,..,$(\tilde{f}^k_{\mathfrak{T}\rho}, E)$ is as follows:

$$R^{CG}(x_s) = \frac{1}{k}\sum_{l=1}^{k} CG^l_s \; ; s = 1, 2, .., m \tag{2.9}$$

Step 8. ***Selection of the best alternative.***
The alternative having minimum ranking value among m corresponding alternatives is the best decision alternative for this fuzzy soft set based group decision-making problem.

If, the best decision alternative is not unique i.e., suppose if x_{m_1}, x_{m_2},..,x_{m_M} have the same ranking value where, $m_1, m_2, .., m_M \in \{1, 2, .., m\}$, then we will go to the next steps.

Step 8.1 Determine the possibility index $PI_{s'}$ of an alternative $x_{s'}$; $s^{'} = m_1, m_2, .., m_M$ as a best decision alternative over all the decision makers as follows:

$$PI_{s'} = \frac{n(x_{s'})}{k}$$

where, $n(x_{s'})$ is the number of decision makers who have recognized $x_{s'}$ as the best alternative.
Based on the possibility index, the best decision alternative will be that alternative who have maximum possibility index among the other alternatives. i.e., select the alternative who have been recognized as the best decision alternative by the maximum number of decision makers.
If, in case, the best decision alternative is not unique, then we have to select any one of them.

Example 2.1. Let us consider four objects as, $X = \{x_1, x_2, x_3, x_4\}$ and four corresponding parameters as, $E = \{e_1, e_2, e_3, e_4\}$ each of which are in positive sense. Now consider three fuzzy soft sets $(\tilde{f}^1, E)$, $(\tilde{f}^2, E)$ and $(\tilde{f}^3, E)$ over X given by three decision makers $D = \{d_1, d_2, d_3\}$ as given in Tables 2.8, 2.9 and 2.10.

Table 2.8: FSS $(\tilde{f}^1, E)$ (Example 2.1)

	e_1	e_2	e_3
x_1	0.8	0.6	0.1
x_2	0.6	0.6	0.7
x_3	0.5	0.4	0.8

Table 2.9: FSS $(\tilde{f}^2, E)$ (Example 2.1)

	e_1	e_2	e_3
x_1	0.1	0.4	0.7
x_2	0.6	0.5	0.6
x_3	0.6	0.4	0.2

Table 2.10: FSS $(\tilde{f}^3, E)$ (Example 2.1)

	e_1	e_2	e_3
x_1	0.7	0.4	0.6
x_2	0.5	0.4	0.7
x_3	0.6	0.5	0.3

Solution: Throughout the solution, for deriving fuzzy distance and fuzzy soft distance, we have used Euclidean distance as given in Section 2.2.

Algorithm I: (Measurement of comprehensive consensus level of a decision maker for every alternative).
Let, the threshold value for comprehensive consensus level is, $\lambda = 0.80$.
Step 1. Three fuzzy soft sets have been given in Tables 2.8, 2.9 and 2.10.

Step 2. Since, every parameter are positive in nature therefore, we do not need the equalization process.
Step 3, 4. Now, by using Equations 2.1 and 2.2, comprehensive consensus level of the decision makers for every alternative are as follows:
$\Theta^1(1) = \mathbf{0.51}$, $\Theta^2(1) = 0.89$, $\Theta^3(1) = \mathbf{0.78}$;
$\Theta^1(2) = \mathbf{0.54}$, $\Theta^2(2) = 0.90$, $\Theta^3(2) = 0.84$;
$\Theta^1(3) = \mathbf{0.68}$, $\Theta^2(3) = 0.88$, $\Theta^3(3) = 0.86$;
From the above values it has been observed that, the comprehensive consensus level of the decision makers d_1, d_2 and d_3 for the alternative x_1 is less than the considered threshold value ($\lambda = 0.8$). Moreover, the comprehensive consensus level of the decision maker d_1 for the alternative x_3 is less than the considered threshold value ($\lambda = 0.8$). Therefore, according to our approach, we have to go to the Algorithm II to reformulate the evaluations of the alternatives so that, the comprehensive consensus level of all the decision makers for every alternative can reach to the considered threshold value.

Algorithm II: (To increase the comprehensive consensus level.)
In Algorithm II, we will derive the total deviation of a decision maker with the other decision makers for the corresponding alternative over each of the parameters. It has been considered that, the threshold value for the total deviation of a decision maker for this fuzzy soft set based group decision-making is 0.4.

Step 1. Here, the decision makers d_1, d_2 and d_3 have less comprehensive consensus for the alternative x_1 and also for the alternative x_3, d_3 decision maker has the less comprehensive consensus level.

Step 2. Then, by using Equation 2.3, the total deviation of the experts d_1, d_2 and d_3 for the alternative x_1 with respect to every parameter are as follows:
$D^1(1)(e_1) = 0.4$; $D^1(1)(e_2) = 0.2$; $D^1(1)(e_3) = \mathbf{0.55}$.
$D^1(2)(e_1) = \mathbf{0.65}$; $D^1(2)(e_2) = 0.1$; $D^1(1)(e_3) = 0.35$.
$D^1(3)(e_1) = 0.35$; $D^1(3)(e_2) = 0.1$; $D^1(3)(e_3) = 0.3$.
Similarly, the total deviation of the expert d_3 for the object or alternative x_1 over every parameter is as follows:
$D^3(1)(e_1) = 0.1$; $D^3(1)(e_2) = 0.05$; $D^3(1)(e_3) = \mathbf{0.55}$.
From the above results it has been observed that, the total deviation of the alternative x_1 with respect to the parameter e_3 for the decision maker d_1 and the total deviation of the same alternative x_1 with respect to the parameter e_1 for the decision maker d_2 are greater than the considered threshold value ($\lambda' = 0.4$). Besides, the total deviation of the alternative x_3 with respect to the parameter e_3 for the decision maker d_1 is greater than the considered threshold value ($\lambda' = 0.4$). Therefore, they need a reformulation.

