M.Sc. 3rd Semester Examination, 2012

PHYSICS

PAPER—PHS-301

Full Marks : 40

Time : 2 hours

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

GROUP – A

[*Marks* : 20]

(*Relativistic Quantum Mechanics*)

Answer *all* questions

1. Answer any *three* bits : 2 × 3

(*a*) Show that the Dirac Matrices anti-commute in pairs and that their trace is equal to zero.

(*Turn Over*)

(*b*) Find the velocity operator for a free particle described through the Dirac Hamiltonian.

(*c*) If P_r be the radial momentum and α_r be the radial velocity for an electron in a central potential and defined as

$$P_r = \frac{\vec{r} \cdot \vec{p} - i\hbar}{r} \quad \text{and} \quad \alpha_r = \frac{\vec{\alpha} \cdot \vec{r}}{r}.$$

Show that

$$\vec{\alpha} \cdot \vec{p} = \alpha_r p_r + \frac{i\hbar\alpha_r \beta k}{r},$$

where,

$$k = \frac{\beta\left(\overrightarrow{\sigma^d} \cdot \vec{L} + \hbar\right)}{\hbar}$$

(*d*) Obtain the eigenvalues of the operator

$$k = \frac{\beta\left(\overrightarrow{\sigma^d} \cdot \vec{L} + \hbar\right)}{\hbar}$$

(*e*) Simplify $\left(\vec{\alpha} \cdot \left(\vec{P} - q\vec{A}\right)\right)^2$.

. Answer any *one* bit : 4

(*a*) Write Dirac Hamiltonian for an electron in a central potential $V(r)$ and show that the spin orbit interaction comes automatically in the Dirac equation.

(*b*) Show that for a Dirac particle, Orbital angular momentum $(\vec{L})$ is not a constant of motion rather the total angular momentum $\vec{J} = \vec{L} + \vec{S}$ is a constant of motion.

. Answer any *one* bit : 10

(*a*) Write the radial equation for a spin zero charged particle in a coulomb field and obtain expression for the energy eigenvalue. Clearly show the additional term occurring in the energy spectrum due to the relativistic treatment.

(*b*) Obtain the plane-wave solution for the spin half particle in the relativistic formalism. Write the four wave functions corresponding to the ± energy and the two spin states in matrix form.

GROUP – B

[*Marks* : 20]

(*Statistical Mechanics*)

Answer Q. No. 1 and any *one* from the rest

1. Answer any *five* bits : 2 × 5

(*a*) Distinguish between ensemble average and quantum mechanical average.

(*b*) If the canonical partition function

$$\phi_N(V,T) = \frac{V^N}{N!}\left(\frac{2\pi m k_B T}{h^2}\right)^{\frac{3N}{2}}$$

Find the equation of state.

(*c*) If the density of states between ν and $\nu + d\nu$ is $g(\nu)d\nu = A\nu^2\, d\nu$. Find the zero point energy of a ϕ_u. Harmonic oscillator.

(*d*) If the grand portion function is given by $\ln \{ = F - A\,H^2 V$. Where F and A are constants, independent of H. Find the magnetization of the system.

(*e*) Prove that for pure state $\hat{\rho}$ (density matrix) is a projection operator.

(*f*) For non-interacting photons radiation pressure is $\frac{1}{3}u$; where u is the energy density, why ?

(*g*) If $E = \pm\frac{1}{2}\mu_B H$ for a spin $\frac{1}{2}$ particle, show that entropy gives rise to concept of negative temperature.

2. (*a*) Deduce an expression of *F-D* distribution function from grand partition function. 6

(*b*) Prove that

$$\rho_{rr'} = \frac{1}{V} e^{\frac{-mK_B T}{2\hbar^2}(\bar{r} - \bar{r}')^2}$$

for a particle in a box. 4

3. (*a*) Prove that particle fluctuation in grand canonical distribution is proportional to $\frac{1}{\sqrt{N}}$. 5

(*b*) A spin $\frac{1}{2}$ particle with 50% in the state $|z_+\rangle$ and 50% in the state $|x_+\rangle$. Find the density matrix and degree of polarization. 2 + 3