M.Sc. 2nd Semester Examination, 2010

PHYSICS

Full Marks : 40

Time : 2 hours

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

PAPER—PH-1201-A

(*Quantum Mechanics*-II)

[*Marks* : 20]

Answer **Q. No. 1, 2** and any *one* from the rest

1. Answer any *two* bits : 2×2

(*a*) Find the matrix elements for transformation matrix which transforms from one orthogonal basis set $\{u_i\}$ to another orthogonal basis set $\{v_\mu\}$.

(*Turn Over*)

(*b*) For a spin $\frac{1}{2}$ particle, find the eigenvalues and normalized eigenvectors of S_x and S_z.

(*c*) Show that the matrix element $\langle n'l'm'|\vec{P}|nlm\rangle$ can be reduced to a term $\langle n'l'm'|\vec{r}|nlm\rangle$. Mention the conditions for the existence of the matrix element $\langle n'l'm'|\vec{r}|nlm\rangle$.

(*d*) If $|\lambda, m\rangle$ is a normalized state such that $J^2|\lambda, m\rangle = \lambda|\lambda, m\rangle$ and $J_z|\lambda, m\rangle = m|\lambda, m\rangle$, then show that $\lambda \geq m^2$. Here J is the angular momentum operator and J_z corresponds to its z-component.

2. Answer any *two* bits : 3×2

(*a*) An electron in an atom is in p-state. Find the possible values of total angular momentum which is the sum of the orbital angular momentum and spin momentum. Write the wave functions of the possible states of the total angular momentum from the component wave functions $|l, m_l\rangle$ and $|s, m_s\rangle$ where $l = 1$ and $s = 1/2$.

(*b*) A charged harmonic oscillator is placed in an uniform electric field $\vec{E}$. The perturbing potential is qEx. Find the correction in the ground state energy level up to second order in E.

$$\text{given } \langle n|x|m\rangle = \begin{cases} \frac{1}{\alpha}\sqrt{\frac{n+1}{2}} & \text{if } m=n+1 \\ \frac{1}{\alpha}\sqrt{\frac{n}{2}} & \text{if } m=n-1 \\ 0 & \text{otherwise} \end{cases}$$

(*c*) Obtain the time evolution of the state vectors and the operators in the interaction picture.

(*d*) Applying the variation function $\phi = Ax(b^2 - x^2)$ to a particle in an infinite one dimensional box of length b, calculate it's energy.

3. (*a*) Establish the stationary perturbation theory for a doubly degenerate state.

(*b*) Write the characteristic equation (no derivation) in the matrix from for the degenerate perturbation theory up to first order for a state which is triply degenerate $H_0|n_1\rangle = \varepsilon_n|n_1\rangle$, $H_0|n_2\rangle = \varepsilon_n|n_2\rangle$, $H_0|n_3\rangle = \varepsilon_n|n_3\rangle$.

(*c*) Write the energy eigenvalues, degree of degeneracy and the corresponding wave functions for a two-dimensional harmonic oscillator for the three lowest states. The wave functions may be given in terms of the wave functions of the one -dimensional harmonic oscillator $u_n(x)$ and $v_m(y)$.

(*d*) Find the first order energy shift for the second excited state of the two-dimensional harmonic oscillator under the perturbing potential $H' = cxy$.

4+1+2+3

4. (*a*) Using the quantum condition find the energy eigenvalues for a harmonic oscillator (in matrix method).

(*b*) Find the form of raising operator $a^\dagger$, lowering operator a and the number operator in terms of $a^\dagger$ and a.

(*c*) Find the matrices for the position and momentum operator. 6+2+2

PAPER—PH-1201-B

(*Mathematical Methods in Physics*)

[*Marks* : 20]

Answer *all* questions

1. Answer any *five* bits : 2×5

(*a*) Find Fourier transform of the function

$$f(x) = 1 \text{ for } |x| < a$$
$$= 0 \text{ for } |x| > a$$

(*b*) Find Laplace transform of

$$t^4 e^{-at}$$

(*c*) Write down Parseval's relations.

(*d*) Prove that

$$\delta(x)\,\delta(y) = \frac{\delta(r)}{2\pi r}$$

in polar co-ordinates.

(*e*) Define Green's function.

(*f*) Write down Dirichlet and Neumann boundary conditions.

(*g*) Write down the generators of SU(2) group.

(*h*) Prove that the transformations $x' = ax + b$, $a \neq 0$ form a Lie group.

2. Answer any *two* bits : 5×2

(*a*) Find Fourier cosine transform of the function

$$f(x) = e^{-x^2}.$$ 5

(*b*) Find the particular integral of

$$u_{xx} - u_{xy} - 6u_{yy} = e^{2x+y} + \sin(2x - 3y).$$ 5

(*c*) $\frac{d^2y}{dx^2} + w^2 y = f(x)$,

and $y(0) = 0$; $y(2) = 0$.

Construct one-dimensional Green's function of the above boundary value problem. 5

(*d*) Consider the symmetry group D_3 of an equilateral triangle : 3 + 2

(*i*) Write down the symmetry operations and work out the group multiplication table.

(*ii*) Identify the classes.