Step 3. Now, we have reformulated the primary evaluations (at $T = 0$ decision round) by using our proposed suggestions for $T = 1$ decision round.

- From, the primary tables (Table 2.8, Table 2.9 and Table 2.10), we have seen that, at $T = 0$, $\tilde{f}^1_{e_3,0}(x_1) = 0.1$. i.e., the primary evaluation of x_1 alternative over e_3 parameter given by the decision maker d_1 is smallest than the other decision makers so, here *Suggestion 1* is appropriate.

- Again, the satisfaction of the alternative x_1 over the parameter e_3 given by the decision maker d_2 is smallest than the other decision makers, so here also *Suggestion 1* is appropriate.

- But, the satisfaction of x_3 alternative over e_3 parameter given by the decision maker d_1 is highest than the decision makers. Therefore, in that case, *Suggestion 2* is appropriate.

Now, by applying these suggestions on the evaluations of the Tables 2.8, 2.9, 2.10 i.e., at $T = 0$ decision round, the new reformulate fuzzy soft sets at $T = 1$ decision round takes the form as given in Table 2.11, Table 2.12 and Table 2.13.

Table 2.11: FSS $(\tilde{f}^1_1, E)$ (Example 2.1) (at T=1)

	e_1	e_2	e_3
x_1	0.8	0.6	0.4
x_2	0.6	0.6	0.7
x_3	0.5	0.4	0.62

Table 2.12: FSS $(\tilde{f}^2_1, E)$ (Example 2.1) (at T=1)

	e_1	e_2	e_3
x_1	0.45	0.4	0.9
x_2	0.6	0.5	0.6
x_3	0.6	0.4	0.2

Table 2.13: FSS $(\tilde{f}^3_1, E)$ (Example 2.1) (at T=1)

	e_1	e_2	e_3
x_1	0.7	0.4	0.6
x_2	0.5	0.4	0.7
x_3	0.6	0.5	0.3

Step 4. Then, by using Equations 2.1 and 2.2, we have derived the the comprehensive consensus level of the decision makers whose comprehensive consensus level at the $T = 0$ decision round were less than the considered threshold value. At $T = 1$ decision round, comprehensive consensus of the experts are as follows:
$\Theta^1(1) = 0.80$; $\Theta^1(2) = 0.80$; $\Theta^3(1) = 0.80$.

Step 5. Since, each of the above comprehensive consensus level of the decision makers is equal to the considered threshold value $\lambda = 0.8$, so now we can go to the selection step.

Selection of the best decision alternative.
By using Algorithm III:
Step 1. Now, the ideal fuzzy soft set corresponding to the three fuzzy soft sets $(\tilde{f}^1_1, E)$,

Table 2.14: Ideal fuzzy soft set $(\tilde{f}^I, E)$ (Example 2.1)

	e_1	e_2	e_3
x_1	0.8	0.6	0.7
x_2	0.6	0.6	0.7
x_3	0.6	0.5	0.62

$(\tilde{f}_1^2, E)$ and $(\tilde{f}_1^3, E)$ at the final $T = 1$ decision round, has been given Table 2.14.
Step 2. Now, the similarity degree of each the individuals $(\tilde{f}_1^1, E)$, $(\tilde{f}_1^2, E)$ and $(\tilde{f}_1^3, E)$ with the ideal fuzzy soft set is as follows: $\rho_1 = 0.35$; $\rho_2 = 0.31$; $\rho_3 = 0.34$.

Step 3. Three weighted fuzzy soft sets from each of the individuals fuzzy soft sets $(\tilde{f}_1^1, E)$, $(\tilde{f}_1^2, E)$ and $(\tilde{f}_1^3, E)$ (given in Tables 2.11, 2.12 and 2.13) have been given in Tables 2.15, 2.16 and 2.17.

Table 2.15: Weighted FSS $(\tilde{f}_{1\rho}^1, E)$ (Example 2.1)

	e_1	e_2	e_3
x_1	0.8	0.6	0.4
x_2	0.6	0.6	0.7
x_3	0.5	0.4	0.62

Table 2.16: Weighted FSS $(\tilde{f}_{1\rho}^2, E)$ (Example 2.1)

	e_1	e_2	e_3
x_1	0.45	0.4	0.9
x_2	0.6	0.5	0.6
x_3	0.6	0.4	0.2

Table 2.17: Weighted FSS $(\tilde{f}_{1\rho}^3, E)$ (Example 2.1)

	e_1	e_2	e_3
x_1	0.7	0.4	0.6
x_2	0.5	0.4	0.7
x_3	0.6	0.5	0.3

Step 4. Then, by using AND operator, the resultant fuzzy soft set from the three weighted fuzzy soft sets $(\tilde{f}_{1\rho}^1, E)$, $(\tilde{f}_{1\rho}^2, E)$ and $(\tilde{f}_{1\rho}^3, E)$ is given in Table 2.18.

