M.Sc. 2nd Semester Examination 2012

PHYSICS

PAPER – PHS- 201

Full Marks : 40

Time : 2 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

Use saparate Scripts for Gr. A & Gr.B

GROUP – A

[*Marks* : 20]

Answer **Q.No.1 & 2** and any **one** from the rest

1. Answer any *two* bits : 2 × 2

(*a*) For Pauli Spin matrices, prove that

$$\sigma_+ \sigma_- = 2(1 + \sigma_z).$$

(*b*) Show that the time-reversal operation is represented by an antiunitary operator.

(*c*) If E_0 is the ground state energy of a system described by the Hamiltonian H, then show that

$$E_0 \leq <\psi | H | \psi>$$

where ψ is any normalized wavefunction of the system.

2. Answer any *two* bits : 3 ×

(*a*) Prove that

$$\frac{1}{2}(L_- L_+ + L_+ L_-) = L^2 - L_z^2.$$

(*b*) The Hamiltonian H and the wave function for 1-D simple harmonic oscillator is given by

$$H = \hbar w \left(a^+ a + \frac{1}{2}\right)$$

$$\psi(0) = \frac{1}{\sqrt{5}} |1\rangle + \frac{2}{\sqrt{5}} |2\rangle$$

at $t = 0$.

Prove that $\langle x(t) \rangle$ oscillates sinusoidally with time.

(*c*) In the interaction picture, obtain the time evolution of the operators and the state vectors.

(*d*) Obtain the matrix of Clebsch-Gordan coefficients for $j_1 = 1$ and $j_2 = \frac{1}{2}$

3. (*a*) Apply the time dependent perturbation theory to obtain Fermi golden rule.

(*b*) If

$$V(x) = \gamma |x|^a$$

Find the energy eigenvalue by WKB method. If $a = 2$ and $a = \infty$; what are their values ? 5 + 5

4. For a charged particle in an electromagnetic field,

(*i*) Write the Hamiltonian

(*ii*) Simplify the Hamiltonian retaining only terms upto first order in vector potential.

(*iii*) Applying time dependent perturbation theory, obtain an expression for transition probability per unit time for absorption.

(*iv*) Explain the selection rules for electrical dipole transition mentioning the approximations involved in it. 1 + 1 + 6 + 2

GROUP – B

[*Marks* : 20]

Answer **Q.No.1** and any **one** from the rest

1. Answer any *five* from the following :

(*a*) If

$$f(x) = e^{-x^2/2}$$

Find the Fourier cosine transform of it.

(*b*) Prove that

$$\hat{L}\left\{\int_0^t f(u)dx\right\} = \frac{F(s)}{S}$$

where $\hat{L}$ represents Laplace transform.

(*c*) If $\hat{L}\, f(t) = F(s)$ then prove that

$$\hat{L}\{f(t-a).\theta(t-a)\} = e^{-as}\, F(s).$$

(*d*) Solve

$\dfrac{\partial^2 z}{\partial y^2} = z$, given that when $y = 0, z = e^x$ and $\dfrac{\partial z}{\partial y} = e^{-x}$.

(*e*) Prove that the order of all elements of a finite group are the divisors of the order of group.

(*f*) If G is a group such that $(AB)^m = A^m B^m$ for three consecutive integers m for all $A, B \in G$, show that G is Abelian group.

(*g*) Define Lie group and Lie algebra.

(*h*) Define continuous groups and their generators.

2. (*a*) Solve by the method of Laplace transform :

$$\frac{d^2 y}{dt^2} + 2\frac{dy}{dt} + 5y = e^t \sin t.$$

with $y(0) = 0 \; ; \; y'(0) = 1.$

(*b*) Show that

$$L^{-1}\left\{\frac{1}{(s^2 + a^2)^2}\right\}$$

$$= \frac{1}{2a^3}(\sin at - at \cos at).$$ 5 + 5

3. (*a*) Construct a Green's function for

$$\frac{d^2 y}{dx^2} - k^2 y = f(x), \quad y(\pm\infty) = 0.$$

(*b*) Develop the irreducible 2×2 matrix representation of the group of operations (rotations and reflections) that transform a square into itself. Give the group multiplication table. 5 + 5