Table 2.18: Resultant fuzzy soft set $(\tilde{f}_1, E)$ (at $T = 1$)

	e_1	e_2	e_3
x_1	0.8	0.6	0.7
x_2	0.6	0.6	0.7
x_3	0.6	0.5	0.62

Step 5. Now, based on the resultant fuzzy soft set $(\tilde{f}_1, E)$, positive ideal and negative ideal solutions are as follows: $x_{I^+} = \{0.18, 0.14, 0.19\}$; $x_{I^-} = \{0.14, 0.12, 0.09\}$.

Step 6, 7. After that, by using Equation 2.6, confidence grade or ranking value of every alternative is as follows: $CG(x_1) = 0.57$; $CG(x_2) = 0.09$; $CG(x_3) = 0.72$.

Step 8. So, we have seen that, by using Algorithm III, best alternative for this group decision-making is, x_2.

By using Algorithm IV:
Step 1. The ideal fuzzy soft set has been given Table 2.14.

Step 2. Now, the similarity degree of each the individuals $(\tilde{f}_1^1, E)$, $(\tilde{f}_1^2, E)$ and $(\tilde{f}_1^3, E)$ with the ideal fuzzy soft set is as follows: $\rho_1 = 0.35$; $\rho_2 = 0.31$; $\rho_3 = 0.34$.

Step 3. Then, the three corresponding weighted fuzzy soft sets $(\tilde{f}_{1\rho}^1, E)$, $(\tilde{f}_{1\rho}^2, E)$ and $(\tilde{f}_{1\rho}^3, E)$ have been given in Tables 2.15, 2.16 and 2.17.

Step 4. Now, the positive ideal and negative ideal solutions for each of the weights fuzzy soft sets are as follows:
$I^+(\tilde{f}_{1\rho}^1) = \{0.28, 0.21, 0.24\}$; $I^+(\tilde{f}_{1\rho}^2) = \{0.19, 0.16, 0.22\}$; $I^+(\tilde{f}_{1\rho}^3) = \{0.24, 0.17, 0.24\}$;
$I^-(\tilde{f}_{1\rho}^1) = \{0.18, 0.14, 0.14\}$; $I^-(\tilde{f}_{1\rho}^2) = \{0.14, 0.12, 0.06\}$; $I^-(\tilde{f}_{1\rho}^3) = \{0.17, 0.14, 0.10\}$.

Step 5, 6. Then, by using Equation 2.8, the confidence grade of the alternatives based on each of the weighted fuzzy soft sets are given below:
$CG^1(x_1) = 0.45$; $CG^2(x_1) = 0.28$; $CG^3(x_1) = 0.29$;
$CG^1(x_2) = 0.36$; $CG^2(x_2) = 0.17$; $CG^3(x_2) = 0.35$;
$CG^1(x_3) = 0.61$; $CG^2(x_3) = 0.77$; $CG^3(x_3) = 0.77$.

Step 7. After that, by using Equation 2.9, the ranking value of every alternative is as follows: $\tilde{R}^{CG}(x_1) = 0.34$; $\tilde{R}^{CG}(x_2) = 0.29$; $\tilde{R}^{CG}(x_3) = 0.72$.

Step 8. Hence, the best alternative is, x_2.
So, we have seen that, by using Algorithm IV, the best decision alternative is x_2.
Hence, overall it can be concluded that, x_2 is the best alternative among three alternatives for this fuzzy soft set based group decision-making.

Example 2.2. Now, we have considered another example. Let, $(\tilde{f}^1, E)$, $(\tilde{f}^2, E)$ and $(\tilde{f}^3, E)$ be three fuzzy soft sets given by three decision makers $D = \{d_1, d_2, d_3\}$ over a universal set $X = \{x_1, x_2, x_3, x_4\}$ with respect to four parameters $E = \{e_1, e_2, e_3, e_4\}$ where, e_1, e_2, e_4 are the positive parameters and e_3 is the negative parameter. Three fuzzy soft sets have been given in 2.19, 2.20 and 2.21. Now, we will select the best decision alternative from four alternatives.
Solution: Algorithm I: Measurement of comprehensive consensus level of the decision

Table 2.19: FSS $(\tilde{f}^1, E)$ (Example 2.2)

	e_1	e_2	e_3	e_4
x_1	0.6	0.7	0.5	0.7
x_2	0.7	0.5	0.8	0.3
x_3	0.2	0.1	0.5	0.5
x_4	0.3	0.3	0.6	0.8

Table 2.20: FSS $(\tilde{f}^2, E)$ (Example 2.2)

	e_1	e_2	e_3	e_4
x_1	0.8	0.6	0.6	0.2
x_2	0.6	0.5	0.8	0.4
x_3	0.3	0.1	0.6	0.6
x_4	0.3	0.4	0.5	0.7

Table 2.21: FSS $(\tilde{f}^3, E)$ (Example 2.2)

	e_1	e_2	e_3	e_4
x_1	0.7	0.6	0.7	0.8
x_2	0.6	0.4	0.7	0.3
x_3	0.2	0.1	0.6	0.5
x_4	0.3	0.5	0.4	0.7

makers for every alternative.
Let, the threshold value for comprehensive consensus level is, $\lambda = 0.70$.
Step 1, 2. Since, e_3 is negative parameter therefore, by using point (i) in Section 2.2, equalized value of all the decision evaluations have been given in Tables 2.22, 2.23, 2.24.

Table 2.22: FSS $(\tilde{f}^1, E)$ after equalization (Example 2.2)

	e_1	e_2	e_3	e_4
x_1	0.6	0.7	0.5	0.3
x_2	0.7	0.5	0.2	0.3
x_3	0.2	0.1	0.5	0.5
x_4	0.3	0.3	0.4	0.8

Table 2.23: FSS $(\tilde{f}^2, E)$ after equalization (Example 2.2)

	e_1	e_2	e_3	e_4
x_1	0.8	0.6	0.4	0.2
x_2	0.6	0.5	0.2	0.4
x_3	0.3	0.1	0.4	0.6
x_4	0.3	0.4	0.5	0.7

Table 2.24: FSS $(\tilde{f}^3, E)$ after equalization (Example 2.2)

	e_1	e_2	e_3	e_4
x_1	0.7	0.6	0.3	0.2
x_2	0.6	0.4	0.3	0.3
x_3	0.2	0.1	0.4	0.5
x_4	0.3	0.5	0.6	0.7

Step 3, 4. Comprehensive consensus level of the decision makers for every alternative are as follows:
$\Theta^1(1) = 0.87$, $\Theta^2(1) = 0.92$, $\Theta^3(1) = 0.93$; $\Theta^4(1) = 0.88$.
$\Theta^1(2) = 0.90$, $\Theta^2(2) = 0.92$, $\Theta^3(2) = 0.92$; $\Theta^4(2) = 0.92$.
$\Theta^1(3) = 0.90$, $\Theta^2(3) = 0.91$, $\Theta^3(3) = 0.94$; $\Theta^4(3) = 0.90$.
Since, the comprehensive consensus of all the decision makers for every alternative is greater than the considered threshold value ($\lambda = 0.7$) so, we will go to the selection step.

Selection of the best alternative.
By using Algorithm III:
The confidence grades or ranking values of the alternatives are, $CG(x_1) = 0.44$; $CG(x_2) = 0.48$; $CG(x_3) = 0.65$; $CG(x_4) = 0.42$.
so, by using Algorithm III, ranking order of the alternatives is, $x_4 > x_1 > x_2 > x_3$.
By using Algorithm IV:
The ranking values of the alternatives are, $\tilde{R}^{CG}(x_1) = 0.42$; $\tilde{R}^{CG}(x_2) = 0.49$; $\tilde{R}^{CG}(x_3) = 0.67$; $\tilde{R}^{CG}(x_4) = 0.44$.

So, by using Algorithm IV, ranking order of the alternatives is, $x_1 > x_4 > x_2 > x_3$.

Hence, we have seen that, through Algorithm III, the best decision alternative is x_4 and through Algorithm IV, the best decision alternative is x_1.

2.4 A case study on sustainable supplier selection in a textile industry

In the recent years, companies have paid attention on sustainable supplier selection in order to maintain environmental and social legislations additionally with its economic development. Therefore, they have incorporated sustainable related parameters for best supplier selection so that, environmental, economic and social aspects can be served simultaneously. Textile and fashion industry is one of the foremost industrial sector which has to adopt with the changing of era. In the global economic development of a country, this industry plays a significant role. But, now a days, people are more interested to use sustainable products to maintain natural resources and to clean our planet. Therefore, textile industries have focused to follow the following agendas to order to maintain the sustainability in producing and selling their products.

- Use of recyclable and biodegradable materials in producing and packing garments instead of unsafe chemical materials and pesticide.
- Use of renewable energy to power machines in manufacturing the products.
- To reduce carbon emission as well as green house gas emission during manufacturing of products.
- To manage the hazardous waste materials.
- To improve overall environmental footprint of textiles.
- To make sure the workers safety and health.
- To make sure good social conditions of the workers.

A supply chain of a textile industry contains basically three parts: supplier, manufacturer and customer as given in Figure 2.3. In this system, supplier selection problem is an emergent issue to enhance the competitive ground of the industry. Accordingly, sustainable supplier selection of a textile industry is an urgent problem in a textile supply chain management system. Now, in this section, we have illustrated a real case study in recognizing best sustainable supplier of a textile industry which is located in Kolkata in the

Figure 2.3: Supply chain of a textile industry

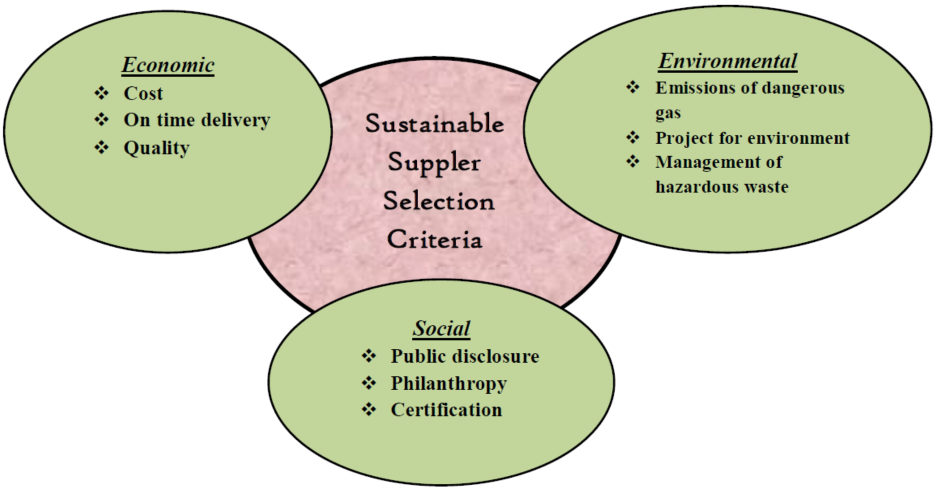

Figure 2.4: Criteria for sustainable supplier selection

state West Bengal in India.

Considered sustainable criteria

Now, we have considered some sustainable criteria to select best sustainable supplier of a textile industry such as, cost, on time delivery, quality, emission of dangerous gas, projects for environment, management of hazardous waste, public disclosure, philanthropy, certification, etc. These criteria can be categorized into three types: economic, environmental and social as given in Figure 2.4. Based on these considered criteria, now we have selected the best sustainable supplier from a set of four suppliers for a textile and fashion industry. A group of three experts have been appointed to conduct this decision-making process. Now assume that, the views of the experts about the suppliers over the considered criteria have been illustrated by fuzzy soft sets. Now, in the following, we have discussed this decision-making problem mathematically.

Mathematical formulation of the case study

Example 2.3. Let, $X = \{S_1, s_2, S_3, S_4\}$ be the set of four suppliers, which has been considered as a universal set and $E = \{cost(e_1),\ On\ time\ delivery(e_2),\ Quality(e_3),\ Emission\ of\ dangerous\ gas(e_4),\ Projects\ for\ environment(e_5),\ Management\ of\ hazardous\ waste(e_6),\ Public\ disclosure(e_7),\ Philanthropy(e_8),\ Certification(e_9)\}$ be a set of nine sustainable supplier selection criteria based on which based supplier will be selected. $D = \{d_1, d_2, d_3\}$ be a set of three experts who have been employed to conduct this decision-making. Now, the assessment of the four suppliers by the three experts have been given in three fuzzy soft sets $(\tilde{f}^1, E)$, $(\tilde{f}^2, E)$, $(\tilde{f}^3, E)$ (Tables 2.25, 2.26 and 2.27). Here, among the nine considered parameters, cost (e_1) and Emission of dangerous gas (e_4) are two negative parameters and rest of the others are positive parameters.

Table 2.25: FSS $(\tilde{f}^1, E)$ provided by d_1 (Example 2.3)

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9
S_1	0.2	0.6	1	0.2	0.6	0.3	0.9	0.1	0.5
S_2	0.4	0.2	0.9	0.4	0.6	0.7	0.1	0.2	0.4
S_3	0.9	0.4	0.5	0.5	0.3	0.2	0.6	0.1	0.4
S_4	0.4	0.7	0.1	0.9	0.4	0.5	0.6	0.2	0.1

Table 2.26: FSS $(\tilde{f}^2, E)$ provided by d_2 (Example 2.3)

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9
S_1	0.3	0.4	0.9	0.1	0.5	0.4	0.8	0.2	0.6
S_2	0.5	0.2	0.8	0.3	0.7	0.8	0.1	0.3	0.4
S_3	0.8	0.5	0.3	0.6	0.3	0.2	0.5	0.1	0.3
S_4	0.3	0.6	0.3	0.9	0.4	0.4	0.7	0.3	0.1

Table 2.27: FSS $(\tilde{f}^3, E)$ provided by d_3 (Example 2.3)

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9
S_1	0.4	0.4	0.8	0.2	0.5	0.3	1	0.2	0.4
S_2	0.5	0.2	0.7	0.2	0.5	0.6	0.2	0.4	0.4
S_3	0.8	0.3	0.4	0.5	0.4	0.3	0.5	0.2	0.3
S_4	0.4	0.6	0.2	0.8	0.4	0.5	0.7	0.1	0.1

Solution: Now, we will recognize the best sustainable supplier by using our proposed fuzzy soft set based group decision-making approach. We have used fuzzy Euclidean distance and fuzzy soft Euclidean distance to determine fuzzy distance and fuzzy soft distance (given in point (ii) and point (iv) in Section 2.2).

Measurement of comprehensive consensus level of a decision maker.
By using Algorithm I:
Let us assume that, the threshold value of the comprehensive consensus level for this decision-making problem is, $\lambda = 0.8$.
Step 1, 2. Since, e_1 and e_4 are the two negative parameters then, to equalize all the evaluations or ratings, we have taken fuzzy complement of the ratings of all the alternatives over the parameters e_1 and e_4. Results have been given in 2.28, 2.29 and 2.30.

Table 2.28: FSS $(\tilde{f}^1, E)$ after equalization (Example 2.3)

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9
S_1	0.8	0.6	1	0.8	0.6	0.3	0.9	0.1	0.5
S_2	0.6	0.2	0.9	0.6	0.6	0.7	0.1	0.2	0.4
S_3	0.1	0.4	0.5	0.5	0.3	0.2	0.6	0.1	0.4
S_4	0.6	0.7	0.1	0.1	0.4	0.5	0.6	0.2	0.1

Table 2.29: FSS $(\tilde{f}^2, E)$ after equalization (Example 2.3)

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9
S_1	0.7	0.4	0.9	0.9	0.5	0.4	0.8	0.2	0.6
S_2	0.5	0.2	0.8	0.7	0.7	0.8	0.1	0.3	0.4
S_3	0.2	0.5	0.3	0.4	0.3	0.2	0.5	0.1	0.3
S_4	0.7	0.6	0.3	0.1	0.4	0.4	0.7	0.3	0.1

Table 2.30: FSS $(\tilde{f}^3, E)$ after equalization (Example 2.3)

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9
S_1	0.6	0.4	0.8	0.8	0.5	0.3	1	0.2	0.4
S_2	0.5	0.2	0.7	0.8	0.5	0.6	0.2	0.4	0.4
S_3	0.2	0.3	0.4	0.5	0.4	0.3	0.5	0.2	0.3
S_4	0.6	0.6	0.2	0.2	0.4	0.5	0.7	0.1	0.1

Step 3,4. The comprehensive consensus level of the experts for the alternatives are as follows:
$\Theta^1(1) = 0.88$; $\Theta^1(2) = 0.88$; $\Theta^1(3) = 0.88$.
$\Theta^2(1) = 0.90$; $\Theta^2(2) = 0.92$; $\Theta^2(3) = 0.90$.
$\Theta^3(1) = 0.91$; $\Theta^3(2) = 0.90$; $\Theta^3(3) = 0.91$.

$\Theta^4(1) = 0.91$; $\Theta^4(2) = 0.91$; $\Theta^4(3) = 0.92$.
Step 5. Since, each $\Theta^l(s) \geq \lambda(= 0.8)$ $(threshold\ value)$; $s = 1, 2, 3, 4, l = 1, 2, 3$ therefore, we will directly go to the Selection step.

Selection process.
By using Algorithm III:
Step 1. The ideal fuzzy soft set $(\tilde{f}^I, E)$ from the three fuzzy soft sets $(\tilde{f}^1, E)$, $(\tilde{f}^2, E)$, $(\tilde{f}^3, E)$ has been provided in Table 2.31.

Table 2.31: Ideal fuzzy soft set $(\tilde{f}^I, E)$ (Example 2.3)

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9
S_1	0.8	0.6	1	0.9	0.6	0.4	1	0.2	0.6
S_2	0.6	0.2	0.9	0.8	0.7	0.8	0.2	0.4	0.4
S_3	0.2	0.5	0.5	0.5	0.4	0.3	0.6	0.2	0.4
S_4	0.7	0.7	0.3	0.2	0.4	0.5	0.7	0.3	0.1

Step 2. The similarity degree of each of the fuzzy soft sets $(\tilde{f}^1, E)$, $(\tilde{f}^2, E)$ and $(\tilde{f}^3, E)$ with the ideal fuzzy soft set are, $\rho_1 = 0.34$; $\rho_2 = 0.33$; $\rho_3 = 0.33$.

Step 3. Now, the weighted fuzzy soft sets from the three corresponding fuzzy soft sets $(\tilde{f}^1, E)$, $(\tilde{f}^2, E)$ and $(\tilde{f}^3, E)$ have been given in Tables 2.32, 2.33 and 2.34.

Table 2.32: Weighted FSS $(\tilde{f}^1_\rho, E)$ (Example 2.3)

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9
S_1	0.272	0.204	0.340	0.272	0.204	0.102	0.306	0.034	0.170
S_2	0.204	0.068	0.306	0.204	0.204	0.238	0.034	0.068	0.136
S_3	0.034	0.136	0.170	0.170	0.102	0.068	0.204	0.034	0.136
S_4	0.204	0.238	0.034	0.034	0.136	0.170	0.204	0.068	0.034

Table 2.33: Weighted FSS $(\tilde{f}^2_\rho, E)$ (Example 2.3)

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9
S_1	0.231	0.132	0.297	0.297	0.165	0.132	0.264	0.066	0.198
S_2	0.165	0.066	0.264	0.231	0.231	0.264	0.033	0.099	0.132
S_3	0.066	0.165	0.099	0.132	0.099	0.066	0.165	0.033	0.099
S_4	0.231	0.198	0.099	0.033	0.132	0.132	0.231	0.099	0.033

Step 4. Resultant fuzzy soft set (using AND operator) has been given in Table 2.35

Table 2.34: Weighted FSS $(\tilde{f}^3_\rho, E)$ (Example 2.3)

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9
S_1	0.198	0.132	0.264	0.264	0.165	0.099	0.330	0.066	0.132
S_2	0.165	0.066	0.231	0.264	0.165	0.198	0.066	0.132	0.132
S_3	0.066	0.099	0.132	0.165	0.132	0.099	0.165	0.066	0.099
S_4	0.198	0.198	0.066	0.066	0.132	0.165	0.231	0.033	0.033

Table 2.35: Resultant fuzzy soft set $(\tilde{f}, E)$ (Example 2.3)

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9
S_1	0.198	0.132	0.264	0.264	0.165	0.099	0.264	0.034	0.132
S_2	0.165	0.066	0.231	0.204	0.165	0.198	0.033	0.068	0.132
S_3	0.034	0.099	0.099	0.132	0.099	0.066	0.165	0.033	0.099
S_4	0.198	0.198	0.034	0.033	0.132	0.132	0.204	0.033	0.033

Step 5. Now, the positive ideal and negative ideal solutions based on Table 2.35 are as follows:
$x_{I^+} = \{0.198, 0.198, 0.264, 0.264, 0.165, 0.198, 0.264, 0.068, 0.132\}$;
$x_{I^-} = \{0.034, 0.066, 0.034, 0.033, 0.099, 0.066, 0.033, 0.033, 0.033\}$.

Step 6, 7. Then, by using Equation 2.6, the confidence grades of the alternatives are as follows: $CG_{S_1} = 0.2142; CG_{S_2} = 0.4461; CG_{S_3} = 0.6386; CG_{S_4} = 0.5584$.

Step 8. So, the final ranking order of the associated alternatives is, $S_1 > S_2 > S_4 > S_3$. Hence, by using Algorithm III, S_1 is the best decision alternative.

By using Algorithm IV:
Step 1, Step 2 and Step 3 are same as Algorithm III. Three weighted fuzzy soft sets have been given in Tables 2.32, 2.33 and 2.34.

Step 4. Now, positive ideal and negative ideal solutions for every weighted fuzzy soft set are as follows:
$I^+(\tilde{f}^1_\rho) = \{0.272, 0.238, 0.340, 0.272, 0.204, 0.238, 0.306, 0.068, 0.170\}$;
$I^+(\tilde{f}^2_\rho) = \{0.034, 0.068, 0.034, 0.034, 0.102, 0.068, 0.034, 0.034, 0.034\}$;
$I^+(\tilde{f}^3_\rho) = \{0.231, 0.198, 0.297, 0.297, 0.231, 0.264, 0.264, 0.099, 0.198\}$;
$I^-(\tilde{f}^1_\rho) = \{0.066, 0.066, 0.099, 0.033, 0.099, 0.066, 0.033, 0.033, 0.033\}$;
$I^-(\tilde{f}^2_\rho) = \{0.198, 0.198, 0.264, 0.264, 0.165, 0.198, 0.330, 0.132, 0.132\}$;
$I^-(\tilde{f}^3_\rho) = \{0.066, 0.066, 0.066, 0.066, 0.132, 0.099, 0.066, 0.033, 0.033\}$.

Step 5, 6, 7. Then, by using Equations 2.8 and 2.9, the ranking value of every associated alternative is as follows:
$\tilde{R}^{CG}(S_1) = 0.2328$; $\tilde{R}^{CG}(S_2) = 0.4698$; $\tilde{R}^{CG}(S_3) = 0.6327$; $\tilde{R}^{CG}(S_4) = 0.5716$.

Step 8. Finally, the final ranking order of these four alternatives is, $S_1 > S_2 > S_4 > S_3$. Hence, by using Algorithm IV, S_1 is the best decision alternative.

Thus, from the above results we have seen that, by using Algorithm III and Algorithm IV, supplier S_1 is best. So, overall it can be concluded that, S_1 is the best sustainable supplier for this textile industry.

2.5 Comparative discussion

Now, in this section, we have studied a comparative discussion to examine the validity and efficiency of our introduced decision-making approach.

Validity of our proposed approach:

In our methodology, before recognizing the best decision alternative, we have measured the consensus of a decision maker with the other associated decision makers for his/her provided description about an alternative and then, some suggestions have been provided to reformulate the description of a decision maker who have less consensus from the considered threshold value about an alternative so that, the consensus of the decision maker can be increased up to the requisite value for the decision-making problem. After that, final decision has been taken. This is the key idea of our proposed approach.

Now, from the existing studies we have seen that, in the references such as, [139] (Roy and Maji's approach), [22] (Basu et al. approach), [4] (Alcantud's approach), etc., some algorithms have founded by which fuzzy soft set based group decision-making problems can be dealt. But, in these existing approaches, researchers did not concerned with the measuring of consensus of a decision maker with other associated decision makers rather they were concerned only with the best final decision selection. Therefore, in our methodology, an additional consensus measuring part has been added. Now, to examine the validity of our proposed approach firstly, we have derived the best decision alternative by our proposed approach without performing the additional consensus measuring and increasing step.

Final ranking values and final ranking order of the associated alternatives based on Examples 2.1, 2.2 and 2.3 (illustrated case study) have been given in Tables 2.36, 2.37 and 2.38 respectively. From these tables we have seen that, in Examples 2.1 and 2.3, best decision alternative is same for each of the methods including our proposed approach (with out measuring consensus). So, from these resulting values it has been educed that, our mentioned decision-making approach is valid.

Table 2.36: Final ranking order of the alternatives based on Example 2.1

Methods	Ranking order of the alternatives	Best decision alternative
Our approach through Algorithm III (except consensus step)	$x_2 > x_3 > x_1$	x_2
Our approach through Algorithm IV (except consensus step)	$x_2 > x_3 > x_1$	x_2
By Roy and Maji's approach [139]	$x_2 > x_3 > x_1$	x_2
By Basu et al. approach [22]	$x_2 > x_3 > x_1$	x_2
By Alcantud's approach [4]	$x_2 > x_3 > x_1$	x_2

Table 2.37: Final ranking order of the alternatives based on Example 2.2

Methods	Ranking order of the alternatives	Best decision alternative
Our approach through Algorithm III (except consensus step)	$x_1 > x_4 > x_2 > x_3$	x_4
Our approach through Algorithm IV (except consensus step)	$x_1 > x_2 > x_4 > x_3$	x_1
By Roy and Maji's approach [139]	$x_1 = x_4 > x_2 = x_3$	x_1, x_4
By Basu et al. approach [22]	$x_1 = x_2 = x_3 = x_4$	x_1, x_2, x_3, x_4
By Alcantud's approach [4]	$x_1 > x_4 > x_2 > x_3$	x_1

Table 2.38: Final ranking order of the alternatives based on Example 2.3

Methods	Ranking order of the suppliers	Best sustainable supplier
Our approach through Algorithm III (except consensus step)	$S_1 > S_2 > S_3 > S_4$	S_1
Our approach through Algorithm IV (except consensus step)	$S_1 > S_2 > S_3 > S_4$	S_1
By Roy and Maji's approach [139]	$S_1 > S_2 > S_4 > S_3$	S_1
By Basu et al. approach [22]	$S_1 > S_2 > S_4 > S_3$	S_1
By Alcantud's approach [4]	$S_1 > S_2 > S_4 > S_3$	S_1

Effectiveness of our proposed approach:

Now, we have illustrated the effectiveness of our proposed group decision-making approach.

- **For Example 2.1.** From Example 2.1 we have seen that, there was a inconsistency in some evaluations of the alternatives and therefore, we have performed consensus level increasing step. After increasing consensus, final results have been given below:
  Through Algorithm III, best alternative is x_2.
  Through Algorithm IV, best alternative is x_2.
  Now, by comparing these results with the Table 2.36, we have observed that, the final

decision alternative is same as x_2 for both the case of 'with consensus' and 'without consensus'.
Though, in this example, for both the cases, results are same, however, for other examples, there may be different results for 'with consensus' and 'with out consensus'.

- **For Example 2.2:** Since, in Example 2.2, comprehensive consensus of every expert for every alternative is greater than the considered threshold value therefore, in this example, we do not need to perform the consensus level increasing step. Again, from Table 2.37, it is observed that, through our proposed Algorithm III, x_4 is the best alternative and through our proposed Algorithm IV, x_1 is the best alternative. Actually, the back ground of these two algorithms is slightly different. In Algorithm III, firstly we have performed an external aggregation and then we have derived the ranking indices of the alternatives on the other hand, in Algorithm IV, firstly we have derived the ranking indices of the alternatives for each of the individual fuzzy soft sets and then we have performed an aggregation on the ranking index values of the alternatives over all the associated fuzzy soft sets. So, when individual decision of a decision maker is more important than the integrated value then, Algorithm IV is more useful than Algorithm III. Because, in that case, by using our proposed Algorithm IV, we can get the best decision alternative based on the opinion of an individual decision maker.

- **For Example 2.3:**
  In this example also, the comprehensive consensus of every expert for every alternative is greater than the considered threshold value therefore, in this example no need to perform the consensus level increasing step.

So, from the above discussions, the main advantages of our proposed decision-making approach can be summarized as follows:
(i) In our proposed algorithm, there is a lot of emphasis on measuring consensus of the decision makers for every alternative which is not founded in the existing approaches. Therefore, by our proposed approach we can get more accurate result than the existing approaches ([4, 22, 139]).
(ii) Moreover, in the best alternative selection part of our proposed algorithm, we have provided two different algorithms, Algorithm III and Algorithm IV where, Algorithm III is suitable for the problems where, integrated opinion of all the decision makers is required on the other hand, Algorithm IV is more better for the problems where, individual opinion of a decision maker is required. So, by our proposed approach, we can select the algorithm for selecting the best decision alternative according to the needs of a problem.

Thus from the over all illustration it is concluded that, the proposed decision-making approach is more useful and effective than the other existing approaches.

2.6 Conclusion

In this chapter, we have used fuzzy soft set theory to handle group decision-making problems. In this regard, a stepwise algorithmic approach has been proposed to determine the best alternative from some alternatives based on a group of decision makers opinions. Our main contributions in this chapter are as follows:

- Our proposed methodology contains mainly three main parts. In first part, we have derived the consensus of a decision maker with the other decision makers for his/her provided opinion about every alternative and further, in second part, we have increased the consensus of a decision maker, who have less consensus at the first part. Finally, at third part, we have selected the best decision alternative. So, if in a real-life related problem, one decision maker may provide an incorrect opinion about an alternative then by our proposed approach, he/she will get a chance to reformulate his/her opinion before selecting the best alternative. This is the main advantage of our proposed approach.

- Further, in the selection step of our proposed approach i.e., in our third part, we have provided two different algorithms for selecting the best alternative where, one is appropriate, when over all opinion of all the decision makers is necessary and another one is appropriate, when individual opinion of a decision maker is necessary. So, by selecting perfect algorithm according to the requirement of a problem, we will get more error less result. This is an another advantage of our proposed approach.

- Moreover, in this chapter we have solved a problem regarding the sustainable supplier selection of textile industry by using our proposed approach which shows a real-life applicability of our proposed decision-making approach.

For further research, one can extend our proposed approach to solve other type of soft sets like, intuitionistic fuzzy soft sets, linguistic valued soft sets, neutrosophic soft sets, etc. Moreover, our proposed approach can applied to other real-life related problems for instance, in disease diagnosis, in companies manager selection, in game theory, etc.

Furthermore, algebraic structures on fuzzy soft set theory have gained a great attention to the researchers. The most significant algebraic structures such as, group, normal subgroup, mapping, etc. have been developed on fuzzy soft sets. However, some algebraic structures including, order of a group, cyclic group, etc. have not yet been introduced. So, for further research on fuzzy soft sets, one can construct these algebraic structures on fuzzy soft sets.